TEACHING GUIDE

Module 1: Quadratic Equations and Inequalities

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of quadratic equations, quadratic inequalities, and rational algebraic equations.

Performance Standard:

The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, quadratic inequalities, and rational algebraic equations and solve them using a variety of strategies.

UNPACKING THE STANDARDS FOR UNDERSTANDING

SUBJECT: Mathematics 9	**LEARNING COMPETENCIES**
QUARTER: First Quarter **TOPIC: Quadratic Equations, Quadratic Inequalities, and Rational Algebraic Equations** **LESSONS:** **1. Illustrations of Quadratic Equations** **2. Solving Quadratic Equations** • **Extracting Square Roots** • **Factoring** • **Completing the Square** • **Using the Quadratic Formula** **3. Nature of Roots of Quadratic Equations** **4. Sum and Product of Roots of Quadratic Equations** **5. Equations Transformable to Quadratic Equations (Including Rational**	1. Illustrate quadratic equations. 2. Solve quadratic equations by: (a) extracting square roots; (b) factoring; (c) completing the square; and (d) using the quadratic formula. 3. Characterize the roots of a quadratic equation using the discriminant. 4. Describe the relationship between the coefficients and the roots of a quadratic equation. 5. Solve equations transformable to quadratic equations (including rational algebraic equations). 6. Solve problems involving quadratic equations and rational algebraic equations. 7. Illustrate quadratic inequalities. 8. Solve quadratic inequalities. 9. Solve problems involving quadratic inequalities.

Algebraic Equations) **6. Applications of Quadratic Equations and Rational Algebraic Equations** **7. Quadratic Inequalities**		
WRITERS: **MELVIN M. CALLANTA** **RICHARD F. DE VERA**	**ESSENTIAL UNDERSTANDING:** Students will understand that quadratic equations, quadratic inequalities, and rational algebraic equations are useful tools in solving real-life problems and in making decisions given certain constraints.	**ESSENTIAL QUESTION:** How do quadratic equations, quadratic inequalities, and rational algebraic equations facilitate finding solutions to real-life problems and making decisions?
	TRANSFER GOAL: Students will be able to apply the key concepts of quadratic equations, quadratic inequalities, and rational algebraic equations in formulating and solving real-life problems and in making decisions.	

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to come up with in this module.

a. Quadratic equations written in standard form
b. Objects or situations in real life where quadratic equations, quadratic inequalities, and rational algebraic equations are illustrated
c. Quadratic equations, quadratic inequalities, and rational algebraic equations that represent real life situations or objects
d. Quadratic equations with 2 solutions, 1 solution, and no solution
e. Solutions of quadratic equations which can be solved by extracting square roots, factoring, completing the square, and by using the quadratic formula
f. A journal on how to determine quadratic equation given the roots, and the sum and product of roots
g. Finding the quadratic equation given the sum and product of its roots
h. Sketch plans or designs of objects that illustrate quadratic equations, quadratic inequalities, and rational algebraic equations
i. Role playing a situation in real life where the concept of quadratic equation is applied

j. Formulating and solving real-life problems involving quadratic equations, quadratic inequalities, and rational algebraic equations
k. Conducting a mathematical investigation on quadratic inequalities
l. Graphing the solution set of quadratic inequalities formulated

Assessment Map

TYPE	KNOWLEDGE	PROCESS/ SKILLS	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	**Pre-Test: Part I** Identifying quadratic equations, quadratic inequalities, and rational algebraic equations	**Pre-Test: Part I** Solving quadratic equations, quadratic inequalities, and rational algebraic equations Describing the roots of quadratic equations Writing the quadratic equations given the roots Solving equations transformable to quadratic equations Graphing the solution sets of quadratic inequalities	**Pre-Test: Part I** Solving problems involving quadratic equations, quadratic inequalities, and rational algebraic equations	**Pre-Test: Part I** Products and performances related to or involving quadratic equations, quadratic inequalities, rational algebraic equations, and other mathematics concepts
	Pre-Test: Part II Situational Analysis Identifying the fixtures or furniture to be designed Determining the mathematics concepts or principles involved in the designs of the fixtures	**Pre-Test: Part II Situational Analysis** Illustrating every part or portion of the fixture including their measures Writing the expressions, equations, or inequalities that describe the situations or problems Solving	**Pre-Test: Part II Situational Analysis** Explaining how to prepare the designs of the fixtures Solving real-life problems	**Pre-Test: Part II Situational Analysis** Making designs of fixtures Formulating equations, inequalities, and problems

		equations and inequalities		
Formative	**Quiz: Lesson 1** Identifying quadratic equations Identifying situations that illustrate quadratic equations	**Quiz: Lesson 1** Representing situations by mathematical sentences Writing quadratic equations in standard form $ax^2 + bx + c = 0$ and identifying the values of a, b, and c	**Quiz: Lesson 1** Differentiating quadratic equations from linear equations Explaining how to write quadratic equations in standard form Justifying why quadratic equations can be written in standard form in two ways Formulating and describing a quadratic equation that represents a given situation	
	Quiz: Lesson 2A Identifying quadratic equations that can be solved by extracting square roots	**Quiz: Lesson 2A** Solving quadratic equations by extracting square roots Writing a quadratic equation that represents the area of the shaded region of a square. Finding the length of a side of a square using the quadratic equation formulated.	**Quiz: Lesson 2A** Explaining how to solve quadratic equations by extracting square roots Justifying why a quadratic equation has at most two roots Explaining why some quadratic equations can be solved easily by extracting square roots Solving real-life problems involving quadratic equations	
	Quiz: Lesson 2B Identifying quadratic equations that	**Quiz: Lesson 2B** Solving quadratic equations by	**Quiz: Lesson 2B** Explaining how to solve quadratic equations by	

	can be solved by factoring	factoring Writing a quadratic equation that represents the area of the shaded region of a rectangular figure. Finding the length and the width of a figure using the quadratic equation formulated.	factoring Explaining why some quadratic equations may be solved more appropriately by factoring Solving real-life problems involving quadratic equations	
	Quiz: Lesson 2C Identifying quadratic equations that can be solved by completing the square	**Quiz: Lesson 2C** Solving quadratic equations by completing the square Writing and solving a quadratic equation that represents the area of the shaded region of a rectangular figure. Finding the particular measure of a figure using the quadratic equation formulated.	**Quiz: Lesson 2C** Explaining how to solve quadratic equations by completing the square Explaining why some quadratic equations may be solved appropriately by completing the square Solving real-life problems involving quadratic equations	
	Quiz: Lesson 2D Determining the values of a, b, and c in a quadratic equation Identifying quadratic	**Quiz: Lesson 2D** Writing quadratic equations in the form $ax^2 + bx + c = 0$ Solving quadratic	**Quiz: Lesson 2D** Explaining how to solve quadratic equations by using the quadratic formula Explaining why all quadratic equations	

	equations that can be solved by using the quadratic formula	equations by using the quadratic formula Writing and solving quadratic equations that represent some given situations	can be solved by using the quadratic formula Solving real-life problems involving quadratic equations	
	Quiz: Lesson 3 Determining the values of a, b, and c in a quadratic equation	**Quiz: Lesson 3** Finding the value of the discriminant of a quadratic equation Describing the roots of a quadratic equation Writing a quadratic equation that represents a given situation	**Quiz: Lesson 3** Explaining how to determine the nature of the roots of quadratic equations Applying the concept of discriminant of quadratic equations in solving real-life problems	
	Quiz: Lesson 4 Determining the values of a, b, and c in a quadratic equation	**Quiz: Lesson 4** Finding the sum and the product of roots of quadratic equations Finding the roots of quadratic equation $ax^2 + bx + c = 0$ using the values of a, b, and c Writing the quadratic equation given the roots Writing a quadratic equation that represents a given situation	**Quiz: Lesson 4** Explaining how to determine the sum and the product of the roots of quadratic equations Explaining how to find the roots of quadratic equation $ax^2 + bx + c = 0$ using the values of a, b, and c Explaining how to determine the quadratic equation given the roots Using the sum and the product of roots of quadratic equations in solving real-life problems	

	Quiz: Lesson 5 Identifying quadratic equations that can be written in the form $ax^2 + bx + c = 0$ Identifying rational algebraic equations that are transformable to quadratic equations	**Quiz: Lesson 5** Transforming equations to quadratic equations in the form $ax^2 + bx + c = 0$ Finding the solutions of equations transformable to quadratic equations in the form $ax^2 + bx + c = 0$ including rational algebraic equations	**Quiz: Lesson 5** Explaining how to transform equations to quadratic equations in the form $ax^2 + bx + c = 0$ Explaining how to solve equations transformable to quadratic equations in the form $ax^2 + bx + c = 0$ Solving equations with extraneous solutions or roots Solving problems involving equations transformable to quadratic equations in the form $ax^2 + bx + c = 0$ including rational algebraic equations	
	Quiz: Lesson 6 Identifying the information given in real-life problems involving quadratic equations	**Quiz: Lesson 6** Solving quadratic equations and rational algebraic equations as illustrated in some real-life problems	**Quiz: Lesson 6** Solving real-life problems involving quadratic equations and rational algebraic equations	
	Quiz: Lesson 7 Identifying quadratic inequalities	**Quiz: Lesson 7** Finding the solution set of quadratic inequalities Graphing the solution set of quadratic inequalities Determining whether a point is a solution to a given inequality	**Quiz: Lesson 7** Explaining how to find the solution set of quadratic inequalities Explaining how to graph the solution set of quadratic inequalities Describing the solution set of quadratic inequalities and	

		Determining the quadratic inequality that is described by a graph	their graphs Explaining how to determine the quadratic inequality that is described by a graph Solving real-life problems involving quadratic inequalities	
Summative	**Post-Test: Part I** Identifying quadratic equations, quadratic inequalities, and rational algebraic equations	**Post-Test: Part I** Solving quadratic equations, quadratic inequalities, and rational algebraic equations Describing the roots of quadratic equations Writing the quadratic equations given the roots Solving equations transformable to quadratic equations Graphing the solution sets of quadratic inequalities	**Post-Test: Part I** Solving problems involving quadratic equations, quadratic inequalities, and rational algebraic equations	**Post-Test: Part I** Products and performances related to or involving quadratic equations, quadratic inequalities, rational algebraic equations, and other mathematics concepts
	Post-Test: Part II Situational Analysis Identifying the locations of establishments, roads, and pathways to be included in the ground plan of the proposed shopping	**Post-Test: Part II Situational Analysis** Writing the expressions, equations, or inequalities that describe the situations or problems	**Post-Test: Part II Situational Analysis** Explaining how to prepare the ground plan of the proposed shopping complex Solving real-life problems	**Post-Test: Part II Situational Analysis** Making a ground plan of the proposed shopping complex Formulating equations, inequalities, and problems

	complex Determining the mathematics concepts or principles involved in the ground plan	Solving equations and inequalities		
Self-Assessment	**Journal Writing:** Expressing understanding of quadratic equations, quadratic inequalities, and rational algebraic equations and their solutions or roots Expressing understanding on finding solutions of quadratic equations, quadratic inequalities, and rational algebraic equations			

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	The learner demonstrates understanding of key concepts of quadratic equations, quadratic inequalities, and rational algebraic equations. Illustrate quadratic equations. Solve quadratic equations by: (a) extracting square roots; (b) factoring; (c) completing the square; (d) using the quadratic formula. Characterize the roots of a quadratic equation using the discriminant. Describe the relationship between the coefficients and the roots of a quadratic equation.	**Paper and Pencil Test** **Part I** items 1, 2, 3, 8, 12, and 15 **Part II** item 3	1 point for every correct response
Process/Skills 25%		**Part I** items 4, 5, 6, 7, 9, 10, 11, 13, 14, and 16 **Part II** items 5 and 6	1 point for every correct response
Understanding 30%		**Part I** items 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, and 28 **Part II** items 1 and 6	1 point for every correct response Rubric for explanation Criteria: Clear Coherent Justified Rubric on Problem Solving

	Solve equations transformable to quadratic equations (including rational algebraic equations). Solve problems involving quadratic equations and rational algebraic equations. Illustrates quadratic inequalities. Solve quadratic inequalities. Solve problems involving quadratic inequalities.		
Product/ Performance 30%	The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, quadratic inequalities, and rational algebraic equations and solve them using a variety of strategies.	**Part II** items 2 and 4	Rubric on Design (Ground Plan) Criteria: 1. Content 2. Clarity of Presentation 3. Accuracy of Measurements Rubric for Equations Formulated and Solved Rubric on Problem Posing/Formulation Criteria: Relevant Authentic Creative Clear Insightful

C. Planning for Teaching-Learning

Introduction:

This module covers key concepts of quadratic equations, quadratic inequalities, and rational algebraic expressions. It is divided into seven lessons namely: Illustrations of Quadratic Equations, Solving Quadratic Equations, Nature of Roots of Quadratic Equations, Sum and Product of Roots of Quadratic Equations, Transforming Equations to Quadratic Equations (including Rational Algebraic Equations), Applications of Quadratic Equations, and Rational Algebraic Equations, and Quadratic Inequalities.

In Lesson 1 of this module, the students will identify and describe quadratic equations and illustrate these using appropriate representations. They will also formulate quadratic equations as illustrated in some real-life situations.

Lesson 2 is divided into four sub-lessons. In this lesson, the students will be given the opportunity to learn the different methods of solving quadratic equations namely: extracting square roots, factoring, completing the square, and using the quadratic formula. They will also determine the method that is more appropriate to use in solving quadratic equations.

After the students have learned to solve quadratic equations, the next thing that they will do is to determine the nature of the roots of these equations using the value of the discriminant. This topic will be covered in Lesson 3.

In Lesson 4, the students will learn about the relationships among the values of a, b, and c in a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, and its roots. In this lesson, the students should be able to come up with the quadratic equation given the roots or vice-versa.

One of the important lessons that students need to learn is that some equations can be transformed to quadratic equations in the form $ax^2 + bx + c = 0$, $a \neq 0$. Some examples of these kinds of equations are rational algebraic equations. The students should be able to identify and solve these equations in Lesson 5.

In Lesson 6, the students will find out the vast applications of quadratic equations as they solve real-life problems involving these. Moreover, they will be given the chance to formulate real-life problems involving quadratic equations and solve these using the appropriate methods.

Another interesting mathematics concept that the students will learn in this module is quadratic inequality. This is the content of Lesson 7. In this lesson, the students will determine the solution set of quadratic inequalities algebraically and graphically. The students will also be given the opportunity to use graphing materials, tools, or computer software like GeoGebra in finding the solution set of quadratic inequalities.

In all the lessons, the students are given the opportunity to use their prior knowledge and skills in learning quadratic equations, quadratic inequalities, and rational algebraic equations. They are also given varied activities to process the knowledge and skills learned and further deepen and transfer their understanding of the different lessons.

As an introduction to the main lesson, show to the students the pictures below then ask them the questions that follow:

Was there any point in your life when you asked yourself about the different real-life quantities such as costs of goods or services, incomes, profits, yields and losses, amount of particular things, speed, area, and many others? Have you ever realized that these quantities can be mathematically represented to come up with practical decisions?

Entice the students to find out the answers to these questions and to determine the vast applications of quadratic equations, quadratic inequalities, and rational algebraic equations through this module.

Objectives:

After the learners have gone through the lessons contained in this module, they are expected to:

a. identify and describe quadratic equations using practical situations and mathematical expressions;

b. use the different methods of finding the solutions of quadratic equations;

c. describe the roots of a quadratic equation using the discriminant;

d. determine the quadratic equation given the sum and the product of its roots and vice-versa;

e. find the solutions of equations transformable to quadratic equations (including rational algebraic equations);

f. formulate and solve problems involving quadratic equations and rational algebraic equations;

g. identify and describe quadratic inequalities using practical situations and mathematical expressions;

h. find the solution set of quadratic inequalities algebraically and graphically; and

i. formulate and solve problems involving quadratic inequalities.

Pre-Assessment:

Check students' prior knowledge, skills, and understanding of mathematics concepts related to Quadratic Equations, Quadratic Inequalities, and Rational Algebraic Equations. Assessing these will facilitate teaching and students' understanding of the lessons in this module.

Answer Key

Part I			**Part II** (Use the rubric to rate students' works/outputs)
1. C	11. D	21. D	
2. C	12. D	22. D	
3. C	13. C	23. B	
4. B	14. B	24. C	
5. A	15. C	25. C	
6. D	16. D	26. A	
7. C	17. C	27. C	
8. B	18. C	28. D	
9. A	19. B		
10. C	20. C		

LEARNING GOALS AND TARGETS:

Students are expected to demonstrate understanding of key concepts of quadratic equations, quadratic inequalities, and rational algebraic equations, formulate real-life problems involving these concepts, and solve these using a variety of strategies. They are also expected to investigate mathematical relationships in various situations involving quadratic equations, quadratic inequalities, and rational algebraic equations.

Lesson 1: ILLUSTRATIONS OF QUADRATIC EQUATIONS

What to KNOW:

Assess students' knowledge of the different mathematics concepts previously studied and their skills in performing mathematical operations. Assessing these will facilitate teaching and students' understanding of quadratic equations. Tell them that as they go through this lesson, they have to think of this important question: *"How are quadratic equations used in solving real-life problems and in making decisions?"*

Ask the students to find the products of polynomials by doing Activity 1. Let them explain how they arrived at each product. Give focus on the mathematics concepts or principles applied to arrive at each product particularly the distributive property of real numbers. Also, ask them to describe and determine the common characteristics of each product, that is, the products are all polynomials of degree 2.

Activity 1: Do You Remember These Products?

Answer Key

1. $3x^2 + 21$
2. $2s^2 - 8s$
3. $w^2 + 10w + 21$
4. $x^2 + 7x - 18$
5. $2t^2 + 9t - 5$
6. $x^2 + 8x + 16$
7. $4r^2 - 20r + 25$
8. $9 - 24m + 16m^2$
9. $4h^2 - 49$
10. $64 - 9x^2$

Show to the students different equations and let them identify which are linear and which are not. Ask them to describe those which are linear equations and differentiate these from those which are not. Let the students describe those equations which are not linear and identify their common characteristics. They should be able to tell that each of those equations which are not linear contains polynomial of degree 2.

Activity 2: Another Kind of Equation!

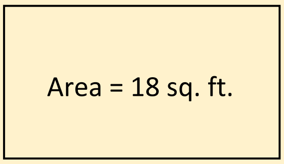
Answer Key

1. Equations which are linear: $2s + 3t = -7$, $8k - 3 = 12$, $c = 12n - 5$, $6p - q = 10$, $9 - 4x = 15$, and $\frac{3}{4}h + 6 = 0$.
2. Linear equations are mathematical sentences with 1 as the highest exponent of the variable.
3. Equations which are not linear: $x^2 - 5x + 3 = 0$, $9r^2 - 25 = 0$, $\frac{1}{2}x^2 + 3x = 8$, $4m^2 + 4m + 1 = 0$, $t^2 - 7t + 6 = 0$, and $r^2 = 144$.
   The highest exponent of the variable/s is 2. The degree of each mathematical sentence is 2.

Provide the students with an opportunity to develop their understanding of quadratic equations. Ask them to perform Activity 3. In this activity, students will be presented with a situation involving a quadratic equation. Out of the given situation, ask the students to formulate an equation relating the area of the bulletin board to its dimensions. By applying their previous knowledge and skills in multiplying polynomials, the students should come up with an equation that contains a polynomial of degree 2. Ask them further to describe the equations formulated and let them use these in finding the dimensions of the bulletin board.

Activity 3: A Real Step to Quadratic Equations

Answer Key

1. 

2. Possible dimensions of the bulletin board: 2 ft. by 9 ft. and 3 ft. by 6 ft.
3. Find two positive numbers whose product equals 18.
   (Note: Area = length x width)
4. Let *w* be the width (in ft.). Then the length is $w + 7$. Since the area is 18, then $w(w+7)=18$. (Other variables can be used to represent the length or width of the bulletin board)
5. Taking the product on the left side of the equation formulated in item 4 yields $w^2+7w=18$. The highest exponent of the variable involved is 2.
6. Yes

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, quadratic equations. Let the students read and understand some important notes on quadratic equations. Tell them to study carefully the examples given.

What to PROCESS:

In this section, let the students apply the key concepts of Quadratic Equations. Tell them to use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

Ask the students to perform Activity 4. In this activity, the students will identify which equations are quadratic and which are not. If the equation is not quadratic, ask them to explain why.

Activity 4: Quadratic or Not Quadratic?

Answer Key

1. Not Quadratic
2. Quadratic
3. Not Quadratic
4. Quadratic
5. Quadratic
6. Quadratic
7. Quadratic
8. Not Quadratic
9. Quadratic
10. Quadratic

In Activity 5, ask the students to identify the situations that illustrate quadratic equations and represent these by mathematical sentences. To further strengthen students' understanding of quadratic equations, you may ask them to cite other real-life situations where quadratic equations are illustrated.

Activity 5: Does It Illustrate Me?

Answer Key

1.	Quadratic;	$w(w+8)=105 \rightarrow w^2+8w=105$ w is the width, in meters, of the swimming pool.	
2.	Not Quadratic;	$x+x+600 \geq 1200 \rightarrow 2x+600 \geq 1200$ x is the cost, in pesos, of the blouse.	
3.	Quadratic;	$2v^2+30v-900=0 \rightarrow v^2+15v-450=0$ v is the speed, in kph, of the bicycle. $15t^2-30t-120=0 \rightarrow t^2-2t-8=0$ t is the time, in hours, of travel of the bicycle.	
3.	Not Quadratic;	$4000x+25000=625000$ x is the number of square meters of lot.	
4.	Quadratic;	$4x^2+38x=92 \rightarrow 2x^2+19x=46$ x is the width, in meters, of the border of flowers.	

One important skill that students need to learn is how to write quadratic equations in standard form, $ax^2+bx+c=0$. Activity 6 gives the students an opportunity to recognize and write equations in this form. Emphasize to them that there are quadratic equations which are not written in standard form. However, these equations can also be written in the form $ax^2+bx+c=0$ using the different mathematics concepts or principles, particularly the distributive property and addition property of equality.

After the students have performed the activity, let them explain how they expressed the equations in the form $ax^2+bx+c=0$. Ask them to discuss with their classmates the different mathematics concepts or principles they applied in writing each quadratic equation in standard form. Let them compare their answers to find out the errors they committed.

Activity 6: Set Me To Your Standard!

Answer Key

Quadratic Equations	a	b	c
1. $-2x^2+3x-7=0$ or $2x^2-3x+7=0$	-2	3	-7
	2	-3	7
2. $-2x^2-6x+5=0$ or $2x^2+6x-5=0$	-2	-6	5
	2	6	-5
3. $x^2+7x+12=0$ or $-x^2-7x-12=0$	1	7	12
	-1	-7	-12
4. $2x^2+5x-7=0$ or $-2x^2-5x+7=0$	2	5	-7
	-2	-5	7
5. $2x^2-6x-15=0$ or $-2x^2+6x+15=0$	2	-6	-15
	-2	6	15
6. $x^2+3x-49=0$ or $-x^2-3x+49=0$	1	3	-49
	-1	-3	49

Answer Key

Quadratic Equations	a	b	c
7. $x^2-8x+24=0$ or $-x^2+8x-24=0$	1	-8	24
	-1	8	-24
8. $x^2+x-2=0$ or $-x^2-x+2=0$	1	1	-2
	-1	-1	2
9. $3x^2-6x=0$ or $-3x^2+6x=0$	3	-6	0
	-3	6	0
10. $x^2+14x-9=0$ or $-x^2-14x+9=0$	1	14	-9
	-1	-14	9

What to REFLECT ON and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of quadratic equations. Provide them with opportunities to think deeply and test further their understanding of the lesson by doing Activity 7. In this activity, the students will differentiate quadratic equations from linear equations, give examples of quadratic equations written in standard form and describe these, and write a quadratic equation that represents a given situation.

Activity 7: Dig Deeper!

Answer Key

1. Quadratic equations are mathematical sentences of degree 2 while linear equations are mathematical sentences of degree 1.
2. (Check students' explanations and the examples they will give. Their answers might be different but all are correct.)
3. Edna and Luisa are both correct. The equations $5-3x=2x^2$ can be written in standard form in two ways, $2x^2+3x-5=0$ or $-2x^2-3x+5=0$.
4. Yes. $2x^2+3x-4=0$ or $-2x^2-3x+4=0$
5. Possible answers:

   a. m

   b. $\dfrac{25000}{m}$

   c. $\dfrac{25000}{m+25}$

   d. $\dfrac{25000}{m+25}=\dfrac{25000}{m}-50$

   $50m^2+1250m-625000=0$

   $m^2+25m-12500=0$

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of quadratic equations by doing a practical task. Let them perform Activity 8. You can ask the students to work individually or in group. In this activity, the students will give examples of quadratic equations written in standard form and name some objects or cite real-life situations where quadratic equations are illustrated like rectangular gardens, boxes, a ball that is hit or thrown, two or more people working, and many others.

Summary/Synthesis/Generalization:

This lesson was about quadratic equations and how they are illustrated in real life. The lesson provided the students with opportunities to describe quadratic equations using practical situations and their mathematical representations. Moreover, they were given the chance to formulate quadratic equations as illustrated in some real-life situations. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next lesson, Solving Quadratic Equations.

Lesson 2A: SOLVING QUADRATIC EQUATIONS BY EXTRACTING SQUARE ROOTS

What to KNOW:

Provide the students with opportunities to relate and connect previously learned mathematics concepts to the new lesson, solving quadratic equations by extracting square roots. As they go through this lesson, tell them to think of this important question: *"How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?"*

The first activity that the students will perform is to find the square roots of some numbers. This lesson had been taken up by the students in their Grade 7 mathematics. Finding the square roots of numbers and the concepts of rational and irrational numbers are prerequisites to the new lesson and the succeeding lessons.

To help students deepen their understanding of the concepts of square roots, rational and irrational numbers, ask them to explain how they arrived at the square roots of numbers. Give emphasis to the number of square roots a positive or a negative number has. Furthermore, let them identify and describe rational and irrational numbers.

Activity 1: Find My Roots!!!

Answer Key

1. 4	4. -8	7. 0.4	10. $\pm\frac{13}{16}$
2. -5	5. 11	8. ±6	
3. 7	6. -17	9. $\frac{4}{5}$	

Ask the students to perform Activity 2. This activity provides the students with an opportunity to recall finding solutions of linear equations, an important skill that they need in order to solve quadratic equations. After the students have performed the activity, let them discuss the mathematics concepts or principles they applied to arrive at the solutions to the equations. Give emphasis to the different properties of equality.

Activity 2: What Would Make a Statement True?

Answer Key

1. $x = 5$	6. $x = -7$
2. $t = 14$	7. $h = 6$
3. $r = -8$	8. $x = 4$
4. $x = 8$	9. $x = 1$
5. $s = 8$	10. $k = 5$

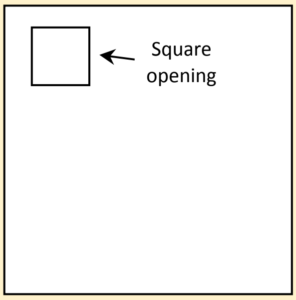
Give the students an opportunity to develop their understanding of solving quadratic equations by extracting square roots. Ask them to perform Activity 3. In this activity, students will be presented with a situation involving a quadratic equation. Ask the students to draw a diagram to illustrate the given situation. Drawing a diagram helps students understand what a given situation is all about.

Let the students relate the different quantities involved in the diagram using expressions and an equation. The students should be able to arrive at a quadratic equation that can be solved by extracting square roots. At this point, students should be able to realize that there are real-life situations that can be represented by quadratic equations. Such quadratic equations can be solved by extracting square roots.

Activity 3: Air Out!!!

Answer Key

1. Possible answer



2. x ; x^2
3. $x^2 - 0.25 = 6$
4. (Let the students answer the question based on what they already know.)

Activity 4 provides students with an opportunity to solve quadratic equations of the form $ax^2 = c$ or $ax^2 - c = 0$. At this point, however, there is still no need to present solving quadratic equations by extracting square roots. Let the students solve first the equations presented in as many ways as they can until they find a shorter way of solving these, that is, solving quadratic equations by extracting square roots.

Activity 4: Learn to Solve Quadratic Equations!!!

Answer Key

1. The quadratic equations given can be written in the form $ax^2 + c = 0$.
2. (Let the students show different ways of solving the equation.)

   $x^2 = 36 \quad \rightarrow \quad x = \pm 6$

   $t^2 - 64 = 0 \quad \rightarrow \quad t = \pm 8$

   $2s^2 - 98 = 0 \quad \rightarrow \quad s = \pm 7$
3. By checking the obtained values of the variable against the equation.
4. Yes

Ask the students to perform Activity 5. Let them find the solutions of three different quadratic equations in as many ways as they can. At this point, the students should realize that a quadratic equation has at most two real solutions or roots.

Activity 5: Anything Real or Nothing Real?

Answer Key

$x^2 = 9$ → $x = \pm 3$

$r^2 = 0$ → $r = 0$

$w^2 = -9$ → no real solutions or roots

2. $x^2 = 9$ has two real solutions or roots.
   $r^2 = 0$ has one real solution or root.
   $w^2 = -9$ has no real solutions or roots.
3. The given equations are in the form $x^2 = k$.
   a. If k is positive, the equation has two solutions or roots.
   b. If k is zero, the equation has one solution or root.
   c. If k is negative, the equation has no real solutions or roots.

Ask the students to give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, solving quadratic equations by extracting square roots. Let the students read and understand some important notes on solving quadratic equations by extracting square roots. Tell them to study carefully the examples given.

What to PROCESS:

Let the students use the mathematical ideas involved in finding the solutions of quadratic equations by extracting square roots and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to solve quadratic equations by extracting square roots by performing Activity 6. Tell them to explain how they arrived at their answers. Give emphasis to the mathematics concepts or principles the students applied in finding the solutions to the equations. Provide the students with opportunities to compare their answers and correct their errors.

Activity 6: Extract Me!!!

Answer Key

1. $x = \pm 4$
2. $t = \pm 9$
3. $r = \pm 10$
4. $x = \pm 12$
5. $s = \pm 5$
6. $x = \pm \frac{15}{2}$
7. $x = \pm 7$
8. $x = 17$; $x = -9$
9. $k = 10$; $k = -24$
10. $s = 8$; $s = -7$

Strengthen further students' understanding of solving quadratic equations by extracting square roots by doing Activity 7. Ask them to explain how they came up with the equation that represents the area of the shaded region and the length of a side of each square. Let them explain why the solutions to the equation they have formulated do not all represent the length of a side of the square. At this point, the students should realize that only positive solutions are used to indicate length.

Activity 7: What Does a Square Have?

Answer Key

1. $s^2 = 169$; $s = 13$ → The length of each side of the square is 13 cm.
2. $(s+5)^2 = 256$; $s = 11$ → The length of each side of the square is 16 cm.

What to REFLECT ON and FURTHER UNDERSTAND:

Provide the students with opportunities to think deeply and test further their understanding of solving quadratic equations by extracting square roots by doing Activity 8 and Activity 9. At this point, the students should realize that quadratic equations may have irrational solutions or roots. If the roots are irrational, let them approximate these roots. Ask them to apply what they have learned in Grade 7 mathematics about approximating square roots.

Activity 8: Extract Then Describe Me!

Answer Key

1. $t = \pm 2$
2. $x = \pm\sqrt{7} \approx 2.646$
3. $r = \pm\sqrt{6} \approx 2.449$
4. $x = \pm\sqrt{150} \approx 12.247$
5. $x = \pm\dfrac{3}{4}$
6. $s = 13$ or $s = -5$

Activity 9: Intensify Your Understanding!

Answer Key

1. Yes, a quadratic equation has at most two solutions.
2. (Let the students give their own examples of quadratic equations with two real solutions, one real solution, or no real solutions then check.)
3. No. The solutions of the quadratic equations $w^2 = 49$ and $w^2 + 49 = 0$ are not the same.
4. If the area of the square table is 3 m^2, then the length of its side is $\sqrt{3}$ m. $\sqrt{3}$ is an irrational number. So it is not possible to use a tape measure to construct a side length of $\sqrt{3}$ m.
5. 3.5 ft.

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of solving quadratic equations by extracting square roots by doing a practical task. Let them perform Activity 10. You can ask the students to work individually or in group. In this activity, the students will describe and give examples of quadratic equations with two real solutions, one real solution, and no real solutions. They will also formulate and solve quadratic equations by extracting square roots.

Summary/Synthesis/Generalization:

This lesson was about solving quadratic equations by extracting square roots. The lesson provided the students with opportunities to describe quadratic equations and solve these by extracting square roots. They were able to find out also how such equations are illustrated in real life. Moreover, they were given the chance to demonstrate their understanding of the lesson by doing a practical task. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the wide applications of quadratic equations in real life.

Lesson 2B: SOLVING QUADRATIC EQUATIONS BY FACTORING

What to KNOW:

Check students' prior mathematical knowledge and skills that are related to solving quadratic equations by factoring. Doing this would facilitate teaching and guide the students in understanding the lesson.

To solve quadratic equations by factoring, it is necessary for the students to recall factoring polynomials which they already studied in Grade 8 mathematics. Activity 1 of this lesson provides this opportunity for the students. However, only those polynomials of degree 2 will be factored since these are the polynomials that are involved in the lesson.

Activity 1: What Made Me?

Answer Key

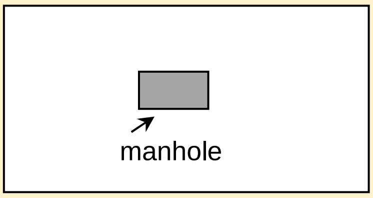
1. $2x(x-4)$
2. $-3s(s-3)$
3. $4x(1+5x)$
4. $5t(1-2t)$
5. $(s+6)(s+2)$
6. $(x-7)(x-3)$
7. $(x+6)(x-1)$
8. $(2r+5)(2r+5)$
9. $(3t-2)(3t+2)$
10. $(2x+7)(x-2)$

Develop students' understanding of the new lesson through a real-life situation. Ask the students to perform Activity 2. Let them illustrate the given situation using a diagram and write expressions and equation that would represent the different measures of quantities involved. Provide the students with an opportunity to think of a way to find the measures of the unknown quantities using the equation formulated.

Activity 2: The Manhole

Answer Key

1. Possible answer



2. Possible answers: w = width in meters; w + 8 = length; $w(w+8)$ = area
3. $w(w+8)-0.5$ or $w^2+8w-0.5$
4. $w^2+8w-0.5=19.5 \quad \rightarrow \quad w^2+8w-20=0$
5. (Let the students show different ways of solving the equation formulated in item 4.)

It might not be possible for the students to solve the equation formulated in Activity 2. If such is the case, ask them to perform Activity 3. This activity will lead them to the solution of the equation formulated. Emphasize in this activity the Zero-Product Property.

Activity 3: Why is the Product Zero?

Answer Key

1. The first two equations are linear equations while the third is a quadratic equation.
2. $x+7=0$ → $x=-7$; $x-4=0$ → $x=4$
   $(x+7)(x-4)=0$ → $x=-7$ or $x=4$
3. Substitute each value of *x* obtained in the equation. If the equation is true for the obtained value of *x*, then that value of *x* satisfies the equation.
4. The values of *x* in the first two equations make the third equation true.
5. Yes. Since $x+7=0$ and $x-4=0$, then their product is also equal to zero. So the value of *x* that makes $x+7=0$ or $x-4=0$ true will also make the equation $(x+7)(x-4)=0$ true.
6. If $(x+7)(x-4)=0$, then either or both $x+7$ and $x-4$ are equal to zero.

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, solving quadratic equations by factoring. Let the students read and understand some important notes on solving quadratic equations by factoring. Tell them to study carefully the examples given.

What to PROCESS:

In this section, let the students use the mathematical ideas involved in finding the solutions of quadratic equations by factoring and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to solve quadratic equations by factoring by performing Activity 4. Tell them to explain how they arrived at their answers. Give emphasis to the mathematics concepts or principles the students applied in finding the solutions to the equations. Provide the students with opportunities to compare their answers and correct their errors.

Activity 4: Factor Then Solve!

Answer Key

1. $x=0$ or $x=-7$	5. $h=-8$ or $h=2$	9. $x=\frac{-9}{2}$ or $x=\frac{9}{2}$
2. $s=0$ or $s=-3$	6. $x=7$ or $x=-2$	10. $s=\frac{3}{2}$
3. $t=-4$	7. $r=-3$ or $r=\frac{-5}{2}$	
4. $x=5$	8. $x=5$ or $x=-5$	

Strengthen further students' understanding of solving quadratic equations by factoring by doing Activity 5. Ask them to explain how they came up with the equation that represents the area of the shaded region and the length and width of the figure. Let them explain why not all solutions to the equation they formulated represent the length and width of the figure. As pointed out in the previous lesson, only positive solutions are used to indicate measures of length.

Activity 5: What Must Be My Length and Width?

Answer Key

1. Length = 17 units; Width = 9 units
2. Length = 19 units; Width = 12 units

What to REFLECT ON and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of quadratic equations. Provide them with opportunities to think deeply and test further their understanding of solving quadratic equations by factoring by doing Activity 6.

Activity 6: How Well Did I Understand?

Answer Key

1. Possible answers: $t^2+12t+36=0$ and $2s^2+8s-10=0$. $2x^2=72$ and $w^2-64=0$ can be solved easily by extracting square roots but can be solved also by factoring.
2. (Evaluate students' responses. They may have different answers.)
3. (Evaluate students' responses. They may have different answers.)
4. a. $x=2$ or $x=-8$ c. $t=\frac{7}{2}$ or $t=5$

   b. $s=-2$ or $s=-8$ d. $x=-4$ or $x=-5$
5. Yes. $14-5x-x^2=0$ can also be written as $x^2+5x-14=0$.
6. By extracting the square root: $(x-4)^2=9 \rightarrow$

   $$x-4=\pm 3$$
   $$x=4\pm 3$$
   $$x=4+3 \text{ or } x=4-3$$
   $$x=7 \text{ or } x=1$$

   By factoring: $(x-4)^2=9 \rightarrow$

   $$(x-4)^2-9=0$$
   $$[(x-4)-3][(x-4)+3]=0$$
   $$(x-4)-3=0 \text{ or } (x-4)+3=0$$
   $$x-4-3=0 \text{ or } x-4+3=0$$
   $$x-7=0 \text{ or } x-1=0$$
   $$x=7 \text{ or } x=1$$

7. Width = 10 inches; Length = 15 inches

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of solving quadratic equations by factoring by doing a practical task. Let them perform Activity 7. You can ask the students to work individually or in a group. In this activity, the students will be placed in a situation presented. They will perform a task and come up with the product being required by the given situation.

Summary/Synthesis/Generalization:

This lesson was about solving quadratic equations by factoring. The lesson provided the students with opportunities to describe quadratic equations and solve these by factoring. They were able to find out also how such equations are illustrated in real life. Moreover, they were given the chance to demonstrate their understanding of the lesson by doing a practical task. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the wide applications of quadratic equations in real life.

Lesson 2C: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

What to KNOW:

Activate students' prior mathematical knowledge and skills and provide them with opportunities to connect these to their lesson Solving Quadratic Equations by Completing the Square. Emphasize to the students the relevance of these mathematical knowledge and skills to their new lesson.

In Activity 1, the students will be asked to solve linear equations and quadratic equations that can be solved by extracting square roots. This activity would further enhance students' skills in solving such equations and enable them to understand better completing the square as a method of solving quadratic equations. Let the students realize the number of solutions a linear equation or a quadratic equation has.

There are quadratic equations whose solutions are irrational or radicals which cannot be expressed as rational numbers. Let the students recognize these solutions. At this stage, however, the students may not be able to simplify these solutions because simplifying radicals has not been taken up yet. You could partly discuss this but you have to focus only on what is needed in the present lesson.

Activity 1: How Many Solutions Do I Have?

Answer Key

1. $x = 5$	6. $x = \dfrac{5}{4}$
2. $s = -24$	7. $x = -4$ or $x = -16$
3. $r = 37$	8. $w = 9 + 2\sqrt{3}$ or $w = 9 - 2\sqrt{3}$
4. $x = \dfrac{23}{6}$	9. $k = \dfrac{1}{4}$ or $k = \dfrac{-5}{4}$
5. $t = \dfrac{31}{7}$	10. $h = \dfrac{3}{5} + \dfrac{\sqrt{2}}{2}$ or $h = \dfrac{3}{5} - \dfrac{\sqrt{2}}{2}$

In solving quadratic equations by completing the square, one of the skills involved is expressing a perfect square trinomial as a square of a binomial. This is a lesson under factoring which the students already studied in Grade 8. To further deepen students' understanding of this lesson, ask them to perform Activity 2. Give the students an opportunity to describe and investigate the relationship among the terms of perfect square trinomials. Also, let them explain how each trinomial is expressed as a square of a binomial.

Immediately follow-up students' understanding of perfect square trinomials by asking them to perform Activity 3. Let them determine the number to be added to the given terms to make an expression a perfect square trinomial. Ask them to explain how they determined each number that was added to the given terms.

Activity 2: Perfect Square Trinomial to Square of a Binomial

Answer Key

1. $(x+2)^2$
2. $(t+6)^2$
3. $(s+5)^2$
4. $(x-8)^2$
5. $(h-7)^2$
6. $(x+9)^2$
7. $\left(t+\frac{1}{3}\right)^2$
8. $\left(r-\frac{7}{2}\right)^2$
9. $\left(s+\frac{3}{8}\right)^2$
10. $\left(w-\frac{5}{2}\right)^2$

Activity 3: Make It Perfect!

Answer Key

1. 1
2. 100
3. 64
4. 144
5. 225
6. $\frac{121}{4}$
7. $\frac{225}{4}$
8. $\frac{441}{4}$
9. $\frac{1}{9}$
10. $\frac{9}{64}$

Provide the students with an opportunity to develop their understanding of solving quadratic equations by completing the square. Ask them to perform Activity 4. In this activity, the students will be presented with a diagram that describes a situation involving a quadratic equation. Let the students use the diagram in formulating expressions and an equation to be used in determining the length of the car park. Once they arrive at the required quadratic equation, provide them with an opportunity to discuss the meaning of "completing the square". Let them find out how they can use completing the square in solving the quadratic equation formulated and in finding the length of the square-shaped car park.

Activity 4: Finish the Contract!

Answer Key

1. s = length, in meters, of a side of the car park; s – 10 = width of the cemented portion
2. $s(s-10)=600 \rightarrow s^2-10s=600$
3. (Evaluate students' responses. They may give different answers.)

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, solving quadratic equations by completing the square. Let the students read and understand some important notes on solving quadratic equations by completing the square. Tell them to study carefully the examples given.

What to PROCESS:

Let the students use the mathematical ideas involved in finding the solutions of quadratic equations by completing the square and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to solve quadratic equations by completing the square by performing Activity 5. Tell them to explain how they arrived at their answers. Give emphasis to the mathematics concepts or principles the students applied in finding the solutions to the equations. Provide the students with opportunities to compare their answers and correct their errors.

Activity 5: Complete Me!

Answer Key

1. $x = -1$ or $x = 3$	6. $x = 7$ or $x = 1$
2. $s = 3$ or $s = -7$	7. $x = 6$ or $x = -1$
3. $t = -1$ or $t = -9$	8. $m = \frac{3}{2}$ or $m = \frac{-17}{2}$
4. $x = 2$ or $x = -16$	9. $r = -2 + \sqrt{3}$ or $r = -2 - \sqrt{3}$
5. $r = 5 + 2\sqrt{2}$ or $r = 5 - 2\sqrt{2}$	10. $w = -3 + 2\sqrt{5}$ or $w = -3 - 2\sqrt{5}$

Strengthen further students' understanding of solving quadratic equations by completing the square by doing Activity 6. Ask them to explain how they came up with the equation that represents the area of the shaded region. Let them explain why not all solutions to the equation they formulated represent the particular measure of each figure. As in the previous lessons, emphasize that only positive solutions are used to indicate measures of length.

Activity 6: Represent then Solve!

Answer Key

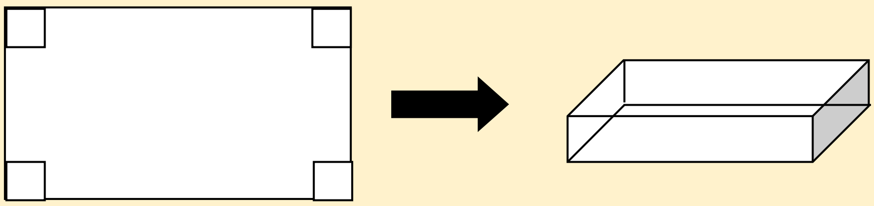
1. $s(s-3) = 88$ or $s^2 - 3s = 88$ or $s^2 - 3s - 88 = 0$; $s = 11$
   Note: The negative solution is disregarded since the problem involves measures of length.
2. $t(t+5) = 176$ or $t^2 + 5t = 176$ or $t^2 + 5t - 176 = 0$; $t = 11$
   Note: The negative solution is disregarded since the problem involves measures of length.

What to REFLECT ON and FURTHER UNDERSTAND:

Provide the students with opportunities to think deeply and test further their understanding of solving quadratic equations by completing the square. Let the students have a closer look at some aspects of quadratic equations and the ways of solving these by doing Activity 7. Give more focus to the real-life applications of quadratic equations.

Activity 7: What Solving Quadratic Equations by Completing the Square Means to Me…

Answer Key

1. Yes. Set the coefficient of the linear term **bx** equal to zero then add to $4x^2 - 25$. The equation $4x^2 - 25 = 0$ becomes $4x^2 + 0x - 25 = 0$. Solve the resulting equation by completing the square.
2. Yes
3. (Evaluate students' responses. They might give different answers.)
4. No. Gregorio did not arrive at the correct solutions. He should first have divided the equation by 2.
5.
   a. 

   b. w = width, in cm; $w + 8$ = length

   c. w, $w - 8$, and 4

   d. $4w(w-8) = 448$ → $4w^2 - 32w = 448$ → $w^2 - 8w = 112$

   e. Solve the equation by completing the square or other methods of solving quadratic equations.

   f. width = $4 + 8\sqrt{2}$ cm; length = $12 + 8\sqrt{2}$ cm

   g. length = $4 + 8\sqrt{2}$ cm; width = $-4 + 8\sqrt{2}$ cm; height = 4 cm

6. 2010

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Let the students work individually or in groups in doing a practical task. Ask them to perform Activity 8. This activity would give the students opportunities to demonstrate their understanding of solving quadratic equations by completing the square. They are expected to come up with sketch plans of open boxes and their respective covers following specific conditions. Using the sketch plans, they will formulate quadratic equations and solve these by completing the square.

Summary/Synthesis/Generalization:

This lesson was about solving quadratic equations by completing the square. The lesson provided the students with opportunities to describe quadratic equations and solve these by completing the square. They were able to find out also how such equations are illustrated in real life. Moreover, they were given the chance to demonstrate their understanding of the lesson by doing a practical task. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the wide applications of quadratic equations in real life.

Lesson 2D: SOLVING QUADRATIC EQUATIONS BY USING THE QUADRATIC FORMULA

What to KNOW:

Ask the students to perform activities that would help them recall the different mathematics concepts previously studied. Provide them with opportunities to connect these concepts to their lesson, Solving Quadratic Equations by Using the Quadratic Formula.

The first activity that the students will do in this lesson is to describe and simplify expressions involving square roots and other mathematics concepts. This activity will familiarize them with these kinds of expressions and how they are simplified. The students will also be able to relate these expressions later to their lesson, solving quadratic equations by using the quadratic formula.

Activity 1: It's Good to be Simple!

Answer Key

1. $\frac{3}{2}$
2. $\frac{1}{2}$
3. $\frac{-6+3\sqrt{2}}{4}$
4. $\frac{-9-2\sqrt{6}}{4}$
5. $\frac{1}{3}$
6. -5
7. $\frac{5+5\sqrt{5}}{8}$
8. $\frac{-5+2\sqrt{3}}{3}$
9. $\frac{-1-\sqrt{2}}{2}$
10. $\frac{1}{3}$

To solve quadratic equation $ax^2 + bx + c = 0$ by using the quadratic formula, it is necessary for the students to determine the values of a, b, and c. These values are required by the quadratic formula in order to solve a given quadratic equation.

There are quadratic equations, however, that are not written in standard form. It is then important for the students to realize that the values of a, b, and c vary depending on how the quadratic equation is written. As in Activity 2, each quadratic equation can be written in standard form in two different ways. This means that there are two possible sets of values of a, b, and c. Let the students explain why this is so.

Activity 2: Follow the Standards!

Answer Key

Quadratic Equations	a	b	c
1. $2x^2 + 9x - 10 = 0$ or $-2x^2 - 9x + 10 = 0$	2	9	-10
	-2	-9	10
2. $-2x^2 + 7x - 2 = 0$ or $2x^2 - 7x + 2 = 0$	-2	7	-2
	2	-7	2
3. $2x^2 - 6x + 1 = 0$ or $-2x^2 + 6x - 1 = 0$	2	-6	1
	-2	6	-1
4. $3x^2 - 7x - 10 = 0$ or $-3x^2 + 7x + 10 = 0$	3	-7	-10
	-3	7	10
5. $2x^2 - 12x - 5 = 0$ or $-2x^2 + 12x + 5 = 0$	2	-12	-5
	-2	12	5
6. $-2x^2 + 5x + 15 = 0$ or $2x^2 - 5x - 15 = 0$	-2	5	15
	2	-5	-15
7. $x^2 + 16x + 48 = 0$ or $-x^2 - 16x - 48 = 0$	1	16	48
	-1	-16	-48
8. $x^2 - 15x + 54 = 0$ or $-x^2 + 15x - 54 = 0$	1	-15	54
	-1	15	-54
9. $3x^2 + 4x - 7 = 0$ or $-3x^2 - 4x + 7 = 0$	3	4	-7
	-3	-4	7
10. $3x^2 - 30x + 85 = 0$ or $-3x^2 + 30x - 85 = 0$	3	-30	85
	-3	30	-85

Provide the students with an opportunity to develop their understanding of solving quadratic equations by using the quadratic formula. Ask them to perform Activity 3. In this activity, the students will be presented with a situation involving a quadratic equation. Let the students formulate expressions and equation that would describe the given situation and ask them to describe these.

Challenge the students to solve the equation that would give the required dimensions of the gardens. Let them use the different methods of solving quadratic equations already presented. At this point, make students realize that some methods for solving quadratic equations are easier to use for a particular equation than others.

Ask the students to perform Activity 4. This activity will provide them with an opportunity to come up with the quadratic formula. Let them find out why the quadratic formula can be used in solving any quadratic equations.

Activity 3: Why do the Gardens Have to be Adjacent?

Answer Key

1. Width = w and Length = l
2. $4w + 2l = 70.5$
   $2lw = 180$
3. Solve for one variable in terms of the other in the first equation then substitute this to the second equation.
4. Possible answers: $l^2 - 35.25l + 180 = 0$ or $w^2 - 17.625w + 45 = 0$
5. The equations formulated are quadratic equations.
6. (Evaluate students' responses. They might give different answers.)

Activity 4: Lead Me to the Formula!

Answer Key

1. $2x^2 + 9x + 10 = 0$

$$\rightarrow \frac{2x^2}{2} + \frac{9x}{2} + \frac{10}{2} = \frac{0}{2}$$

$$x^2 + \frac{9x}{2} + 5 = 0$$

$$x^2 + \frac{9x}{2} = -5$$

$$x^2 + \frac{9x}{2} + \frac{81}{16} = -5 + \frac{81}{16}$$

$$\left(x + \frac{9}{4}\right)\left(x + \frac{9}{4}\right) = \frac{-80}{16} + \frac{81}{16}$$

$$\left(x + \frac{9}{4}\right)^2 = \frac{1}{16}$$

$$x + \frac{9}{4} = \pm\sqrt{\frac{1}{16}}$$

$$x = -\frac{9}{4} \pm \frac{1}{4}$$

$$x = \frac{-9 \pm 1}{4}$$

2. $x = \frac{-5}{2}$ or $x = -2$
3. The solutions are irrational and not equal.
4. (Monitor students' work.)
5. $ax^2 + bx + c = 0$
6. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
7. Yes

Let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, solving quadratic equations by using the quadratic formula. Before they perform the next set of activities, let them read and understand first some important notes on solving quadratic equations by using the quadratic formula. Tell them to study carefully the examples given.

What to PROCESS:

At this point, ask the students to use the mathematical ideas involved in finding the solutions of quadratic equations by using the quadratic formula and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to solve quadratic equations by using the quadratic formula by performing Activity 5. Tell them to explain how they used the quadratic formula in finding the solutions to the quadratic equations. Furthermore, let them describe the quadratic equations and their solutions. Provide the students with opportunities to compare their answers and correct their errors.

Activity 5: Is the Formula Effective?

Answer Key

1. $x = -1$ or $x = -9$
2. $x = 7$ or $x = 5$
3. $x = 2$ or $x = -7$
4. No real roots
5. $x = \frac{-7 + \sqrt{65}}{2}$ or $x = \frac{-7 - \sqrt{65}}{2}$
6. No real roots
7. $x = \frac{1}{2}$
8. $x = 0$ or $x = \frac{4}{3}$
9. $x = 2\sqrt{2}$ or $x = -2\sqrt{2}$
10. $x = -1 + \frac{\sqrt{10}}{2}$ or $x = -1 - \frac{\sqrt{10}}{2}$

Strengthen further students' understanding of solving quadratic equations using the quadratic formula by doing Activity 6. Ask the students to represent some real-life situations by quadratic equations. Let them find out if the quadratic equations formulated can be used to determine the unknown quantities in each situation. Let them explain why the solutions to the equation they have formulated do not all represent a particular measure that is involved in each situation.

Activity 6: Cut to Fit!

Answer Key

1. Plywood 1: $x(2x) = 4.5$
   Plywood 2: $x(2x - 1.4) = 16$
   Plywood 3: $x(5 - x) = 6$

2.

Plywood 1:	$2x^2 - 4.5 = 0$	a = 2	b = 0	c = -4.5
	$-2x^2 + 4.5 = 0$	a = -2	b = 0	c = 4.5
Plywood 2:	$2x^2 - 1.4x - 16 = 0$	a = 2	b = -1.4	c = -16
	$-2x^2 + 1.4x + 16 = 0$	a = -2	b = 1.4	c = 16
Plywood 3:	$x^2 - 5x + 6 = 0$	a = 1	b = -5	c = 6
	$-x^2 + 5x - 6 = 0$	a = -1	b = 5	c = -6

Answer Key

3. Plywood 1: $2x^2 - 4.5 = 0$ $\qquad x = \frac{3}{2}$ or $x = -\frac{3}{2}$

   Plywood 2: $2x^2 - 1.4x - 16 = 0$ $\qquad x = \frac{16}{5}$ or $x = -\frac{5}{2}$

   Plywood 3: $x^2 - 5x + 6 = 0$ $\qquad x = 3$ or $x = 2$

4. $\frac{2}{3}$ ft., $\frac{16}{5}$ ft., and 2 ft., respectively

5. 3 ft., 5 ft., and 3 ft., respectively

What to REFLECT ON and FURTHER UNDERSTAND:

Provide the students with opportunities to think deeply and test further their understanding of solving quadratic equations using the quadratic formula by doing Activity 7. Give more focus to the real-life applications of quadratic equations.

Activity 7: Make the Most Out of It!

Answer Key

1. Yes, both have roots $2 \pm \frac{\sqrt{22}}{2}$.
2. (Evaluate students' responses.)
3. a. $x = \frac{-2 \pm \sqrt{-32}}{2}$. The solutions are not real numbers.

   b. $x = \frac{-4 \pm \sqrt{-40}}{4}$. The solutions are not real numbers.

   c. $x = \frac{7}{2}$ or $x = \frac{3}{2}$. The solutions are real numbers.

   d. $x = -\frac{1}{2} \pm \frac{\sqrt{33}}{6}$. The solutions are real numbers.
4. (Evaluate students' responses.)
5. (Evaluate students' responses.)

Answer Key

6. a. let x = the width, in meters, of the car park
   $x + 120$ = the length of the car park

   b. $x(x+120) = 6400$

   c. Transform to quadratic equation $ax^2 + bx + c = 0$ then solve.

   d. The length is 160 m and the width is 40 m.

   e. No. $320x80 \neq 12800$

7. The width is $\frac{-3+\sqrt{929}}{20}$ m or approximately 1.37 m and the length is $\frac{3+\sqrt{929}}{20}$ or approximately 3.35 m

8. The length of a side of the base of the box is 8 cm.

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of solving quadratic equations by using the quadratic formula by doing a practical task. Let them perform Activity 8 individually or in groups. In this activity, the students will make a floor plan of a new house given some conditions. Using the floor plan, they will formulate quadratic equations and solve these by using the quadratic formula.

Summary/Synthesis/Generalization:

This lesson was about solving quadratic equations using the quadratic formula. The lesson provided the students with opportunities to describe quadratic equations and solve these using the quadratic formula. They were able to find out also how such equations are illustrated in real life. Moreover, they were given the chance to demonstrate their understanding of the lesson by doing a practical task. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the wide applications of quadratic equations in real life.

Lesson 3: THE NATURE OF THE ROOTS OF A QUADRATIC EQUATION

What to KNOW:

Check students' prior knowledge of the different mathematics concepts and mathematical skills needed in understanding the nature of roots of quadratic equations. Tell the students that as they go through this lesson, they have to think of this important question: *"How does the nature of roots of quadratic equation facilitate in understanding the conditions of real-life situations?"*

The roots of quadratic equations can be real numbers or not real numbers. Hence, students need to recall the concept of real numbers and be able to describe these. They should be able to explain also why a number is not real. Activity 1 of this lesson provides this opportunity for the students.

Activity 1: Which are Real? Which are Not?

Answer Key

1. (Evaluate students' responses.)
2. Real numbers: 24.5, $\frac{7}{8}$, $\frac{5}{12}$, 289, $\sqrt{25}$, $\frac{\sqrt{15}}{9}$, $\sqrt{35}$

   Not real numbers: $\sqrt{-15}$ $\frac{\sqrt{-21}}{9}$
3. Rational numbers: 24.5, $\frac{7}{8}$, $\frac{5}{12}$, 289, $\sqrt{25}$

   Irrational numbers: $\frac{\sqrt{15}}{9}$, $\sqrt{35}$
4. Perfect square numbers : 289

   Not perfect square numbers $\frac{7}{8}$, $\frac{5}{12}$, $\sqrt{25}$, $\frac{\sqrt{15}}{9}$, $\sqrt{35}$
5. Perfect square number is a number that can be expressed as a square of a rational number.

It is not always necessary to determine first the roots of a quadratic equation in order to describe them. The roots of a quadratic equation can also be described using the value of the discriminant which can be obtained using the expression $b^2 - 4ac$. To use this expression, it is necessary for the students to determine the values of a, b, and c in a quadratic equation. However, emphasize to the students that they need to write first the quadratic equation in standard form before they identify these values of a, b, and c. Let them perform Activity 2.

After identifying the values of a, b, and c in Activity 2, you may ask the students to use these values in evaluating the expression $b^2 - 4ac$. Let them do the same in Activity 3.

Activity 2: Math in A, B, C?

Answer Key

1. $x^2 + 5x - 4 = 0$	a = 1	b = 5	c = -4
2. $4x^2 + 8x - 3 = 0$	a = 4	b = 8	c = -3
3. $4x^2 - 10x + 1 = 0$	a = 4	b = -10	c = 1
4. $3x^2 - 8x - 15 = 0$	a = 3	b = -8	c = -15
5. $3x^2 - 42x - 12 = 0$	a = 3	b = -42	c = -12

Activity 3: What's My Value?

Answer Key

1. $b^2 - 4ac = 9$
2. $b^2 - 4ac = 169$
3. $b^2 - 4ac = 0$
4. $b^2 - 4ac = 12$
5. $b^2 - 4ac = -576$

Let the students realize that given the values of a, b, and c, the value of $b^2 - 4ac$ can be obtained. Ask them to perform Activity 4. In this activity, the students will write the quadratic equation given the values of a, b, and c and solve this using any of the methods presented in the previous lessons. Let them explain how they came up with the roots of each equation.

Immediately after the students do Activity 4, ask them to perform Activity 5. Tell them to write their answers for Activities 3 and 4 in the table provided. Let the students describe the roots of the quadratic equation and relate these to the value of its discriminant. The students should realize at this stage that the value of the discriminant of a quadratic equation can be used to describe its roots.

Activity 4: Find My Equation and Roots

Answer Key

$ax^2 + bx + c = 0$	Roots
1. $x^2 + 5x + 4 = 0$	$x = -1$ or $x = -4$
2. $2x^2 + x - 21 = 0$	$x = -\frac{7}{2}$ or $x = 3$
3. $4x^2 + 4x + 1 = 0$	$x = -\frac{1}{2}$
4. $x^2 - 2x - 2 = 0$	$x = 1 + \sqrt{3}$ *or* $x = 1 - \sqrt{3}$
5. $9x^2 + 16 = 0$	no real roots

Activity 5: Place Me on the Table!

Answer Key

1. Complete the table below.

Equation	$b^2 - 4ac$	Roots
1. $x^2 + 5x + 4 = 0$	9	-1 or -4
2. $2x^2 + x - 21 = 0$	169	$-\frac{7}{2}$ or 3
3. $4x^2 + 4x + 1 = 0$	0	$\frac{1}{2}$
4. $x^2 - 2x - 2 = 0$	12	$1 + \sqrt{3}$ *or* $1 - \sqrt{3}$
5. $9x^2 + 16 = 0$	- 576	no real roots

2. If $b^2 - 4ac$ is zero, the roots are real and equal.
   If $b^2 - 4ac$ is positive and a perfect square, the roots are rational.
   If $b^2 - 4ac$ is positive but not a perfect square, the roots are irrational.
   If $b^2 - 4ac$ is negative, there are no real roots.

3. Real and equal roots: $4x^2 + 4x + 1 = 0$
   Rational roots: $2x^2 + x - 21 = 0$, $x^2 + 5x + 4 = 0$ and $4x^2 + 4x + 1 = 0$
   Irrational roots: $x^2 - 2x - 2 = 0$
   No real roots: $9x^2 + 16 = 0$

Develop students' understanding of the nature of roots of quadratic equations through a real-life situation involving a quadratic equation. Ask the students to solve the problems given in Activity 6. This activity provides the students with an opportunity to realize that the occurrence of a particular event may not always be possible for a given condition. For example, a ball cannot reach a height of 160 ft. following the initial conditions of the problem. Hence, s = 160 is not a realistic value of *s* in the equation $s = 100t - 16t^2$.

Activity 6: Let's Shoot that Ball!

Answer Key

1. 24 feet
2. 0.55 and 5.70 seconds
3. 6.25 seconds
4. No. The highest point is when t = 3.125 s, when the ball has a height of 156.25 feet.

Before the students perform the next activities, let them give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, nature of roots of quadratic equations. Let the students read and understand some important notes on the nature of roots of quadratic equations. Tell them to study carefully the examples given.

What to PROCESS:

Let the students use the mathematical ideas involved in determining the nature of roots of quadratic equations and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to determine the nature of roots of quadratic equations using the discriminant by performing Activity 7. Tell them to explain how they arrived at their answers. Ask them further how the value of the discriminant facilitates in determining the nature of roots of quadratic equations.

Activity 7: What is My Nature?

Answer Key

1. discriminant : 0 nature of the roots: rational and equal
2. discriminant: 1 nature of the roots: rational and not equal
3. discriminant: 36 nature of the roots: rational and not equal
4. discriminant: -15 nature of the roots: not real
5. discriminant: 24 nature of the roots: irrational and not equal
6. discriminant: 4 nature of the roots: rational and not equal
7. discriminant: -23 nature of the roots: not real
8. discriminant: -288 nature of the roots: not real
9. discriminant: 336 nature of the roots: irrational and not equal
10. discriminant: 64 nature of the roots: rational and not equal

Strengthen further students' understanding of the nature of roots of quadratic equations by doing Activity 8. Present to the students a situation involving a quadratic equation. In the given situation, ask the students to determine whether the dimensions of the table are rational numbers without actually computing for the roots.

Activity 8: Let's Make A Table!

Answer Key

1. Length = $p + 1$
2. $p(p+1) = 6$ *or* $p^2 + p - 6 = 0$
3. Yes. A positive discriminant implies two distinct roots
4. 3 m by 2 m

What to REFLECT ON and FURTHER UNDERSTAND:

Provide the students with opportunities to think deeply and test further their understanding of the nature of roots of quadratic equations by doing Activity 9. Let them explain how the nature of the roots of quadratic equations are determined then give examples to illustrate. Moreover, ask them to solve problems involving the discriminants of quadratic equations.

Activity 9: How Well Did I Understand the Lesson?

Answer Key

1. a. The roots are real numbers and equal.

   b. The roots are rational and not equal.

   c. The roots are irrational and not equal.

   d. There are no real roots.

2. Find the value of the discriminant of the quadratic equation.

3. Yes

4. Yes, examples:
   $x^2 + 4x + 4 = 0$ $\quad$ $x^2 + 6x + 9 = 0$ $\quad$ $x^2 + 10x + 25 = 0$

5. a. $-16t^2 + 29t + 6 = 20$ or $-16t^2 + 29t - 14 = 0$

   b. No, the discriminant is negative.

Before the students move on to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of the nature of roots of quadratic equations by doing a practical task. Let them perform Activity 10. You can ask the students to work individually or in groups. In this activity, the students will be asked to solve a particular real-life problem involving the discriminant of quadratic equations and then cite similar or other situations where this mathematics concept is applied.

Activity 10: Will It or Will It Not?

Answer Key

1. a. 2.4 meters

   b. 1.75 sec (going down) and 0.29 sec (going up)

   It will take 2.22 sec to touch the ground.

   c. No. The discriminant of the resulting equation is negative.

   d. Yes

   e. Example: A ball is thrown from an initial height of 1.5 m with an initial velocity of 12 m per second.

   f. *Example:* What will be the height of the ball from the ground after 2.5 seconds?
   Answer: 0.875 m.

Summary/Synthesis/Generalization:

This lesson was about the nature of the roots of quadratic equations. The lesson provided the students with opportunities to describe the nature of the roots of quadratic equations using the discriminant even without solving the equation. More importantly, they were able to find out how the nature of the roots of quadratic equations is illustrated in real-life situations. Their understanding of this lesson and other previously learned mathematical concepts and principles will facilitate their understanding of the succeeding lessons.

Lesson 4: SUM AND PRODUCT OF ROOTS OF QUADRATIC EQUATIONS

What to KNOW:

Have students perform mathematical tasks to activate their prior mathematical knowledge and skills then let them connect these to their new lesson, sum and product of roots of quadratic equations. Emphasize to the students this important question: *"How do the sum and product of roots of quadratic equation facilitate in understanding the required conditions of real-life situations?"*

Start the lesson by asking the students to add and multiply rational numbers. These are the basic skills that students need to learn about the relationships among the values of a, b, and c in a quadratic equation $ax^2 + bx + c = 0$ and its roots. Ask them to perform Activity 1. Let them explain how they arrived at their answers and how they applied the different mathematics concepts or principles in performing each operation.

Activity 1: Let's do Addition and Multiplication!

Answer Key

1. 22	4. $\frac{1}{8}$	7. -28	10. $\frac{3}{10}$
2. 5	5. $-\frac{3}{2}$	8. 72	
3. -23	6. 120	9. $-\frac{6}{35}$	

Provide the students with opportunities to enhance their skill in finding the roots of quadratic equations by doing Activity 2. Let them use the different methods of solving quadratic equations which were already presented in the previous lessons. Finding the solutions of a quadratic equation facilitates in determining the relationships among its roots and its terms. Once the roots are known, the students can then relate these to the terms of the quadratic equation.

Activity 2: Find My Roots!

Answer Key

1. $x = -1$ or $x = -2$	6. $h = -\frac{1}{5}$ or $h = \frac{2}{3}$
2. $s = 2$ or $s = 3$	7. $s = \frac{3}{4}$ or $s = -\frac{1}{3}$
3. $r = -4$ or $r = 2$	8. $t = -\frac{1}{3}$ or $t = \frac{3}{2}$
4. $t = -6$	9. $m = \frac{4 + 2\sqrt{7}}{3}$ or $m = \frac{4 - 2\sqrt{7}}{3}$
5. $x = -\frac{5}{2}$ or $x = -\frac{3}{2}$	10. $w = 4$ or $w = -\frac{5}{2}$

In Activity 3 of this lesson, the students will be asked to determine the values of a, b, and c of quadratic equations written in standard form and their respective roots. Let the students find the sum and product of these roots and relate the results to the values of a, b, and c. At this point, the students should realize that the sum and the product of roots of a quadratic equation are equal to $-\frac{b}{a}$ and $\frac{c}{a}$, respectively. The students should also learn that the quadratic equation can be determined given its roots or the sum and product of its roots.

Activity 3: Relate Me to My Roots!

Answer Key

1. a. a = 1, b = 7, and c = 12
   b. a = 2, b = -3, and c = -20
2. a. $x = -3$ or $x = -4$ b. $x = -\frac{5}{2}$ or $x = 4$
3. Complete the table

Quadratic Equation	Sum of Roots	Product of Roots
$x^2 + 7x + 12 = 0$	-7	12
$2x^2 - 3x - 20 = 0$	$\frac{3}{2}$	-10

4. The sum of the roots of quadratic equation is equal to $-\frac{b}{a}$ and the product is equal to $\frac{c}{a}$.
5. Yes. 6. Yes

Present to the students a real-life illustration of the relationships among the roots and the terms of a quadratic equation. Let them perform Activity 4. In this activity, the students should realize that the dimensions of the garden represent the roots of the quadratic equation. Hence, the sum of the roots represents one-half of the perimeter of the garden and the product of the roots represents its area.

Activity 4: What the Sum and Product Mean to Me…

Answer Key

1. Let x = the width of the garden.
   The area of the garden is given by the equation $x(23 - x) = 132$
2. The equation is a quadratic equation.
3. $x(23 - x) = 132$ → $x^2 - 23x + 132 = 0$
   The roots of the equation are 11 and 12. These represent the dimensions of the garden.
4. The sum of the roots is 23. This is half the perimeter of the garden.
5. The product of the roots is 132. This is equal to the area of the garden.

Ask the students to give a brief summary of the activities done before performing the next set of activities. Let them relate or connect their responses in the activities given to their new lesson, the sum and product of the roots of quadratic equations. Tell the students to read and understand some important notes on the sum and product of roots of quadratic equations and to study carefully the examples given.

What to PROCESS:

In this section, let the students use the mathematical ideas involved in finding the sum and product of roots of quadratic equation and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to find the sum and product of roots of quadratic equations by performing Activity 5. Let them explain how they arrived at their answers. Furthermore, ask them the significance of knowing the sum and product of roots of quadratic equations.

Activity 5: This is My Sum and this is My Product. Who Am I?

Answer Key

	Sum	Product	Roots
1.	-4	3	$x=-1$ or $x=-3$
2.	-2	-3	$x=1$ or $x=-3$
3.	-4	-21	$x=3$ or $x=-7$
4.	$-\frac{3}{2}$	-1	$x=\frac{1}{2}$ or $x=-2$
5.	$\frac{10}{3}$	$-\frac{8}{3}$	$x=4$ or $x=-\frac{2}{3}$
6.	-2	$\frac{3}{4}$	$x=-\frac{3}{2}$ or $x=-\frac{1}{2}$
7.	$\frac{2}{3}$	$-\frac{8}{9}$	$x=\frac{4}{3}$ or $x=-\frac{2}{3}$
8.	$\frac{3}{4}$	$-\frac{9}{8}$	$x=\frac{3}{2}$ or $x=-\frac{3}{4}$
9.	$\frac{19}{10}$	$\frac{3}{5}$	$x=\frac{2}{5}$ or $x=\frac{3}{2}$
10.	$\frac{3}{2}$	0	$x=0$ or $x=\frac{3}{2}$

Activity 6 is the reverse of Activity 5. In this activity, the students will be asked to determine the quadratic equation given the roots. However, the students should not focus only on getting the sum and product of roots to arrive at the required quadratic equation. Challenge the students to find other ways of determining the equations given the roots.

Activity 6: Here are the Roots. Where is the Trunk?

Answer Key

1. $x^2 - 14x + 45 = 0$	6. $x^2 + 9x = 0$
2. $x^2 - 18x + 80 = 0$	7. $x^2 - 7x + 11.25 = 0$
3. $x^2 - 9x + 18 = 0$	8. $x^2 + 6x + 9 = 0$
4. $x^2 + 18x + 80 = 0$	9. $36x^2 + 36x + 5 = 0$
5. $x^2 - 12x - 45 = 0$	10. $12x^2 - x - 6 = 0$

Strengthen further the students' understanding of the sum and product of roots of quadratic equations by doing Activity 7. Ask the students to use a quadratic equation to represent a given real-life situation. Let them apply their knowledge of the sum and product of roots of quadratic equations to determine the measures of the unknown quantities.

What to REFLECT ON and FURTHER UNDERSTAND:

Provide the students with opportunities to think deeply and test further their understanding of the sum and product of roots of quadratic equations by doing Activity 8. Give more focus to the real-life applications of quadratic equations.

Activity 7: Fence my Lot!

Answer Key

1. $2l + 2w = 90$
   $lw = 450$
2. (Evaluate students' responses.)
3. Form the quadratic equation that describes the given situation then solve. The equation is $x^2 - 45x + 450 = 0$.
4. Length = 30 m; Width = 15 m

Activity 8: Think of These Further!

Answer Key

1. There are two ways of finding the quadratic equation given its roots:

   First is method: We use the equations describing the roots to come up with two binomials whose product is zero. If the resulting equation is simplified, it becomes a quadratic equation in the form $ax^2 + bx + c = 0$.

   Second method: We get the sum and product of the roots and substitute these in the equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, where $\frac{b}{a}$ is the sum of the roots and $\frac{c}{a}$ is the product of the roots.

   (Evaluate students' responses to b and c.)

2. No. The given information is not sufficient.
3. Since the sum is -5, then the other roots is -12.
   $(x + 12)(x - 7) = 0 \quad \rightarrow \quad x^2 + 5x - 84 = 0$
4. No. The given information is not sufficient.
5. $x^2 + 20x + 51 = 0$
6. 3 ft. by 7 ft.

Before the students move on to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of the sum and product of roots of quadratic equations by doing a practical task. Let them perform Activity 9. Ask them to write a journal about their understanding of the lesson, give examples of finding quadratic equations given the roots, and pictures of real-life objects that illustrate the applications of the sum and product of roots of quadratic equations. Let them use these pictures and the measures of quantities involved to determine the quadratic equations.

Summary/Synthesis/Generalization:

This lesson was about the sum and product of roots of quadratic equations. In this lesson, the students were able to relate the sum and product of the roots of the quadratic equation $ax^2 + bx + c = 0$ to the values of a, b, and c. Furthermore, this lesson has given them opportunity to find the quadratic equation given the roots. Students' understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the succeeding lessons.

Lesson 5: EQUATIONS TRANSFORMABLE TO QUADRATIC EQUATIONS

What to KNOW:

Provide activities that would assess students' knowledge of the different mathematics concepts previously studied and their skills in performing mathematical operations. Students' responses to these activities would facilitate teaching and their understanding of equations transformable to quadratic equations. Present to the students this important question for them to think of: *"How does finding solutions of quadratic equations facilitate in solving real-life problems?"*

Enhance further the students' skills in solving quadratic equations by asking the students to perform Activity 1. Let them use the different methods of finding solutions to quadratic equations. Ask them to explain how they arrived at their answers. Focus on the mathematics concepts and principles that the students applied in solving the equations.

Activity 1: Let's Recall

Answer Key

1. $x = 2$	3. $r = 2$ or $r = -7$	5. $n = 2$ or $n = -3$
2. $s = 5$ or $s = -2$	4. $m = -2$ or $m = -\frac{1}{2}$	6. $p = -\frac{4}{3}$ or $p = -1$

There are equations involving rational algebraic expressions that can be transformed to quadratic equations. To transform these to quadratic equations, there is a need for the students to recall addition and subtraction of rational algebraic equations. Activity 2 provides the students with an opportunity to add or subtract rational algebraic expressions and write the results in their simplest forms.

Activity 2: Let's Add and Subtract!

Answer Key

1. $\frac{2x^2+5}{5x}$	3. $\frac{2x^2 + 3x + 3}{3x}$	5. $\frac{3x^2-5x+10}{2x^2-4x}$
2. $\frac{-2x^2 + x + 20}{5x}$	4. $\frac{x-1}{6x}$	6. $\frac{x^2 - 2}{x^2 + 3x + 2}$

Provide the students with an opportunity to develop their understanding of equations transformable to quadratic equations. Ask them to perform Activity 3. In this activity, the students will be presented with a situation involving a rational algebraic equation. Let the students formulate expressions and equations that would describe the given situation. Ask them to describe the equation and to find ways of solving it.

Activity 3: How Long Does it Take You to Finish your Job?

Answer Key

1. $\frac{x}{2}$ hrs
2. $\frac{1}{x}$ and $\frac{2}{x}$
3. $\frac{1}{x}+\frac{2}{x}=\frac{1}{4}$
4. The equation formed is a rational algebraic equation.
5. (Evaluate students' responses. They might give different ways of solving the problem.)

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, equations transformable to quadratic equations. Let the students read and understand some important notes on equations transformable to quadratic equations. Tell them to study carefully the examples given.

What to PROCESS:

In this section, let the students use the mathematical ideas involved in finding the solutions of equations transformable to quadratic equations and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to transform some equations to quadratic equations by performing Activity 4. Let them explain how they applied the different mathematics concepts and principles in transforming each equation to quadratic equation. Provide the students with opportunities to compare their answers and correct their errors.

Activity 4: View Me In Another Way!

Answer Key

1. $x^2+5x-2=0$
2. $s^2+12s+21=0$
3. $2t^2-2t+4=0$
4. $5r^2+20r+15=0$
5. $2m^2-22m+60=0$
6. $8x^2+25x-200=0$
7. $3t^2+14t-4=0$
8. $x^2-x-5=0$
9. $s^2-6s-43=0$
10. $7r^2+22r-41=0$

Present to the students the equations in Activity 5 and let them solve these. Ask the students to explain how they arrived at the solutions to the equations and how they applied the different mathematics concepts and principles in solving each. Provide the students with an opportunity to find and present other ways of solving the equation. Do the same for Activity 6.

At some points in the procedure that the students followed in solving the equations, ask them if there are quadratic equations formed. If there are any, let them explain how they arrived at these equations and how they solved these.

Let the students determine if the solutions or roots thay they obtained make the equation true. If there are extraneous roots, ask them to explain why these could not be possible roots of the equation.

Activity 5: What Must Be The Right Value?

Answer Key

Equation 1: $x = 7$ or $x = 3$

Equation 2: $x = \frac{2+\sqrt{14}}{2}$ or $x = \frac{2-\sqrt{14}}{2}$

Equation 3: $x = 1$ or $x = \frac{1}{2}$

Activity 6: Let's Be True!

Answer Key

1. $x = -7$ or $x = 4$
2. $s = 0$ or $s = 6$
3. $t = 5$ or $t = 2$
4. $r = 2$ or $r = -3$
5. $x = 3$ or $x = -2$
6. $x = -2+\sqrt{10}$ or $x = 2-\sqrt{10}$
7. $x = \frac{-1+\sqrt{17}}{2}$ or $x = \frac{-1-\sqrt{17}}{2}$
8. $x = -5$ or $x = 1$
9. $x = 4$ or $x = -1$
10. $x = \frac{19+\sqrt{161}}{2}$ or $x = \frac{19-\sqrt{161}}{2}$

Strengthen further students' understanding of solving equations transformable to quadratic equations by doing Activity 7. Ask them to represent each quantity involved in the situation using expression or equation. Let the students describe and solve the equation. Tell them to explain how they arrived at their answer and how they applied the different mathematics concepts or principles to come up with the solutions.

Activity 7: Let's Paint The House!

Answer Key

1. $(m + 5)$ hours
2. $\frac{1}{m}$, $\frac{1}{m+5}$
3. $\frac{1}{m} + \frac{1}{m+5} = \frac{1}{6}$
4. The equation formed is a rational algebraic equation.
5. The solution is 10 hours for Mark and 15 hours for Jessie.

What to REFLECT ON and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of equations transformable to quadratic equations. Provide them with opportunities to think deeply and test further their understanding of solving these kinds of equations by doing Activity 8. Give more focus on the real-life applications of quadratic equations.

Activity 8: My Understanding of Equations Transformable to Quadratic Equations

Answer Key

1. (Evaluate students' responses.)
2. (Evaluate students' responses.)
3. (Evaluate students' responses.)
4. b. $\frac{x^2 - 5x}{x - 5} = 15 - 2x$ has extraneous roots.
5. 20 minutes and 30 minutes.

Provide the students with opportunities to express their understanding of equations transformable to quadratic equations by doing Activity 9. Give attention to the difference between rational algebraic equations and quadratic equations and the methods or procedures in finding their roots including the extraneous roots.

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Activity 9: A Reality of Rational Algebraic Equation

Give the students opportunities to demonstrate their understanding of solving equations transformable to quadratic equations by doing a practical task. Ask them to cite a real-life situation that illustrates a rational algebraic equation transformable to a quadratic equation. Let the students formulate and solve the equation that represents the situation cited.

Summary/Synthesis/Generalization:

This lesson was about the solutions of equations that are transformable to quadratic equations including rational algebraic equations. This lesson provided the students with opportunities to transform equations into the form $ax^2 + bx + c = 0$ and to solve these. Moreover, this lesson provided them with opportunities to solve real-life problems involving rational algebraic equations transformable to quadratic equations. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the succeeding lessons.

Lesson 6: SOLVING PROBLEMS INVOLVING QUADRATIC EQUATIONS

What to KNOW:

Determine students' prior mathematical knowledge and skills that are needed for them to understand the different applications of quadratic equations. As the students go through this lesson, let them think of this important question: *"How are quadratic equations used in solving real-life problems and in making decisions?"*

Start the lesson by asking the students to solve quadratic equations. Tell them to perform Activity 1. Emphasize to the students that solving quadratic equations is a skill that they need for them to solve problems involving this mathematics concept.

Activity 1: Find My Solutions!

Answer Key

1. $x = 0$ or $x = \frac{5}{2}$
2. $t = 0$ or $t = 8$
3. $x = 0$ or $x = -\frac{1}{2}$
4. $r = 2$ or $r = -13$
5. $h = 4$ or $h = 10$
6. $m = -\frac{4}{3}$ or $m = 5$
7. $k = 9$ or $k = -5$
8. $t = 7$ or $t = -\frac{7}{2}$
9. $w = 4$ or $w = -\frac{1}{3}$
10. $u = \frac{3}{2}$ or $u = -\frac{5}{2}$

Another skill that students need to develop is representing a real-life situation by an equation. Let them perform Activity 2. In this activity, the students should be able to understand very well the problem, identify the unknown quantities, and write quadratic equations to represent the relations among these quantities.

Activity 2: Translate into…

Answer Key

1. $l^2 - 45l + 350 = 0$
2. $w^2 - 32w + 240 = 0$
3. $x(2x + 3) = 160$
4. $x(3x + 3) = 126$
5. $\frac{1}{x} + \frac{1}{x - 20} = \frac{1}{90}$

Provide the students with an opportunity to develop their skills in solving problems involving quadratic equations. Ask them to perform Activity 3. In this activity, the students will be presented with a situation involving a quadratic equation. Let the students formulate expressions and equations that would describe the given situation and ask them to describe these.

Challenge the students to solve the equation that would give the required dimensions of the floor. Let them use the different methods of solving quadratic equations already presented.

Activity 3: What Are My Dimensions?

Answer Key

1. x, $(x+5)$
2. $x(x+5)=84$
3. The equation is a quadratic equation that can be written in the form $ax^2+bx+c=0$.
4. Transform the quadratic equation to the form $ax^2+bx+c=0$ .and solve for its roots.
5. Width is 7 m and length is 12 m
6. (Evaluate students' responses.)

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their lesson, solving problems involving quadratic equations. Let the students read and understand some important notes on quadratic equations and their applications to solving real-life problems. Tell them to study carefully the examples given.

What to PROCESS:

In this section, let the students use the mathematical ideas involved in solving problems involving quadratic equations and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to perform Activity 4. In this activity, the students will write quadratic equations that would represent some real-life situations. Let the students compare and discuss their answers. Give them an opportunity to check their errors if there are any.

You can also ask the students to identify first the unknown quantity in each situation and solve for it using the equation formulated.

Activity 4: Let Me Try!

Answer Key

1. a. 3 seconds or 4.5 seconds
   b. No. The discriminant of the resulting equation is negative.

2. a. x, $x(x+36)$
   b. $x(x+36)=5{,}152$
   c. Write the quadratic equation in standard form then solve.
   d. Length = 92 m and Width = 56 m
   e. No. Doubling the length and width results in 4 times the area.

3. a. let w = the width of the pool
   l = the length of the pool
   b. Perimeter: $2l+2w=86$; Area: $lw=450$
   c. (Evaluate students' responses.)
   d. Length = 25 m and Width = 18 m
   e. (Evaluate students' responses.)
   f. (Evaluate students' responses.)

What to REFLECT ON and FURTHER UNDERSTAND:

Let the students think deeply and test further their understanding of the applications of quadratic equations by doing Activity 5. Ask them to solve problems involving quadratic equations with varying conditions.

Activity 5: Find Those Missing!

Answer Key

1. Length = 12 m; Width = 7m
2. 4 m
3. 70 kph and 50 kph
4. Jane: approximately 18 hrs, 15 minutes
   Maria: approximately 14 hrs, 15 minutes
5. 7%

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of the applications of quadratic equations by doing a practical task. Let them perform Activity 6 and Activity 7. You can ask the students to work individually or in a group. In Activity 6, the students will make a design or sketch plan of a table than can be made out of ¾ inch by 4 feet by 8 feet plywood and 2 inches by 3 inches x 8 feet wood. Using the design or sketch plan, they will formulate and solve problems involving quadratic equations.

In Activity 7, the students will cite and role play a real-life situation where the concept of a quadratic equation is applied. They will also formulate and solve problems out of this situation.

Summary/Synthesis/Generalization:

This lesson was about solving real-life problems involving quadratic equations. The lesson provided the students with opportunities to see the real-life applications of quadratic equations. Moreover, they were given opportunities to formulate and solve quadratic equations based on real-life situations. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the succeeding lessons.

Lesson 7: QUADRATIC INEQUALITIES

What to KNOW:

Find out how much students have learned about the different mathematics concepts previously studied and their skills in performing mathematical operations. Checking these will facilitate teaching and students' understanding of Quadratic Inequalities. Tell them that as they go through this lesson, they have to think of this important question: *"How are quadratic inequalities used in solving real-life problems and in making decisions?"*

Provide the students with an opportunity to enhance further their skills in finding solutions to mathematical sentences previously studied. Let them perform Activity1. In this activity, the students will solve linear inequalities in one variable and quadratic equations. These mathematical skills are prerequisites to learning quadratic inequalities.

Ask the students to explain how they arrived at the solution/s and how they applied the mathematics concepts or principles in solving each mathematical sentence. Let them describe and compare those mathematical sentences with only one solution and those with more than one solution.

Activity 1: What Makes Me True?

Answer Key

1. $x > 3$	6. $x = -3$ or $x = -2$
2. $r < 13$	7. $t = 7$ or $t = 1$
3. $s \geq 7$	8. $r = 2$ or $r = -9$
4. $t \leq 5$	9. $h = 4$ or $h = \frac{-3}{2}$
5. $m < 4$	10. $s = \frac{2}{3}$ or $s = -\frac{2}{3}$

Let the students differentiate quadratic equations from other mathematical sentences by performing Activity 2. In this activity, the students should be able to describe quadratic equations and recognize the different inequality symbols being used in mathematical sentences. At this point, the students should realize that there are mathematical sentences that contain polynomials of degree 2 but are not quadratic equations.

Activity 2: Which Are Not Quadratic Equations?

Answer Key

1. $x^2 + 9x + 20 = 0$; $15 - 6h^2 = 10$; $4x^2 - 25 = 0$; and $m^2 = 6m - 7$
2. Quadratic equations are mathematical sentences of degree 2 that can be written in the form $ax^2 + bx + c = 0$.
3. $2s^2 + 7s + 5 > 0$; $2t^2 < 21 - 9t$; $r^2 + 10r \leq -16$; and $3w^2 + 12w \geq 0$
   The mathematical sentences contain inequality symbols.
4. The highest exponent of the variable in each mathematical sentence is 2.
   The mathematical sentences make use of inequality symbols while the quadratic equations make use of equality symbol.

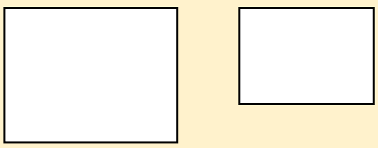
Provide the students with an opportunity to develop their understanding of quadratic inequalities. Ask them to perform Activity 3. In this activity, the students will be presented with a situation involving quadratic inequality. Let the students formulate mathematical sentences that will describe the given situation and ask them to describe these. Challenge them to find the solutions to these mathematical sentences.

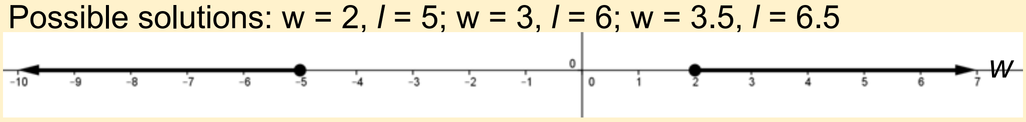
Ask the students to draw and interpret the graph that represents the solution set of one of the mathematical sentences formulated. Let them find out if all solutions that can be obtained from the graph are true to the given situation. Tell them to explain their answer.

Activity 3: Let's Do Gardening!

Answer Key

1. Possible answers:



2. Possible answers: 2 m by 4 m or 1.5 m by 3.5 m
3. Area of the first garden: (2 m)(4 m) = 8 m^2
   Area of the first garden: (1.5 m)(3.5 m) = 5.25 m^2
4. The area of the smallest garden is 3 m^2. This occurs when the length is 3 m and the width is 1 m. There is no theoretical limit to the largest garden. It can be as large as what can fit in Mr. Bayani's vacant lot.
5. $w(w + 2) \geq 3$, where w is the width of each garden.
6. Yes. Look for values of w that would make the mathematical sentence true.
7. $w(w + 3) \geq 10$, where w is the width of each garden.
   Possible solutions: w = 2, *l* = 5; w = 3, *l* = 6; w = 3.5, *l* = 6.5
8. 

9. No. The negative solutions cannot be used since the situation involves measures of length.

Before proceeding to the next activities, let the students give a brief summary of the activities done. Provide them with an opportunity to relate or connect their responses in the activities given to their new lesson, quadratic inequalities. Let the students read and understand some important notes on quadratic inequalities. Tell them to study carefully the examples given.

What to PROCESS:

Let the students use the mathematical ideas they have learned about quadratic inequalities and their solution sets and the examples presented in the preceding section to answer the succeeding activities.

Ask the students to determine whether a mathematical sentence is a quadratic inequality or not. Let them perform Activity 4. Tell them to describe what quadratic inequalities are and how they are different from linear inequalities. Also ask the students to give examples of quadratic inequalities.

Activity 4: Quadratic Inequalities or Not

Answer Key

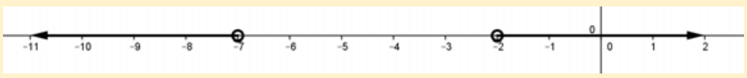
1. Quadratic Inequality
2. Not Quadratic Inequality
3. Quadratic Inequality
4. Quadratic Inequality
5. Not Quadratic Inequality
6. Not Quadratic Inequality
7. Quadratic Inequality
8. Quadratic Inequality
9. Quadratic Inequality
10. Not Quadratic Inequality

Let the students find the solution sets of some quadratic inequalities and graph these. Ask them to explain how they determined the solution set and the mathematics concepts or principles they applied in solving each inequality. Furthermore, let the students show the graph of the solution set of each inequality and ask them to describe this. Challenge the students to determine if it is possible for a quadratic inequality not to have a real solution. Ask them to justify their answer.

Activity 5: Describe My Solutions!

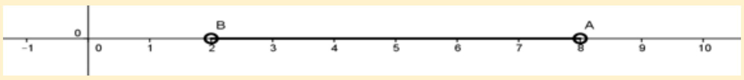
Answer Key

1. $\{x : x < -7 \text{ or } x > -2\}$



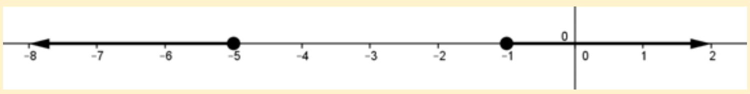
x

2. $\{r : 2 < r < 8\}$



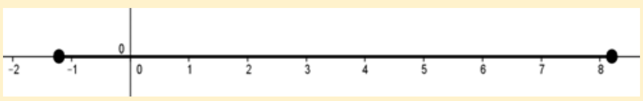
r

3. $\{x : x \leq -5 \text{ or } x \geq -1\}$



x

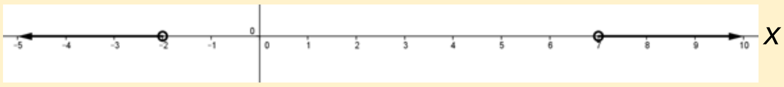
4. $\left\{m : \dfrac{7-\sqrt{89}}{2} \leq m \leq \dfrac{7+\sqrt{89}}{2}\right\}$



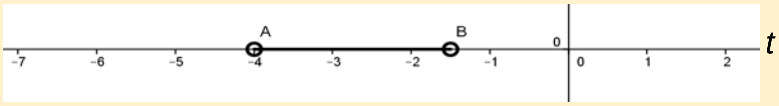
m

Answer Key

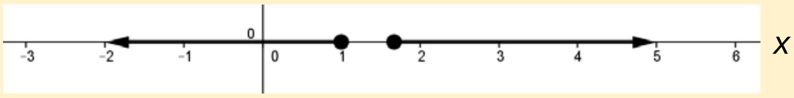
5. $\{x : x < -2 \text{ or } x > 7\}$



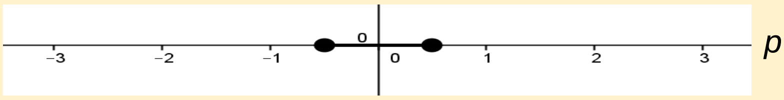
6. $\left\{t : -4 < t < -\dfrac{3}{2}\right\}$



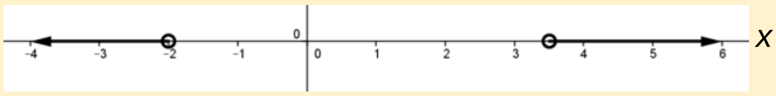
7. $\left\{x : x \leq 1 \text{ or } x \geq \dfrac{5}{3}\right\}$



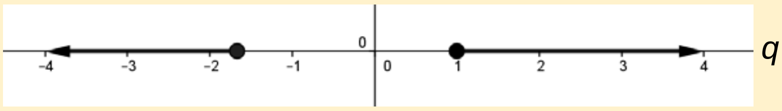
8. $\left\{p : -\dfrac{1}{2} \leq p \leq \dfrac{1}{2}\right\}$



9. $\left\{x : x < -2 \text{ or } x > \dfrac{7}{2}\right\}$



10. $\left\{q : q \leq -\dfrac{5}{3} \text{ or } x \geq 1\right\}$



Let the students determine whether the coordinates of a point are solutions to a given inequality. Tell them to perform Activity 6. Ask them to justify their answer. At this stage, the students should be able to strengthen their understanding of the number of solutions a quadratic inequality has. They should realize that a quadratic inequality has an infinite number of solutions.

Activity 7 is related to Activity 6. In this activity, the students will select from the list of mathematical sentences the inequality that is described by a graph. To do this, the students are expected to select some points on the graph. Then, they will determine if the coordinates of each point make an inequality true. At this point, the students should be able to describe the graphs of quadratic inequalities in two variables involving “less than”, “greater than”, “less than or equal to”, and “greater than or equal to”.

Activity 6: Am I a Solution or Not?

Answer Key

1. Not a Solution
2. Not a Solution
3. A Solution
4. A Solution
5. Not a Solution
6. Not a Solution
7. A Solution
8. A Solution
9. Not a Solution
10. A Solution

Activity 7: What Represents Me?

Answer Key

1. $y \geq -x^2 - 2x + 8$
2. $y < 2x^2 + 7x + 5$
3. $y \leq 2x^2 + 7x + 5$
4. $y > -x^2 - 2x + 8$

Strengthen further students' understanding of quadratic inequalities by doing Activity 8. Ask them to represent a real-life situation by a quadratic inequality. Let them find out if the quadratic inequality formulated can be used to determine the unknown quantities in the given situation.

Activity 8: Make It Real!

Answer Key

1. Width = x and Length = x + 36
2. $x(x+36) < 2040$ or $x^2 + 36x < 2040$
3. Possible Answers: Width = 15 ft. and Length = 51 ft. → Area = 765 ft.^{2}
   Width = 22 ft. and Length = 58 ft. → Area = 1276 ft.^{2}
   Width = 30 ft. and Length = 66 ft. → Area = 1980 ft.^{2}
4. No. It is not realistic to have a conference hall whose width is too narrow.

What to REFLECT ON and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of quadratic inequalities. Provide them with opportunities to think deeply and test further their understanding of quadratic inequalities by doing Activity 9. Give more focus on the real-life applications of quadratic inequalities.

Activity 9: How Well I Understood…

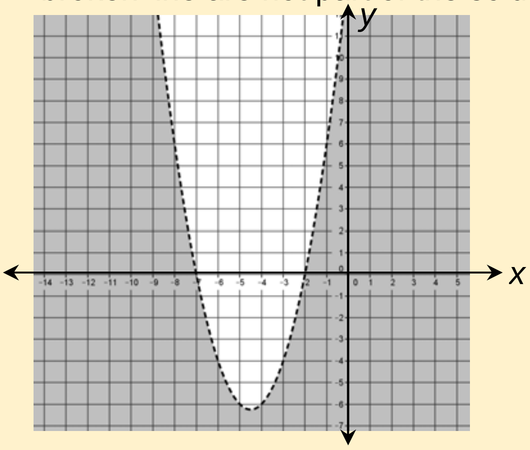
Answer Key

1. Quadratic inequalities are inequalities that contain polynomials of degree 2 and can be written in any of the following forms.

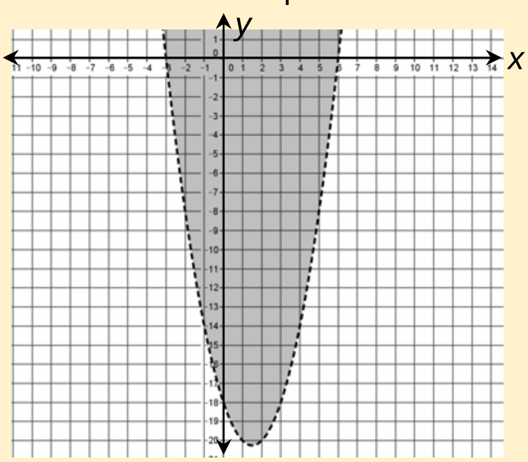
$ax^2 + bx + c > 0$	$ax^2 + bx + c \geq 0$
$ax^2 + bx + c < 0$	$ax^2 + bx + c \leq 0$

where a, b, and c are real numbers and a ≠ 0.

2. (Evaluate students' answers.)
3. (Evaluate students' responses.)
4.
   a. The solution set of $y < x^2 + 9x + 14$ consists of the coordinates of all points on the shaded region of the graph below. All coordinates of points on the broken line are not part of the solution set.

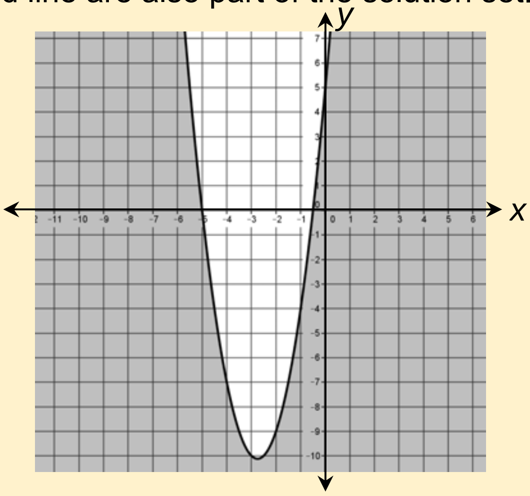


   b. The solution set of $y > x^2 - 3x - 18$ consists of the coordinates of all points on the shaded region of the graph below. All coordinates of points on the broken line are not part of the solution set.

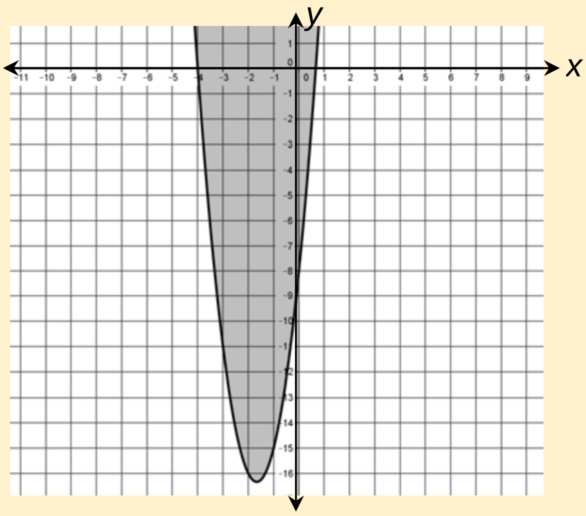


Answer Key

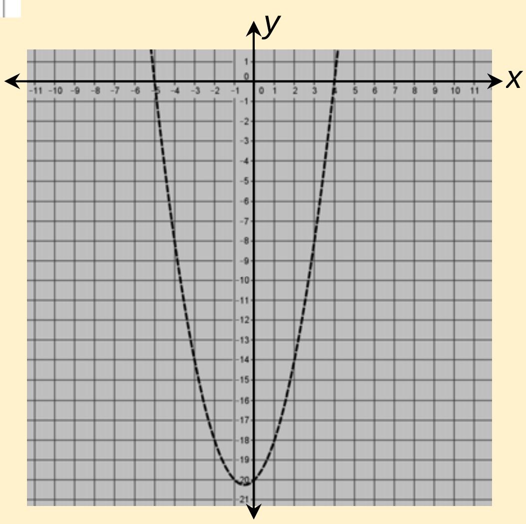
c. The solution set of $y \le 2x^2 + 11x + 5$ consists of the coordinates of all points on the shaded region of the graph below. All coordinates of points on the solid line are also part of the solution set.



d. The solution set of $y \ge 3x^2 + 10x - 8$ consists of the coordinates of all points on the shaded region of the graph below. All coordinates of points on the solid line are also part of the solution set.

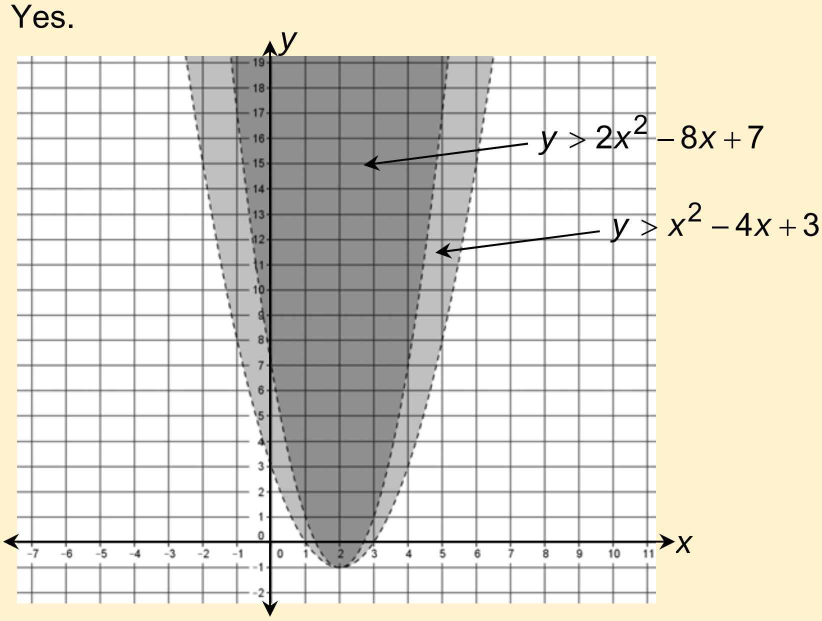


5. No. All coordinates of points on the broken line are not part of the solution set of both inequalities.



Answer Key

6. Yes.



7. Possible dimensions: width = 4 cm, length = 7 cm, and height = 5 cm
width = 3 cm, length = 6 cm, and height = 5 cm

8. a. Width = *w* and Length = *w* + 40

b. $12w(w + 40)$ or $12w^2 + 480w$

$12w^2 + 480w \leq 4800$ or $w^2 + 40w \leq 400$, where w is the width

c. Greatest possible dimensions: width = 8 cm, length = 48 cm, and height = 12 cm

d. $12w^2 + 480w > 4800$ or $w^2 + 40w > 400$

e. Possible dimensions: width = 9 cm, length = 49 cm, and height = 12 cm
width = 12 cm, length = 52 cm, and height = 12 cm

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Refer to the Assessment Map.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of quadratic inequalities by doing a practical task. Let them perform Activity 10 and Activity 11. You can ask the students to work individually or in a group.

In Activity 10, the students will conduct a mathematical investigation on some quadratic inequalities. They will prepare a written report of their investigation following a specified format. In this activity, the students are expected to come up with some conjectures and test these to arrive at a set of conclusions.

For activity 11, the students are expected to formulate quadratic inequalities involving the dimensions of the floors of rooms, and the measures and costs of tiles. They will find then graph the solutions sets of these inequalities.

Summary/Synthesis/Generalization:

This lesson was about quadratic inequalities and their solution sets and graphs. The lesson provided the students with opportunities to describe quadratic inequalities and their solution sets using practical situations, mathematical expressions, and their graphs. Moreover, they were given the opportunity to draw and describe the graphs of quadratic inequalities and to demonstrate their understanding of the lesson by doing a practical task. Their understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next lesson, Quadratic Functions.

POST-ASSESSMENT

Part I

Directions: Find out how much you already know about this module. Choose the letter that you think best answers the question. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. Which of the following is the standard form of quadratic equations?

   A. $ax^2 + bx + c < 0, a \neq 0$ C. $Ax + By + C = 0$

   B. $ax^2 + bx + c = 0, a \neq 0$ D. $y = mx + b$

2. Which of the following is a quadratic equation?

   A. $3m - 7 = 12$ C. $2x^2 - 7x \geq 3$

   B. $-5n^2 + 4n - 1$ D. $t^2 + 5t - 14 = 0$

3. In the quadratic equation $5w^2 + 9w - 10 = 0$, which is the quadratic term?

   A. w^2 B. $9w$ C. $5w^2$ D. -10

4. Which of the following rational algebraic equations is transformable to a quadratic equation?

   A. $\frac{7}{2} + \frac{5}{s+2} = \frac{7}{8s}$ C. $\frac{2t-1}{5} + \frac{2}{3} = \frac{3t}{4}$

   B. $\frac{1}{m} + \frac{5}{m+1} = 6m$ D. $\frac{w+1}{2} - \frac{w+2}{4} = 7$

5. How many real roots does the quadratic equation $3x^2 + 7x + 10 = 0$ have?

   A. 0   B. 1   C. 2   D. 3

6. The roots of a quadratic equation are -4 and -5. Which of the following quadratic equations has these roots?

   A. $x^2 + x - 20 = 0$ C. $x^2 + 9x + 20 = 0$

   B. $x^2 - x + 20 = 0$ D. $x^2 - 9x + 20 = 0$

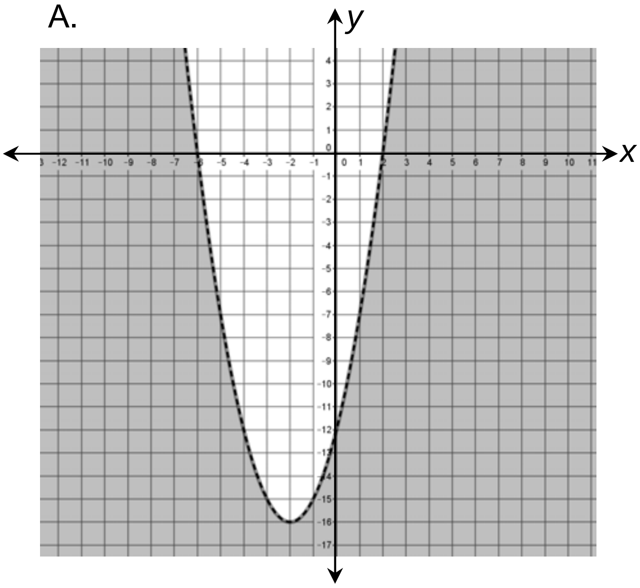
7. Which of the following mathematical statements is a quadratic inequality?

   A. $5t^2 - r - 20 = 0$ C. $4x^2 + 2x - 2 \geq 0$

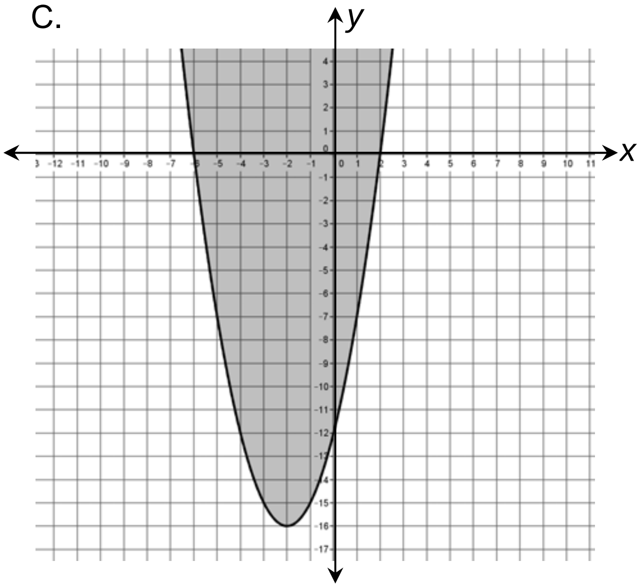
   B. $9h + 18 < 0$ D. $m^2 + 9m + 16 = 0$

8. Which of the following shows the graph of $y \le x^2 + 4x - 12$?

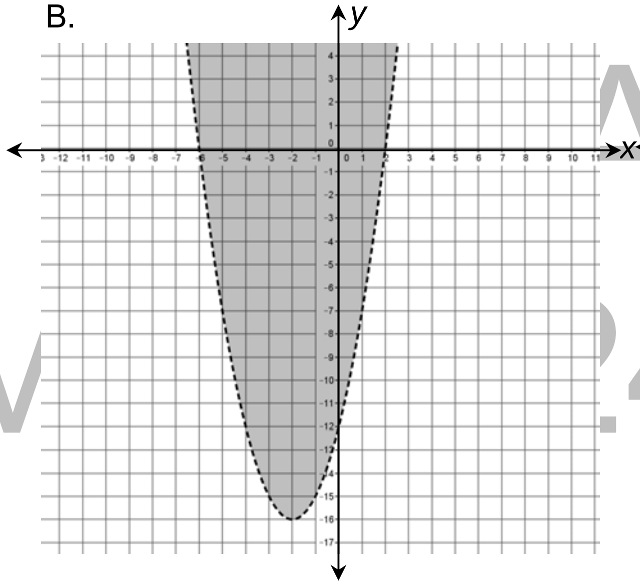
A.



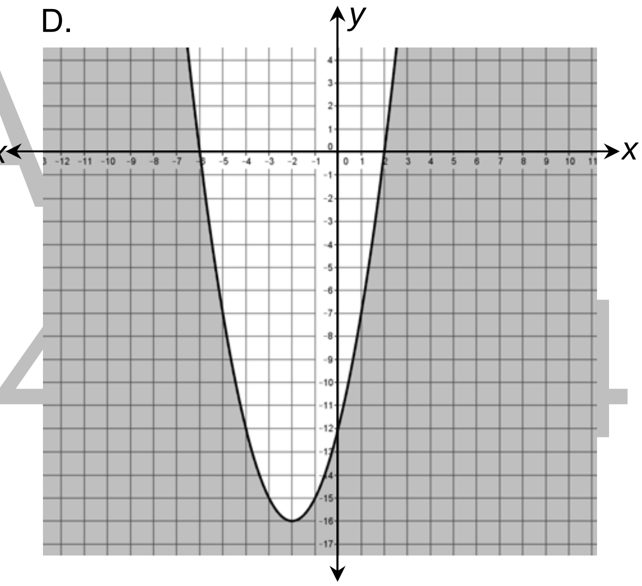
C.



B.



D.



9. Which of the following values of x make the equation $x^2 + x - 20 = 0$ true?

I. -5 II. 4 III. 5

A. I and II B. II and III C. I and III D. I, II, and III

10. Which of the following quadratic equations has no real roots?

A. $6m^2 + 4m = 3$

B. $t^2 - 5t - 9 = 0$

C. $2s^2 - 4s = -4$

D. $4r^2 + 2r - 5 = 0$

11. What is the nature of the roots of the quadratic equation if the value of its discriminant is negative?
    A. The roots are not real.
    B. The roots are irrational and not equal.
    C. The roots are rational and not equal.
    D. The roots are rational and equal.

12. One of the roots of $2x^2 - 3x - 9 = 0$ is 3. What is the other root?
    A. $\frac{3}{2}$ B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. $-\frac{3}{2}$

13. What are the roots of the quadratic equation $x^2 - 3x - 10 = 0$?
    A. 2 and 5 B. -2 and 5 C. -10 and 1 D. 10 and -1

14. What is the sum of the roots of the quadratic equation $3x^2 + 15x - 21 = 0$?
    A. 15 B. 5 C. -5 D. -7

15. Which of the following quadratic equations can be solved easily by extracting square roots?
    A. $6x^2 - 9x + 12 = 0$ C. $4m^2 - 64 = 0$
    B. $5n^2 + 7n - 51 = 0$ D. $v^2 - 3v - 10 = 0$

16. Which of the following coordinates of points belong to the solution set of the inequality $y < 3x^2 + 3x - 6$?
    A. (-1,2) B. (1,-2) C. (-2,7) D. (-1,-1)

17. A 5 cm by 5 cm square piece of cardboard was cut from a bigger square cardboard. The area of the remaining cardboard was 60 cm^2. If *s* represents the length of the bigger cardboard, which of the following expressions gives the area of the remaining piece?
    A. $s - 25$ B. $s^2 - 25$ C. $s^2 + 25$ D. $s^2 + 60$

18. The length of a wall is 17 m more than its width. If the area of the wall is less than 60 m^2, which of the following could be its length?
    A. 3 m B. 16 m C. 18 m D. 20 m

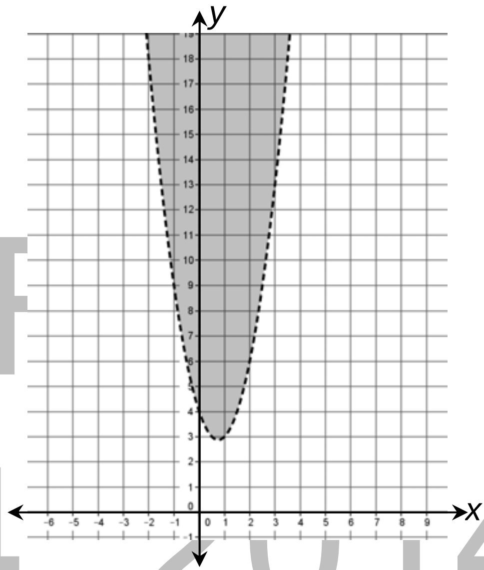
19. The length of a garden is 5 m longer than its width and the area is 36 m^2. How long is the garden?
    A. 4 m B. 5 m C. 9 m D. 13 m

20. A car travels 30 kph faster than a truck. The car covers 540 km in three hours less than the time it takes the truck to travel the same distance. How fast does the car travel?
    A. 60 kph B. 90 kph C. 120 kph D. 150 kph

21. A 10 cm by 17 cm picture is mounted with border of uniform width on a rectangular frame. If the total area of the border is 198 cm^2, what is the length of side of the frame?
A. 23 cm B. 20 cm C. 16 cm D. 13 cm

22. Louise G Electronics Company would like to come up with an LED TV such that its screen is 1200 square inches larger than the present ones. Suppose the length of the screen of the larger TV is 8 inches longer than its width and the area of the smaller TV is 720 square inches. What is the length of the screen of the larger LED TV?
A. 24 in. B. 32 in C. 40 in. D. 48 in.

23. The figure on the right shows the graph of $y > 2x^2 - 3x + 4$. Which of the following is true about the solution set of the inequality?
I. The coordinates of all points on the shaded region belong to the solution set of the inequality.
II. The coordinates of all points along the parabola as shown by the broken line do not belong to the solution set of the inequality.
III. The coordinates of all points along the parabola as shown by the broken line belong to the solution set of the inequality.



A. I and II B. I and III C. II and III D. I, II, and III

24. It takes Darcy 6 days more to paint a house than Jimboy. If they work together, they can finish the same job in 4 days. How long would it take Darcy to finish the job alone?
A. 3 days B. 4 days C. 10 days D. 12 days

25. An open box is to be formed out of a rectangular piece of cardboard whose length is 16 cm longer than its width. To form the box, a square of side 5 cm will be removed from each corner of the cardboard. Then the edges of the remaining cardboard will be turned up. If the box is to hold at most 2,100 cm^3, what mathematical statement would represent the given situation?
A. $w^2 + 4w \leq 480$ C. $w^2 + 4w \geq 420$
B. $w^2 - 4w \leq 420$ D. $w^2 - 4w \leq 480$

26. The length of a garden is 4 m more than twice its width and its area is 38 m^2. Which of the following equations represents the given situation?

A. $2x^2 + 4x = 19$

B. $x^2 + 2x = 19$

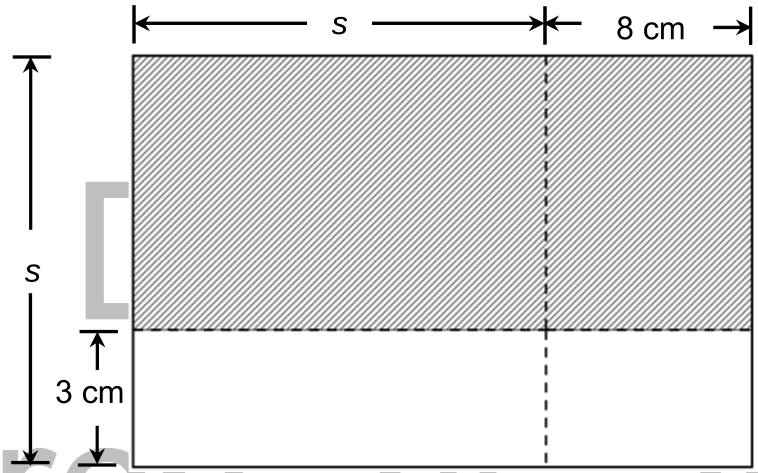
C. $x^2 + x = 19$

D. $x^2 + 2x = 38$

27. From 2004 through 2012, the average weekly income of an employee in a certain company is estimated by the quadratic expression $0.15n^2 + 3.35n + 2220$, where n is the number of years after 2004. In what year was the average weekly income of an employee equal to Php2,231.40?

A. 2004 B. 2005 C. 2006 D. 2007

28. In the figure below, the area of the shaded region is 312 cm^2. What is the length of the longer side of the figure?



A. 8 cm B. 13 cm C. 24 cm D. 37 cm

Part II

Directions: Read and understand the situation below then answer or perform what are asked.

A new shopping complex will be built in a 10-hectare real estate. The shopping complex shall consist of the following:

a. department store;
b. grocery store;
c. cyber shops (stores that sells computers, cellular phones, tablets, and other gadgets and accessories);
d. hardware store including housewares;
e. restaurants, food court or fast food chains;
f. bookstore;
g. car park;
h. children's playground;
i. game zone;
j. appliance center; and
k. other establishments

The company that will put up the shopping complex asked a group of architects to prepare the ground plan for the different establishments. Aside from the buildings to be built, the ground plan must also include pathways and roads.

1. Suppose you are one of those architects assigned to do the ground plan. How would you prepare the plan?

2. Prepare a ground plan to illustrate the proposed shopping complex.

3. Using the ground plan prepared, determine all the mathematics concepts involved.

4. Formulate problems involving these concepts particularly quadratic equations and inequalities.

5. Write the quadratic equations and inequalities that describe the situations or problems.

6. Solve the problems, equations, and inequalities formulated.

Rubric for Design

4	3	2	1
The design is accurately made, presentable, and appropriate.	The design is accurately made and appropriate.	The design is not accurately made but appropriate.	The design is made but not appropriate.

Rubric for Equations and Inequalities Formulated and Solved

4	3	2	1
Equations and inequalities are properly formulated and solved correctly.	Equations and inequalities are properly formulated but not all are solved correctly.	Equations and inequalities are properly formulated but are not solved correctly.	Equations and inequalities are properly formulated but are not solved.

Rubric on Problems Formulated and Solved

Score	Descriptors
6	Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate.
5	Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
4	Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes.
3	Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details.

2	Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension.
1	Poses a problem but demonstrates minor comprehension, not being able to develop an approach.

Source: D.O. #73 s. 2012

Answer Key

Part I

1. B
2. D
3. C
4. A
5. A
6. C
7. C
8. D
9. A
10. C
11. A
12. D
13. B
14. C
15. C
16. B
17. B
18. C
19. C
20. B
21. A
22. D
23. A
24. D
25. D
26. B
27. D
28. C

Part II (Use the rubric to rate students' works/outputs)

GLOSSARY OF TERMS

1. Discriminant – This is the value of the expression $b^2 - 4ac$ in the quadratic formula.

2. Extraneous Root or Solution – This is a solution of an equation derived from an original equation. However, it is not a solution of the original equation.

3. Irrational Roots – These are roots of equations which cannot be expressed as quotient of integers.

4. Quadratic Equations – These are mathematical sentences of degree 2 that can be written in the form $ax^2 + bx + c = 0$.

5. Quadratic Formula – This is an equation that can be used to find the roots or solutions of the quadratic equation $ax^2 + bx + c = 0$. The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

6. Quadratic Inequalities – These are mathematical sentences that can be written in any of the following forms: $ax^2 + bx + c > 0$, $ax^2 + bx + c < 0$, $ax^2 + bx + c \geq 0$, and $ax^2 + bx + c \leq 0$.

7. Rational Algebraic Equations – These are mathematical sentences that contain rational algebraic expressions.

8. Rational Roots – These are roots of equations which can be expressed as quotient of integers.

9. Solutions or Roots of Quadratic Equations – These are the values of the variable/s that make quadratic equations true.

10. Solutions or Roots of Quadratic Inequalities – These are the values of the variable/s that make quadratic inequalities true.

DEPED INSTRUCTIONAL MATERIALS THAT CAN BE USED AS ADDITIONAL RESOURCES FOR THE LESSON: QUADRATIC EQUATIONS AND INEQUALITIES

1. EASE Modules Year II Modules 1, 2 and 3

2. BASIC EDUCATION ASSISTANCE FOR MINDANAO (BEAM) Mathematics 8 Module 4 pp. 1-55

REFERENCES AND WEBSITE LINKS USED IN THIS MODULE:

References:

Bellman, Allan E., et. Al. Algebra 2 Prentice Hall Mathematics, Pearson Prentice Hall, New Jersey USA, 2004.

Brown, Richard G., Mary P. Dolciani, Robert H. Sorgenfrey and William L. Cole. Algebra, Structure and Method, Book I, Houghton Mifflin Company, Boston MA, 1990.

Brown, Richard G., Mary P. Dolciani, Robert H. Sorgenfrey, and Robert B. Kane. Algebra, Structure and Method Book 2. Houghton Mifflin Company, Boston, 1990.

Chapin, Illingworth, Landau, Masingila and McCracken. Prentice Hall Middle Grades Math, Tools for Success, Prentice-Hall, Inc., Upper Saddle River, New Jersey, 1997.

Clements, Douglas H., Kenneth W. Jones, Lois Gordon Moseley and Linda Schulman. Math in my World, McGraw-Hill Division, Farmington, New York, 1999.

Coxford, Arthur F. and Joseph N. Payne. HBJ Algebra I, Second Edition, Harcourt Brace Jovanovich, Publishers, Orlando, Florida, 1990.

Fair, Jan and Sadie C. Bragg. Prentice Hall Algebra I, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1991.

Gantert, Ann Xavier. Algebra 2 and Trigonometry. AMSCO School Publications, Inc., 2009.

Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. Algebra 1, Applications, Equations, and Graphs. McDougal Littell, A Houghton Mifflin Company, Illinois, 2004.

Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. Algebra 2, Applications, Equations, and Graphs. McDougal Littell, A Houghton Mifflin Company, Illinois, 2008.

Smith, Charles, Dossey, Keedy and Bettinger. Addison-Wesley Algebra, Addison-Wesley Publishing Company, 1992.

Wesner, Terry H. and Harry L. Nustad. Elementary Algebra with Applications. Wm. C. Brown Publishers. IA, USA.

Wilson, Patricia S., et. al. Mathematics, Applications and Connections, Course I, Glencoe Division of Macmillan/McGraw-Hill Publishing Company, Westerville, Ohio, 1993.

WEBSITE Links as References and for Learning Activities:

Algebra.help. Solve by Using the Quadratic Equation Lessons. (2011). Retrieved November 7, 2013, from http://www.algebrahelp.com/lessons/equations/quadratic/

Algebra II: Quadratic Equations. (2011). Retrieved November 7, 2013, from http://library.thinkquest.org/20991/alg2/quad.html

At Home Tuition. Quadratic Equations. (2010). Retrieved November 7, 2013, from http://www.athometuition.com/QuadraticEquationsFormula.php

Beginning Algebra (v. 1.0) Section 6.6 Solving Equations by Factoring. (2012). Retrieved November 7, 2013, from http://2012books.lardbucket.org/books/beginning-algebra/s09-06-solving-equations-by-factoring.html

Beginning Algebra (v. 1.0) Section 9.1 Extracting Square Roots. (2012). Retrieved November 7, 2013, from http://2012books.lardbucket.org/books/beginning-algebra/s12-01-extracting-square-roots.html

Beginning Algebra (v. 1.0) Section 9.2 Completing the Square. (2012). Retrieved November 7, 2013, from http://2012books.lardbucket.org/books/beginning-algebra/s12-02-completing-the-square.html

Beginning Algebra (v. 1.0) Section 9.3 Quadratic Formula. (2012). Retrieved November 7, 2013, from http://2012books.lardbucket.org/books/beginning-algebra/s12-03-quadratic-formula.html

Dawkins, Paul. Paul's Online Math Notes. Quadratic Equations: A Summary. (2013). Retrieved November 7, 2013, from http://tutorial.math.lamar.edu/Classes/Alg/QuadraticApps.aspx

Dendane, Abdelkader. Solutions to Quadratic Equations with Rational Expressions. (2012). Retrieved November 7, 2013, from http://www.analyzemath.com/Algebra2/algebra2_solutions.html

Dendane, Abdelkader. Solve Quadratic Equations Using Discriminant. (2012). Retrieved November 7, 2013, from http://www.analyzemath.com/Equations/Quadratic-1.html

Dendane, Abdelkader. Solve Quadratic Equations with Rational Expressions-Problems. (2012). Retrieved November 7, 2013, from http://www.analyzemath.com/Algebra2/Algebra2.html

Discovery Education WEBMath. Solve a Quadratic Equation by Factoring. (2013). Retrieved November 7, 2013, from http://www.webmath.com/quadtri.html

High Points Learning Inc. iCoachMath.Com. Discriminant. (2013). Retrieved November 7, 2013, from http://www.icoachmath.com/math_dictionary/discriminant.html

Houghton Mifflin Harcourt Cliffs Notes. Solving Quadratic Inequalities. (2013). Retrieved November 7, 2013, from http://www.cliffsnotes.com/math/algebra/algebra-ii/quadratics-in-one-variable/solving-quadratic-inequalities

Lesson Using quadratic equations to solve word problems. Retrieved November 7, 2013, from http://www.algebra.com/algebra/homework/quadratic/lessons/Using-quadratic-equations-to-solve-word-problems.lesson

Math-Help-Ace.com.Quadratic Equation Solver - Finding The Quadratic Equation With Given Roots, Examples, Exercise. (2008). Retrieved November 7, 2013, from http://www.math-help-ace.com/Quadratic-Equation-Solver.html

Math is Fun Completing the Square. (2012). Retrieved November 7, 2013, from http://www.mathsisfun.com/algebra/completing-square.html

Math is Fun Real World Examples of Quadratic Equations. (2012). Retrieved November 7, 2013, from http://www.mathsisfun.com/algebra/quadratic-equation-real-world.html

Math is Fun Solving Quadratic Inequalities. (2012). Retrieved November 7, 2013, from http://www.mathsisfun.com/algebra/inequality-quadratic-solving.html

Math Warehouse How to Solve a Quadratic Inequality. Retrieved November 7, 2013, from http://www.mathwarehouse.com/quadratic-inequality/how-to-solve-and-graph-quadratic-inequality.php

Math Warehouse Solve a Quadratic Equation by Factoring. Retrieved November 7, 2013, from http://www.mathwarehouse.com/quadratic/solve-quadratic-equation-by-factoring.php

NySphere International, Inc. Algebra: Quadratic Equations Word Problems. (2013). Retrieved November 7, 2013, from http://www.tulyn.com/algebra/quadratic-equations/wordproblems

Oswego City School District Regents exam Prep Center. Algebra2 Trig. Completing the Square. (2012). Retrieved November 7, 2013, from http://www.regentsprep.org/Regents/math/algtrig/ATE12/indexATE12.htm

Oswego City School District Regents exam Prep Center. Algebra2 Trig. Discriminant. (2012). Retrieved November 7, 2013, from http://www.regentsprep.org/Regents/math/algtrig/ATE3/discriminant.htm

Oswego City School District Regents exam Prep Center. Algebra2 Trig. Nature of Roots (Sum and Product). (2012). Retrieved November 7, 2013, from http://www.regentsprep.org/Regents/math/algtrig/ATE4/natureofroots.htm

Oswego City School District Regents exam Prep Center. Algebra2 Trig. Quadratic Inequalities. (2012). Retrieved November 7, 2013, from http://www.regentsprep.org/regents/math/algtrig/ate6/quadinequal.htm

Oswego City School District Regents exam Prep Center. Algebra2 Trig. Solving Quadratic Equations with the Quadratic Formula. (2012). Retrieved November 7, 2013, from http://www.regentsprep.org/Regents/math/algtrig/ATE3/quadformula.htm

Pindling.org. Math by Examples. College Algebra. "Chapter 1.4 Applications of Quadratic Equations". (2006). Retrieved November 7, 2013, from http://www.pindling.org/Math/CA/By_Examples/1_4_Appls_Quadratic/1_4_Appls_Quadratic.html

Quadratic Equations Lesson. Retrieved November 7, 2013, from http://www.algebra.com/algebra/homework/quadratic/lessons/quadform/

Seward, Kim. WTAMU and Kim Seward Virtual Math Lab College Algebra Tutorial 23A. Quadratic Inequalities. (2010). Retrieved November 7, 2013, from http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut23_quadineq.htm

She Loves Math Quadratic Inequalities. (2013). Retrieved November 7, 2013, from http://www.shelovesmath.com/algebra/intermediate-algebra/quadratic-inequalities/

Slideshare Inc. Quadratic Equation Word Problems. (2013). Retrieved November 7, 2013, from http://www.slideshare.net/jchartiersjsd/quadratic-equation-word-problems

Solving Quadratic Equations by Using the Square Root Property. Retrieved November 7, 2013, from http://www.personal.kent.edu/~bosikiew/Algebra-handouts/solving-sqroot.pdf

The Purple Math Forums Completing the Square Solving Quadratic Equations. (2012). Retrieved November 7, 2013, from http://www.purplemath.com/modules/sqrquad.htm

The Purple Math Forums. Quadratic World Problems: Projectile Motion. (2012). Retrieved November 7, 2013, from http://www.purplemath.com/modules/quadprob.htm

The Purple Math Forums Solving Quadratic Equations by Factoring. (2012). Retrieved November 7, 2013, from http://www.purplemath.com/modules/solvquad.htm

The Purple Math Forums Solving Quadratic Equations by Taking Square Roots. (2012). Retrieved November 7, 2013, from http://www.purplemath.com/modules/solvquad2.htm

The Purple Math Forums. The Quadratic Formula Explained. (2012). Retrieved November 7, 2013, from http://www.purplemath.com/modules/quadform.htm

TutorVista.com. (2013). Algebra 1 Quadratic Equation. Retrieved November 7, 2013, from http://math.tutorvista.com/algebra/quadratic-equation.html

uwlax.edu. Retrieved November 7, 2013, from http://www.uwlax.edu/faculty/hasenbank/archived/mth126fa08/notes/11.10%20-%20Quadratic%20Applications.pdf

WEBSITE Link for Videos:

easymathk12. Retrieved November 7, 2013, from http://www.youtube.com/watch?v=I7FI4T19uIA

Math Vids. Solving Rational Equation to Quadratic 1. (2013). Retrieved November 7, 2013, from http://mathvids.com/lesson/mathhelp/1437-solving-rational-equation-to-quadratic-1

WEBSITE Links for Images:

Smith, Cameron. Kentucky girls basketball star scores all of her team's first 40 points in win. (2012). Retrieved November 7, 2013, from http://sports.yahoo.com/blogs/highschool-prep-rally/kentucky-girls-basketball-star-scores-her-team-first-181307906.html

TEACHING GUIDE

Module 2: Quadratic Functions

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of quadratic functions.

Performance Standard:

The learner is able to investigate thoroughly the mathematical relationship in various situations, formulate real-life problems involving quadratic functions and solve them using a variety of strategies.

UNPACKING OF STANDARDS FOR UNDERSTANDING

<table>
<tr>
<td rowspan="3">SUBJECT:
Math Grade 9
QUARTER
First Quarter
TOPIC:
Quadratic functions
LESSONS:
Introduction to Quadratic Functions
Graphs of Quadratic Functions
Finding the Equation of Quadratic Function
Applications of Quadratic Functions

WRITER:
Leonides E. Bulalayao</td>
<td colspan="2">LEARNING COMPETENCIES</td>
</tr>
<tr>
<td colspan="2">1. Model real-life situation using quadratic functions.
2. Represent a quadratic function using: a) table of values, b) graph, and c) equation.
3. Transform the quadratic function in general form $y = ax^2 + bx + c$ into standard form (vertex form) $y = a(x - h)^2 + k$ and vice versa.
4. Graph a quadratic function and determine the following: a) domain, b) range, c) intercepts, d) axis of symmetry e) vertex, f) direction of the opening of the parabola.
5. Analyze the effects of changing the values of a, h, and k of a quadratic function its graph.
6. Determine the equations of a quadratic function given: a) a table of values, b) graph, and c) zeros.
7. Solve problems involving quadratic functions.</td>
</tr>
<tr>
<td>ESSENTIAL UNDERSTANDING:
Students will understand that quadratic functions are useful tools in solving real-life problems and in making decisions</td>
<td>ESSENTIAL QUESTION
How do quadratic functions facilitate finding solutions on real-life problems and in making decisions?</td>
</tr>
<tr>
<td></td>
<td colspan="2">TRANSFER GOALS:
The students will be able to apply the key concepts of quadratic functions in formulating and solving real-life problems and in making decisions.</td>
</tr>
</table>

B. **Planning for Assessment**

Product/Performance

The following are products and performances that students are expected to come up within this module.

a) Quadratic functions drawn from real-life situations
b) Objects or situations in real life where quadratic functions are illustrated.
c) Quadratic functions that represent real life situations or objects.
d) Conduct Mathematical investigations on the transformation of the graph of a quadratic function.
e) Make a combination notes on recognizing the quadratic function and in transforming quadratic function from standard forms to general form and vice versa.
f) Construct a principle pattern organizer on the concepts of deriving equations of quadratic functions.
g) Design a restaurant interior that demonstrates students' understanding of quadratic functions.
h) Make a presentation of world's famous parabolic arches that shows the relevance of mathematics through the parabolic curve.
i) Find the equations of world's parabolic bridges.
j) Make a fund raising project that demonstrates students' understanding of a quadratic function.
k) Make a parabolic model of bridges that shows students' evidence of learning of quadratic function.

ASSESSMENT MAP

TYPE	KNOWLEDGE	PROCESS/ SKILLS	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	**Pre-Test: Part I** Identifying quadratic functions given the equations. Identifying zeros of quadratic function from the graph.	**Pre-Test: Part I** Transforming the quadratic function from general form into standard form. Describing the transformation of the graph of the quadratic function.	**Pre-Test: Part I** Finding the equation of the quadratic function given the different conditions in relation to its zeros. Solving problems involving quadratic functions	
			Analyzing the graph that corresponds to the given equation	
	Part II	Part II	Part II	Part II

	Identifying the information given in a problem	Representing the situation using a mathematical equation.	Explaining how a mathematical statement is derived from a given situation	Formulating and solving problems involving quadratic functions
Formative	**Quiz: Lesson 1** Distinguishing quadratic functions from linear and other functions	**Quiz: Lesson 1** Transforming the quadratic function in general form into standard form (vertex form) .	**Quiz: Lesson 1** Representing situations using quadratic functions.	Formulating and solving problems involving quadratic functions
	Quiz: Lesson 2 Giving the properties of the graph of a quadratic function	**Quiz: Lesson 2** Graphing a quadratic function and determining its characteristic. Matching the graph of a quadratic function and its equation.	**Quiz: Lesson 2** Analyzing and explaining the effects of changes in the variables a, h, and k in the graph of a quadratic function in standard form. Analyzing problems involving a quadratic function and solve it through graphing. Analyzing the effects of variables a, b and c in the graph of a quadratic function in general form. Justifying the best option from two given situations involving quadratic functions	Designing a curtain in a restaurant that involves quadratic curves
	Quiz: Lesson 3 Identifying the zeros of a quadratic function.	**Quiz: Lesson 3** Finding the equation of a quadratic function from table of values, graph, and zeros.	**Quiz: Lesson 3** Analyzing the pattern and formulate the equation of a quadratic function. Solving problems involving a quadratic functions Making and justifying the	Conducting a research on parabolic bridges and analyzing the given data to determine the equation of the quadratic functions.

			best decision based on the solved problems involving quadratic function	
	Quiz: Lesson 4 Identifying the given information in the problem	**Quiz: Lesson 4** Representing the given situation by *a* mathematical expression/ statement	**Quiz: Lesson 4** Solving problems involving quadratic functions. Analyzing patterns and determining the equation of the quadratic functions.	Proposing a well-planned fundraising activity.
Summative	**Post-Test: Part I** Identifying quadratic functions and their graphs.	**Post-Test: Part I** Finding the maximum point of a quadratic function. Finding the equation of a quadratic function. Describing the transformation of the graph of the quadratic function	**Post-Test: Part I** Solving problems involving quadratic functions Analyzing the graph that correspond to the given equation	**Post-Test: Part I** Products and performances related to or involving systems of linear equations and inequalities in two variables
	Part II Conducting bridge simulations.	**Part II** Constructing a scale model	**Part II** Explaining how a concepts of a quadratic function be used in constructing a scale model. Determining the equation of the scale model. Analyzing the situations while creating a scale model.	Bridge project designs and a scale model. Write-up and presentation
Self-Assessment	**Journal Writing:** Reflections on what they have learned for each session. Combination notes on key concepts of quadratic functions.			

Assessment Matrix (Summative Test)

<table>
<tr><th>Levels of Assessment</th><th>What will I assess?</th><th>How will I assess?</th><th>How Will I Score?</th></tr>
<tr><td>Knowledge 15%</td><td rowspan="4">The learner demonstrates understanding of key concepts of quadratic functions.

1. Model real-life situations using quadratic functions.

2. Represents a quadratic function using: a) table of values, b) graph, and c) equation.

3. Transforms the quadratic function defines by $y = ax^2 + bx + c$ into the form $y = a(x- h)^2 + k$.

4. Graphs a quadratic function and determine s the following: a) domain, b) range, c) intercepts, d) axis of symmetry e) vertex, f) direction of the opening of the parabola.

5. Analyzes the effects of changing the values of a, h, and k of a quadratic function in its graph.

6. Determines the equation of a quadratic function given: a) a table of values, b) graph, and c) zeros.

7. Solves problems involving quadratic functions.</td><td>Paper and Pencil Test

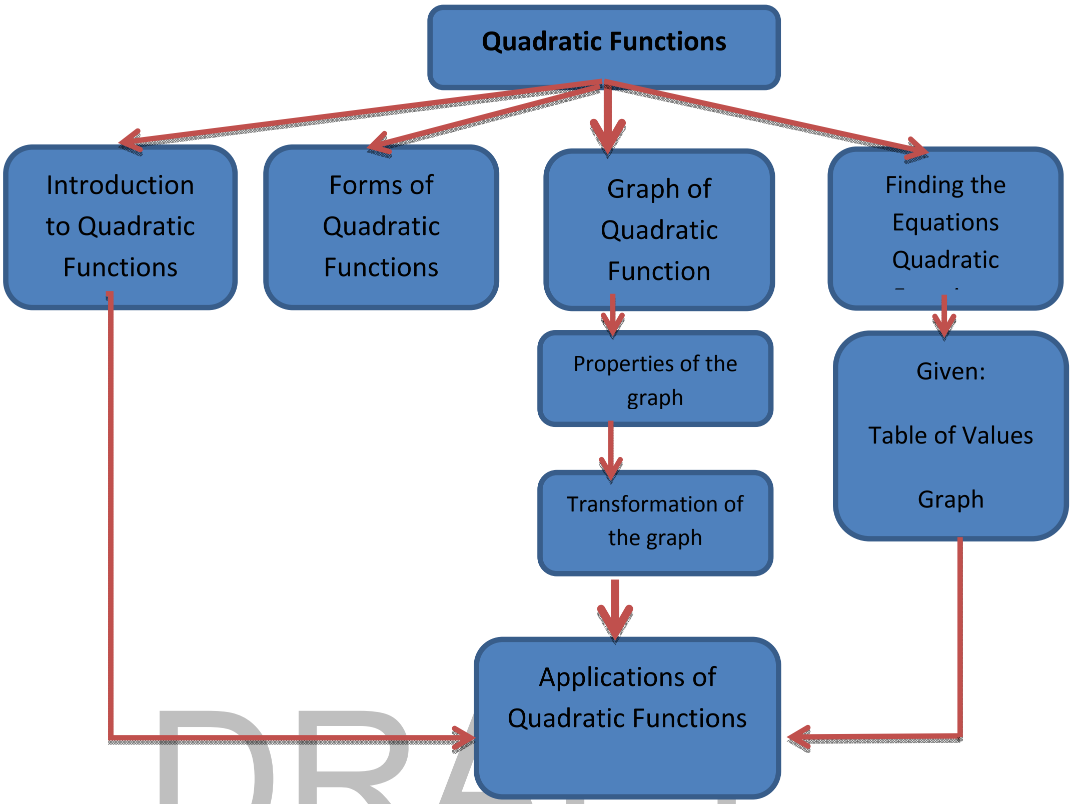
Part I items 1, 2, and5</td><td>1 point for every correct response</td></tr>
<tr><td>Process/Skills 25%</td><td>Part I items 3, 4, 6, 7, and 8.</td><td>1 point for every correct response</td></tr>
<tr><td>Understanding 30%</td><td>Part I items 9, 10, 11, 12, 13, and 14.</td><td>1 point for every correct response</td></tr>
<tr><td>Product/ Performance 30%</td><td>Part II

Bridge Project

Bridge designs Scale model.

Write up and presentation</td><td>Rubric</td></tr>
</table>

Module Map



Introduction:

This module covers key concepts of quadratic functions. It is divided into four lessons namely: Introduction to Quadratic Functions, Graphs of Quadratic Functions, Finding the Equation of Quadratic Function and Applications of Quadratic Functions.

In Lesson 1 of this module, the students will model real-life problems using quadratic functions. They will differentiate quadratic function from linear or other functions. They will transform a quadratic function in general form into standard form and vice versa and lastly, they will illustrate some real-life situations that model quadratic functions.

Lesson 2 is about graphs of quadratic functions. In this lesson, the students will be able to determine the domain, range, intercepts, axis of symmetry and the opening of the parabola. They will be given the opportunity to investigate and analyze the transformation of the graph of a quadratic function. The students will also be given the opportunity to use any graphing materials, or any graphing softwares like Graphcalc, GeoGebra, Wingeom, and the like in mathematical investigation activities. Lastly, the students will apply the mathematical concepts they learned in solving real-life problems.

In Lesson 3, the students will learn to determine the zeros of a quadratic function in different methods. Illustrative examples will be presented. They will also learn to derive the equation of a quadratic function given a table of values, graphs, and zeros. Furthermore, they will be given a chance to apply the concepts learned in real-life problems and also to formulate their own real-life problems involving quadratic functions.

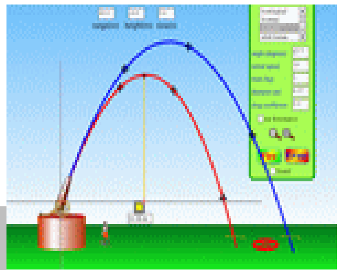
Lesson 4 is about applications of quadratic functions to real life. In this lesson, the students will be familiarized with the most common applications of quadratic function. The students will be able to solve real-life problems involving the quadratic function. They will formulate real-life problems and solve them in a variety of strategies using the concepts of quadratic functions.

To sum up, in all the lessons in this module, the students are given the opportunity to use their prior knowledge and skills in learning quadratic functions. Varied activities are given for the students to determine the mathematical concepts of quadratic functions and to master the knowledge and skills needed to apply in solving problems. Mathematical investigations are also given to develop mathematical thinking skills of the students and to deepen their understanding of the lesson. Activities in formulating real-life problems are also presented in some activities.

As an introduction to the main lesson, show to the students the pictures below then ask them the questions that follow:

http://www.popularmechanics.com/cm/popularmechanics/images/y6/free-throw-0312-mdn.jpg

http://sites.davidson.edu/mathmovement/wp content/uploads/2011/11/trajectories.png



http://web.mnstate.edu/lindaas/phys160/lab/Sims/projectileMotion.gif -throw-0312-mdn.jpg

Have you ever asked yourself why PBA star players are good in free throws? How do angry bird expert players hit their targets? Do you know the secret key in playing this game? What is the maximum height reached by a falling object given a particular condition?

Motivate the students by assuring them that they will be able to answer the above questions and they will learn a lot of applications of the quadratic functions as they go on with the lessons.

Objectives:

After the learners have gone through the lessons contained in this module, they are expected to:

1. model real-life situations using quadratic functions;
2. represent a quadratic function using: a) table of values, b) graph, and c) equation;
3. transform the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x- h)^2 + k$;
4. graph a quadratic function and determine the following: a) domain, b) range, c) intercepts, d) axis of symmetry e) vertex, f) direction of the opening of the parabola;

5. analyze the effects of changing the values of *a*, *h*, and *k* of a quadratic function its graph;
6. determine the equation of a quadratic function given: a) a table of values, b) graph, and c) zeros;
7. Solve problems involving quadratic functions.

Teacher's Note and Reminders
Discuss the purpose of pre-assessment to the students.

Pre-Assessment:

To begin this module, check students' prior knowledge, skills, and understanding of mathematics concepts involving quadratic functions. The results of this assessment will be your basis for planning the learning experiences to be provided for the students.

Answer Key

Part I

1. a	2. b	3. d	4. b	5. a	6. c	7. b
8. d	9. b	10. c	11. a	12. b	13. b	14. a

Part II

Task 1

Task 2

Performance Task of the students might be assessed using this suggested rubric.

Rubrics on Problem Solving

	4	3	2	1
Identifying Relevant Information	Important information needed to solve problem are clearly identified.	Important information from unimportant information are separated.	Sometimes need help in identifying important information in the problem.	Missed the important information in the problem.

Analyzing Problems	All the characteristics of a problem are carefully analyzed.	Some of the characteristics of a problem are carefully analyzed.	Not all the characteristics of a problem are analyzed.	Some of the characteristics of a problem not analyzed.
Content /Key Points	Relationships between variables are clearly presented. Table of values and graphs are shown.	Relationships between variables are presented. Information/ Data needed are determined.	Relationships between variables hardly recognized Information/ Data needed are not enough.	Relationships between variables cannot be recognized. Lack of data/Information.
Mathematical Solution	Calculated the correct answer. Work shown is logical. Calculations are completely correct and answers properly labelled.	Calculated a correct answer Calculations are correct.	Work is partially shown. Minor errors may be evident. Calculations contain minor errors.	Attempted to solve the problem. A limited amount of work shown.

LEARNING GOALS AND TARGETS:

Students are expected to demonstrate an understanding of key concepts of the quadratic function, formulate real-life problems involving the concepts of quadratic functions, and solve these using a variety of strategies. They are also expected to investigate mathematical relationships in various situations involving quadratic functions.

Lesson 1: INTRODUCTION TO QUADRATIC FUNCTIONS

What to KNOW:

Assess students' knowledge by recalling the different mathematics concepts previously learned and their skills in dealing with functions. Assessing these will guide you in planning the teaching and learning activities needed to understand the concepts of a quadratic function. Tell them that as they go through this lesson, they have to think of this important question: *"How are quadratic functions used in solving real-life problems and in making decisions?"*

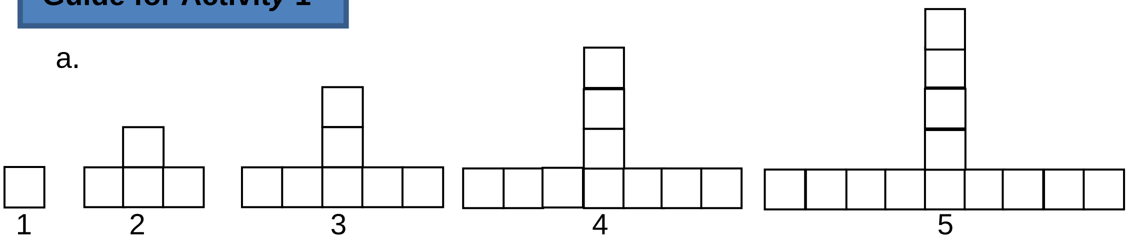
You may start this lesson by giving **Activity 1** and let the students recall the different ways of representing a linear function. Remember that a function can be represented using:

a) table of values
b) ordered pairs
c) graphs
d) equation
e) diagram

Give focus to different ways of representing a function and remind the students that they will be using some of those ways in representing a quadratic function.

Guide for Activity 1

a.



b. Table of Values

Figure number (x)	1	2	3	4	5
Number of blocks (y)	1	4	7	10	13

e. Equation/Pattern $y = 3(x - 1) + 1 \Rightarrow y = 3x - 2$

f. The graph of the equation consists of ordered pairs of b.

Before giving **Activity 2** to your students, give some exercises on functional relationships.
For example:

1. $A(r) = \pi r^2$

   Area of a circle A(r) is a function of the radius.

2. $A(s) = s^2$

   Area of a square A(s) is a function of its side.

Let the students work in groups of 4-5 members. **Activity 2** is an introductory activity to quadratic function. Let the students do the activity and guide them to formulate equations leading to quadratic functions. Ask them to graph the obtained points to have an initial feature of the graph of the quadratic function. Ask them to compare the obtained equation and the linear function. Let them describe the graph.

Guide for Activity 2

a. $2w + l = 80$ Equation of the 3 sides
b. $l = 80 - 2w$
c. $A = w(80-2w)$
   $A = 80w - 2w^2$ Area of the parking lot.
d. Table of values

Width (w)	5	10	15	20	25	30	35
Area (A)	350	600	750	800	750	600	350

f. The independent variable is the width (w) and the dependent variable is the area (A).

h. Graph should be done by the students.

Discuss these key concepts.

Teacher's Notes and Reminders

A quadratic function is one whose equation can be written in the form $\mathbf{y = ax^2 + bx + c}$. where ***a, b*** and ***c*** are real numbers and $\boldsymbol{a \neq 0}$.

In an equation a function, the highest exponent of the variable *x* is called the **degree of a function.** Thus, the degree of the quadratic function is 2.

Show to the students the different equations in **Activity 3** and let them identify which are quadratic and which are not. Ask them to describe those which are quadratic and differentiate these from those which are not.

Activity 3 Answer Key

1. quadratic function
2. not quadratic function
3. quadratic function
4. not quadratic function
5. not quadratic function
6. not quadratic function
7. quadratic function
8. quadratic function
9. quadratic function
10. not quadratic function

Note: *For an alternative learning activities, please consider Activity 2 " Did You Know?" found on p. 34 of BEAM Module 3, Learning Guide 6.*

Provide the students with opportunities to differentiate a quadratic function from a linear function by giving them the Activity 4. In this activity, students will investigate one difference between a linear function and a quadratic function by exploring patterns in changes in y. Guide the students in doing the activity.

Guide for Activity 4

1. The function f(x) = 2x+1 is a linear function and the second function g(x) = x^2 + 2x -1 is a quadratic function.
3. The differences between two adjacent x-values in each table are all equal to 1.
4.

f(x) = 2*x* + 1

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

2 2 2 2 2 2

g(x) = x^2 + 2*x* -1

x	-3	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7	14

-3 -1 1 3 5 7

2 2 2 2 2

7. Graphs should be done by the students.
8. The graph of a linear function is a straight line while the graph of a quadratic function is a curve.

Discuss these key concepts with the students and explain the illustrative examples.

Teacher's Notes and Reminders

In a **linear function**, equal differences in x produce equal differences in y. However, in **quadratic function**, equal differences in x do not lead to equal first differences in y; instead the **second differences** in y are equal. The graph of a linear function is a **straight line** while the graph of a quadratic function is a smooth curve called ***parabola***.

Discuss the illustrative example. (Refer to LM p. 9)

Teacher's Notes and Reminders

The general form of a quadratic function is $\boldsymbol{y = ax^2 + bx + c}$ and the standard form or vertex form is $\boldsymbol{y = a(x - h)^2 + k}$ where (h, k) is the vertex.

Present to the class the illustrative examples. (Refer to LM p. 10-11)

Ask the students to perform **Activity 5**. In this activity, the students will be able to apply what they have learned in the illustrative examples presented above.

Guide for Activity 5

Discuss with them the procedure in transforming a quadratic function from the general form $\boldsymbol{y = ax^2 + bx + c}$ into the **standard form** $\boldsymbol{y = a(x - h)^2 + k}$. Emphaize to them that the standard form of a quadratic function can sometimes be more convenient to use when working on problems involving the vertex of the graph of the function.

A. 1. $y = (x – 2)^2 -14$

2. $y = 3\left(x\frac{2}{3}\right)^2 - \frac{1}{3}$

Present to the class the illustrative example in transforming a quadratic function from standard form **$y = a(x - h)^2 + k$** into general form ***$y = ax^2 + bx + c$***. (Refer to LM p. 12-13)

Ask the students to perform **Activity 6**. In this activity, the students will be able to apply what they have learned in the illustrative examples presented above. Let them recall the concepts of special products.

Guide for Activity 6

A.

Steps	Task
1. Expand $(x – 1)^2$	$x^2 – 2x + 1$
2. Multiply the perfect square trinomial by 2	$2(x^2 – 2x + 1)$
3. Simplify	$2x^2 – 4x + 2$
4. Add 3	$2x^2 – 4x + 2 + 3$
5. Result	$2x^2 – 4x + 5$

B. 1. $y = 2x^2 – 16x + 37$

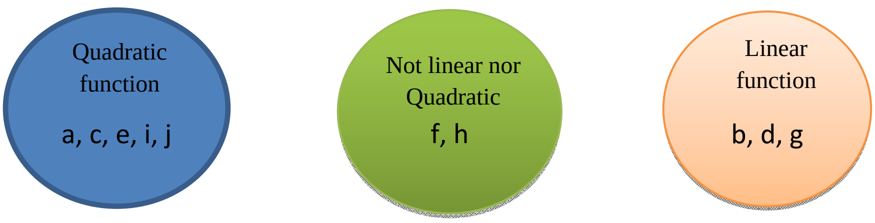
2. $y = 3x^2 – 3x + \frac{7}{4}$

What to PROCESS:

In this section, let the students apply the key concepts of a quadratic function. Tell them to use the mathematical ideas and the skills they learned from the activities and from the examples presented in the preceding section to answer the activities provided.

Ask the students to perform **Activity 7**. In this activity, the students will be able to classify the given equations. Ask the students how quadratic functions differ from other functions and what makes a function quadratic.

Activity 7 Answer Key



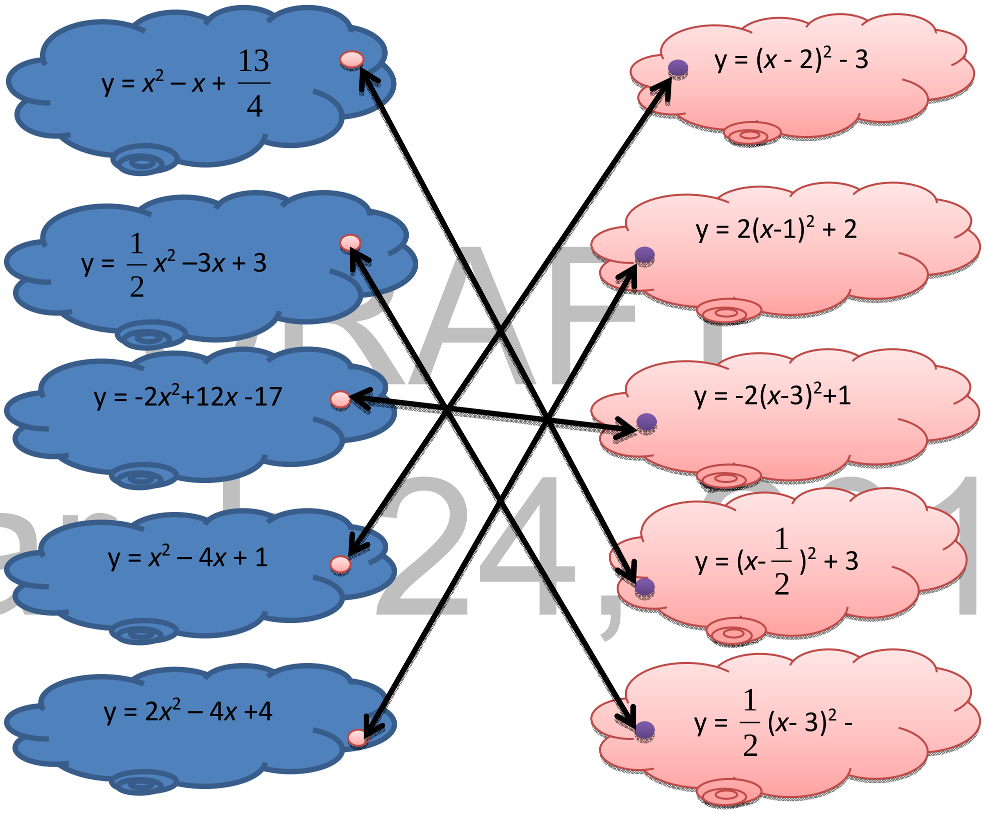
One important skill that students need to learn is finding patterns or relationships. **Activity 8** gives the students an opportunity to develop their mathematical thinking by finding patterns from given figures. Ask the students to identify which of the set of figures describe a quadratic function. Let the students justify their answer.

Activity 8 Answer Key

a. $y = x^2 - 1$, quadratic b. $y = \frac{1}{2}x^2 + \frac{1}{2}x$, quadratic c. $y = 2x - 1$, not quadratic

Ask the students to perform **Activity 9.** This activity provides the students an opportunity to transform the quadratic function $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

Activity 9 Answer Key



Ask the students to perform **Activity 10.** This activity provides the students an opportunity to practice writing the equation of quadratic function $y = a(x - h)^2 + k$ in the form $y = ax^2 + bx + c$.

Activity 10 Answer Key

BOX:

M	$y = x^2 - x + \frac{7}{4}$
A	$y = 3x^2 + 12x + \frac{23}{2}$
T	$y = x^2 - 2x - 15$
H	$y = 2(x+1)^2 - 2$
I	$y = x^2 - 2x - 3$
S	$y = 2x^2 + 5x - 3$
F	$y = x^2 - 6x + 14$
U	$y = -2x^2 + 12x - 17$
N	$y = x^2 - 36$

Note: *For an alternative learning activity, please consider Activity 4 " Hundreds of Pi's" found on p. 37 of BEAM Module 3, Learning Guide 6.*

Let the students work in pairs to answer the problem in **Activity 11.** Let them think of the different strategies to answer the problem and to justify their claim.

Activity 11 Answer Key

Is it yes when the bullet hits the top of the antenna.

Find the height of the bullet when x = 150, which is the distance of the antenna from the firing place.

Substituting in $y = -\frac{1}{1500}x^2 + \frac{2}{15}x$

$$y = -\frac{1}{1500}(150)^2 + \frac{2}{15}(150)$$

$$y = -15 + 20$$
$$y = 5$$

Thus, the height of the bullet is 5m, which is the same as the height of the antenna.

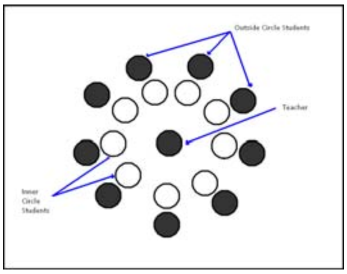
What to REFLECT and UNDERSTAND

Let the students perform the activities in this section. Some of the activities lead them to reflect and to deepen their understanding on the lesson.

In A**ctivity 12**, the students will be able to assess their knowledge in identifying the quadratic function. This activity provides the students an opportunity to collaborate with their classmates and share their ideas or what they learned in the previous sessions. The directions are given below.

Inside Outside Circle (Kagan, 1994)
Directions:

Students are divided into two groups, usually by numbering off.
One group forms a circle and turns around to face outward.
The other group of the students creates an outside circle by facing a peer from the inner circle.
The teacher provides prompts or discussion topics. If the teacher stands in the center, he or she can monitor student responses.
After allowing time for discussion, the teacher has the students in the outside circle move one or more to the right or left, therefore greeting a new partner. Steps 4 and 5 are repeated with the new set of partners until time or questions/topics are exhausted.



Guide questions for this activity can be found in **LM p. 18**. Since you cannot attend to each group's discussion, provide a post-discussion activity to emphasize the key concepts of quadratic functions involved in the activity.

Allow the students to work on **Activity 13** individually to master the skills in transforming a quadratic function into different forms. It will serve as a self-assessment activity for the students.

Give **Activity 14** to your students for you to check their understanding of the concepts. Ask the students to apply the concepts they learned to present the solution to the problem. Let the students find the pattern, relationship, draw the table of values, graph and give the equation.

Assess the performance of the students using the rubric below.

Rubrics on Investigating Patterns

	4	3	2	1
Presentation of solution	Used mathematical language, graphs, table of values and/or charts appropriately. Solution is presented in a clear and orderly manner.	Used mathematical language, graphs, table of values, diagrams. Solution is presented clearly.	Used mathematical language, graphs, table of values. Solution is presented in an unclear manner.	Used a little mathematical language, graphs, table of values. Presented the problem in an unclear manner.
Completion	Successfully completed all parts of the task.	Completed most parts of the task.	Completed some parts of the task.	Work is incomplete.

What to TRANSFER

Give the students opportunities to demonstrate their understanding of quadratic functions by solving real-life problems and by doing a practical task. Let them perform **Activity 15**. You can ask the students to work individually or in a groups.

You can assess the performance of the students using this rubric or you can make your own rubric in assessing the performance or output of the students.

Rubrics on Problem Solving

	4	3	2	1
Identifying Relevant Information	Important information needed to solve problem are clearly identified.	Important information and unimportant information are separated.	Sometimes need help in identifying important information in the problem.	Missed the important information in the problem.
Analyzing Problems	All the characteristics of a problem are carefully analyzed.	Some of the characteristics of a problem are carefully analyzed.	Not all the characteristics of a problem are analyzed.	Some of the characteristics of a problem not analyzed.
Content /Key Points	Relationships between variables are clearly presented. Table of values and graphs are shown.	Relationships between variables are presented. Information/ Data needed are determined.	Relationships between variables hardly recognized Information/ Data needed are not enough.	Relationships between variables cannot be recognized Lack of data/Information.

Mathematical Solution	Calculated the correct answer. Work shown is logical. Calculations are completely correct and answers are properly labelled.	Calculated a correct answer Calculations are correct.	Work is partially shown. Minor errors may be evident. Calculations contain minor errors.	Attempted to solve the problem. A limited amount of work shown.

Let the students perform the task in **Activity 16**. This activity provides the students with an initial idea of the importance of quadratic functions in creating designs. Ask them to use the mathematical concepts that they *have* learned.

Rubrics on Exploring Parabolic Designs

	4	3	2	1
Parabolic designs observed	Presented the three parabolic designs observed in the community. Explanation on the designs were shown.	Presented the three parabolic designs observed in the community.	Presented only two parabolic designs in the community.	Only one parabolic design was presented.
Mathematical Concepts	Complete understanding of the mathematical concepts and their use in creating the design are evident.	Substantial understanding of the mathematical concepts and their use in creating the design are evident.	Understanding of the mathematical concepts needed and their use in creating the designs are evident.	Limited understanding of the concepts and their use in creating the design are shown.
Designs	Design is complex, detailed and show imagination and creativity.	Design is simple and show some imagination and creativity.	Design is limited and show repetition of single ideas.	Design is simple.

Lesson 2 Graphs of Quadratic Function

What to KNOW:

Assess students' knowledge by recalling the different mathematics concepts previously learned especially the process of transforming the quadratic function $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$. Tell them that as they go through this lesson, they have to think of this important question: *"How are the graphs of quadratic functions used in solving real-life problems and in making decisions?"*

You may start this lesson by giving **Activity 1**. In this activity, the students will be able to sketch the graph of a quadratic function by completing the table of values. This activity provides students the opportunity to determine the properties of the graph of a quadratic function. Guide them in doing the **Activity 1**

Based on the results of the activity, post-discussion is to be done emphasizing the properties of the graph of a quadratic function.

Activity 1 Answer Key

a. $y = x^2 - 2x - 3$ $y = -x^2 + 4x - 1$

$y = (x - 1)^2 - 4.$ $y = -(x - 2)^2 + 3.$

b. Complete the table of values for x and y

$y = x^2 - 2x - 3$

x	-3	-2	-1	0	1	2	3	4	5
y	12	5	0	-3	-4	-3	0	5	12

$y = -x^2 + 4x - 1$

x	-4	-3	-2	-1	0	1	2	3
y	-33	-22	-13	-6	-1	2	3	2

d. The value of *a* has something to do with the opening of the parabola. If a > 0, the parabola opens upward but if a < 0, the parabola opens downward.

h. Can you identify the domain and range of the functions?

$y = x^2 - 2x - 3$ Domain: Set of all real numbers   Range: $y \geq -4$

$y = -x^2 + 4x - 1$ Domain: Set of all real numbers   Range: $y \leq 3$

Teacher's Notes and Reminders

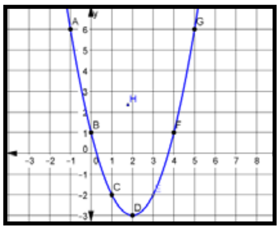
The graph of a quadratic function $y = ax^2 + bx + c$ is called **parabola**. You have noticed that the parabola opens **upward or downward**. It has a turning point called vertex which is either the lowest point or the highest point of the graph. If the value of a > 0, it opens upward and has a **minimum point** but if a < 0, the parabola opens downward and has a **maximum point**. There is a line called the **axis of symmetry** which divides the graph into two parts such that one-half of the graph is a reflection of the other half. If the quadratic function is expressed in the form $y = a(x - h)^2 + k$, the vertex is the point **(h, k)**. The line **x = h** is the axis of symmetry and **k** is the minimum or maximum value of the function. The **domain** of a quadratic function is the set of all real numbers. The **range** depends on whether the parabola opens upward or downward. If it opens upward, the range is the set $\{y : y \geq k\}$; if it opens downward, then the range is the set $\{y : y \leq k\}$.

Let the students perform **Activity 2**. This activit*y* provides the students opportunit*y* to draw the graph of a quadratic function in another way. Tell the students that they can also use this method in graphing a quadratic function.

Guide for Activity 2

1. The vertex form of the quadratic function is $y = (x - 2)^2 - 3$. The vertex is (2, -3).

4. Graph



Let the students do **Activity 3**. In this activity, the students will be able to develop their mathematical thinking skills by solving a number problem. Let the students analyze their ideas and let them process the information on getting the correct answer. Guide them to do the activity particularly in formulating the equations. Let the students think of the properties of the graph of a quadratic function to solve the problem.

Guide for Activity 3

a. Table of values

Number (n)	0	1	2	3	4	5	And so on
Product (P)	0	38	72	102	128	150	

c. The vertex is (10, 200). Let the students observe the graph and try to relate the answer they obtained in the puzzle to the vertex of the graph. Let the students write their observation or conclusion.

d. 20 –n

e. Function: $P(n) = (20 - n)(2n) = 40n - 2n^2$

f. Quadratic function

g. $P(n) = -2(n - 10)^2 + 200$

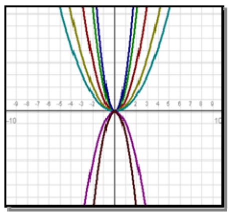
h. 200

i. 10

j. Answers may vary.

Let the students perform the **Activity 4**. In this activity, the students will be given an opportunity to investigate the effects of *a. h*, and *k* in the graph of the quadratic function $y = a(x - h)^2 + k$. Let the students observe the graphs and make generalizations based on their observations.

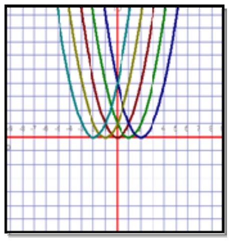
Guide for Activity 4



Part A

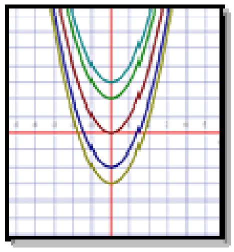
In the graph of $y = a(x - h)^2 + k$, the larger the $|a|$ is, the *narrower* is the graph. If $a > 0$, the parabola opens upward but if $a < 0$, the parabola opens downward

Part B.



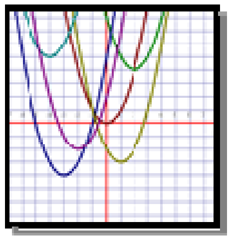
To graph $y = a(x - h)^2$, slide the graph of $y = ax^2$ h units horizontally. If h > 0, slide it to the right, if h < 0, slide it to the left. The graph has vertex (h, 0) and its axis is the line x = h.

Part C



To graph $y = ax^2 + k$, slide the graph of $y = ax^2$ vertically **k** units. If **k > 0** slide it upward; if k < 0, slide it downward. The graph has vertex (0, k) and its axis of symmetry is the line $x = 0$ (y – axis).

Part D



To graph $y = a(x - h)^2 + k$, slide the graph of $y = ax^2$ h units horizontally and k units vertically. The graph has vertex (h, k) and its axis of symmetry is the line x = h.

Note:

If a < 0, the parabola opens downward. The same procedure can be applied in transforming the graph of quadratic function.

Vertex of the graph of a quadratic function:

In standard form $f(x) = a(x - h)^2 + k$, the vertex (h, k) can be directly obtained from the values of h and k.

In general form $f(x) = ax^2 + bx + c$, the vertex (h, k) can be obtained using the formulas h = $\frac{-b}{2a}$ and k = $\frac{4ac - b^2}{4a}$.

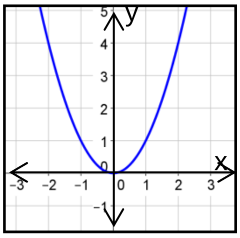
What to PROCESS:

In this section, let the students apply the key characteristics of the graph of a quadratic function. Tell them to use these mathematical ideas and the skills they learned from the examples presented in the preceding section to answer the activities provided.

Ask the students to perform **Activity 5**. In this activity, the students will be given an opportunity to graph a quadratic function and identify the vertex (minimum point/maximum point), opening of the graph, equation of the axis of symmetry, domain, and range.

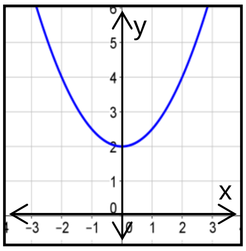
Activity 5 Answer Key

1. f(x) = x^2



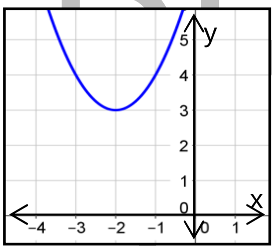
Vertex: *(0, 0)*
opening of the graph: *upward*
Vertex is a *minimum* point
Equation of the axis of symmetry: *x = 0*
Domain: *All real numbers* Range: $y \geq 0$

3. f(x) = $\frac{1}{2}x^2 + 2$



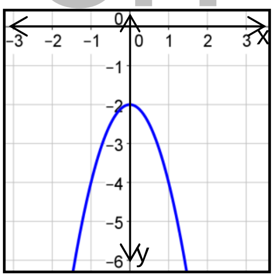
Vertex: *(0, 2)*
opening of the graph: *upward*
Vertex is a *minimum* point
Equation of the axis of symmetry: *x = 0*
Domain: *All real numbers* Range: $y \geq 2$

5. f(x) = $(x + 2)^2 + 3$



Vertex: *(-2, 3)*
opening of the graph: *upward*
Vertex is a *minimum* point
Equation of the axis of symmetry: *x = -2*
Domain: *All real numbers* Range: $y \geq 3$

7. f(x) = $-2x^2 - 2$



Vertex: *(0, -2)*
opening of the graph: *downward*
Vertex is a *maximum* point
Equation of the axis of symmetry: *x = 0*
Domain: *All real numbers* Range: $y \leq -2$

Knowledge about the concepts of the graph of a quadratic function is very important in solving real-life problems. Let the students do the **Activity 6**. This activity enables the students to apply the properties of quadratic function to work on real-life problems.

Guide for Activity 6

a. Quadratic function

c. 19 ft
d. The ordinate of the vertex represents the maximum height.

Let the students find the vertex of the function in another way. Emphasize the importance of the vertex of a quadratic function in finding the maximum height reached by the object.

To master the skills in graphing and the properties of the graph of a quadratic function, let the students perform **Activity 7**. It provides the learners with an opportunity to test their ability to identify graph of the given quadratic function.

Activity 7 Answer Key

1) mismatch — The correct equation is $y = x^2 + 4$
2) match
3) mismatch — The correct equation is $y = 2(x - 3)^2 + 1$
4) match
5. match

After the activity, ask the students the properties they applied to determine easily if the quadratic function and the graph are match or mismatch.

Activity 8 allows the students to apply what they have learned in transforming the graph of a quadratic function into standard form. Let the student perform the activity.

What to REFLECT and UNDERSTAND

Let the students perform the activities in this section to deepen their understanding of the graph of a quadratic function. The activity provides the students with an opportunity to solve problems involving quadratic functions.

Skills in analyzing the graph are very useful in solving real-life problems involving quadratic functions. Allow the students to perform **Activity 9**. This activity provides the students with an opportunity to apply the properties of the graph of a quadratic function to solve real-life problems.

Activity 9 Answer Key

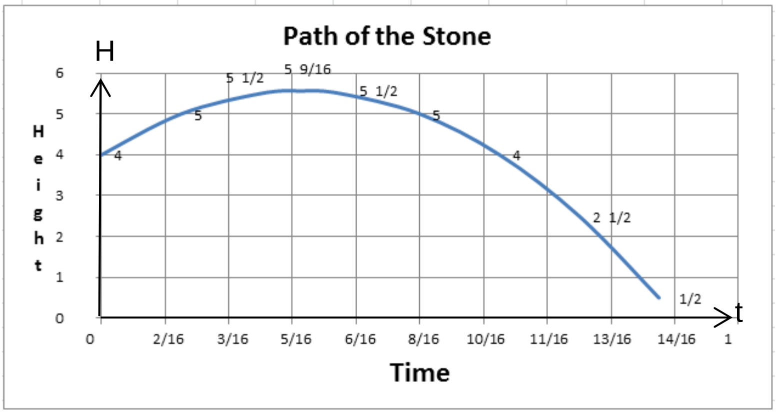
Problem 1:

a) parabola
b) 0
c) 10 m
d) 6
e) around 5 m
f) 12 m

Let the students solve problem number 2 in **Activity 9**. This activity can be used as an assessment tool to determine if the students really understand the lesson.

Problem 2:

a.



b. $5\dfrac{9}{16}$ feet

c. $\dfrac{5}{16}$ seconds

Activity 10 is a clock partner activity. This activity will give the students opportunity to conceptualize the concepts of transformation of the graph of $y = a(x - h)^2 + k$.

Write your name in your clock. Make an appointment with 12 of your classmates-one for each hour on the clock. Be sure both of you record the appointment on your clock. Make an appointment only if there is an open slot at that hour on both of your clocks.

Give **Activity 11** to the students to check their understanding regarding the lesson. It will serve as a self-assessment activity for the students.

Making comparison is one way of checking the student's understanding of the concepts. Give **Activity 12** to the students and let them compare the two given situations. This activity enables the students to develop their reasoning skills and ability to make a wise decision.

Activity 12 Answer Key

1. A 2. A 3. A

Conducting math investigation is one of the useful methods to develop the mathematical thinking skills of the students. **Activity 13** provides the students with an opportunity to deal with a mathematical investigation. Let the students investigate the effects of the variable *a, b*, and *c* in the general form of a quadratic function $y = ax^2 + bx + c$. Ask the students to make a conclusion based on their observation. Allow them to check their work.

Activity 13 Answer Key

Given the graph of $y = ax^2 + bx + c$, the effects are as follows:

- Changing ***c*** moves it up and down.
- Changing ***b*** affects the location of the vertex with respect to the y-axis. When **b** = 0, the vertex of the parabola lies on the y-axis. Changing **b** does not affect the shape of the parabola. The graphs with positive values of *b* have shifted down and to the left, those with negative values of *b* have shifted down and to the right.
- Changing ***a*** alters the opening of the parabola. If a > 0, the parabola opens upward and if a < 0, the parabola opens downward. The larger the value of $|a|$ is, the narrower is the curve.

What to TRANSFER

Let the student work in groups of 5-6 members to perform **Activity 14** and to show the extent of what they have learned in this lesson. You may use this rubric to assess their product/performance.

Rubrics on Designing a Curtain

	4	3	2	1
Proposed Plan	Proposed plan is well organized. Information and data needed in the plan are complete with varied explanations.	Proposed plan is organized. Information and data needed in the plan are complete.	Proposed plan is organized. Information and data needed in the plan are presented but others are missing.	Proposed plan is organized. Lack of Information and data.
Proposed Budget	Proposed budget was clearly presented.	Proposed budget are presented.	Proposed budget are not clearly stated.	No budget proposal.
Designs	Design is complex, and shows imagination and creativity. Parabolic curves are clearly seen.	Design is simple but shows some imagination and creativity. Parabolic curves are observed.	Design is limited but parabolic curves are observed.	Design is simple.
Mathematical Concepts	Mathematical concepts applied in creating the design are evident. Lines of symmetry as well as the height of parabolic curves are appropriately proportioned to the room.	Mathematical concepts are applied in creating the design. Lines of symmetry as well as the height of parabolic curves are evident.	Mathematical concepts applied in creating the design are evident. Lines of symmetry as well as the height of parabolic curves are not clearly seen in the design	Mathematical concepts applied in creating the design are evident. .

Activity 15 can be done if you have internet access and computer units available. Let your students do activity 15. In this activity, students will be able to have an opportunity to experience a simple research project on parabolic arches.

This activity also provides student a chance to develop their technology literacy skills.

You may assess the students' performance in this activity using this suggested rubric:

Rubrics on Parabolic Arches Webquest

	4	**3**	**2**	**1**
Content	Presented more than three parabolic arches. Information regarding each parabolic arch is presented including the purpose of constructing the arch.	Presented three parabolic arches. Information regarding each parabolic arch is presented including the purpose of constructing the arch.	Presented only two parabolic arches including some information regarding the arches.	Presented only one parabolic arch.
Presentation	Included several pictures timelines, and charts in the presentation. The graphics are clear, labeled correctly. Used digital format in the presentation.	Included several pictures in the presentation. The graphics are labeled correctly. Used digital format in the presentation.	Included several pictures in the presentation. The graphics are labeled. Used digital format in the presentation.	Included minimal pictures in the presentation. The graphics are labeled. Used digital format in the presentation.
Mathematical Concepts	Used the properties of the graph of a quadratic function in describing the parabolic arches. Presented the arches using Cartesian Plane.	Used the properties of the graph of a quadratic function in describing the parabolic arches.	Slightly used the properties of the graph of a quadratic function in describing the parabolic arches.	Just describe the parabolic arch
References	Information is supported by many good resources. References are stated in the paper and thepresentation.	Information is supported by good resources. References are stated in the paper.	Some information is supported by resources. References are not properly stated in the paper.	No references.

Lesson 3 Finding the Equation of a Quadratic Function

What to KNOW

Before you start this lesson, make a simple "checking of understanding" activity regarding the concepts previously discussed. This will help you in teaching this lesson.

Begin this lesson by recalling the roots of a quadratic equation and relate them later with the zeros of the quadratic function. In **Activity 1,** let the students find the roots of the given quadratic equations in 3 methods.

a) Factoring
b) Completing the square
c) Quadratic formula

Note: Procedure in finding the roots can be seen in module 1 lesson 2.
Let the students examine the graph in **Activity 2**. In this activity, the students will be able to see the zeros of the quadratic function using the graph. Guide the students in understanding the concept of zeros of a quadratic function and the x - intercepts.

Guide for Activity 2

a. The graph is a parabola.
b. The vertex is (1, -4). The axis of symmetry is x = 1.
c. -1 and 3
d. x – intercepts
e. 0

Discuss the illustrative examples in finding the zeros of a quadratic function. (See LM p. 39-40. Clarify the definitions of zeros of the functions and the x – intercepts.

The next activity will give students the opportunity to derive the equation of a quadratic function from a table of values. Let the students work on **Activity 3**. This activity asks students to determine the equation of a quadratic function derived from tables of values or given the three points on the curve. The latter gives the students an opportunity for the student to recall the process of finding the solution of a system of linear equation in 3 variables.

Guide for Activity 3A

a. Let the students draw the graph.

b. The value of y when x is -2 is 0. Similarly, the value of y when x is 1 is 0. Thus, the zeros are -2 and 1.

c. Zeros are x = -2 and x = 1
$x + 2 = 0$ or $x - 1 = 0$
$(x + 2)(x – 1) = 0$
$x^2 + x – 2 = 0$
d. The equation of the quadratic function is $f(x) = x^2 + x – 2$.

Ask the students to think of an alternative way of finding the equation of a quadratic function. Remind them of the procedure in getting it by using the sum and product of the roots **(Module 1, Lesson 4).**

In **Activity 3B** , guide the students to arrive at the equation of a quadratic function given the 3 points from the table of values.

Activity 3B Answer Key

a. Answers may vary.
b. Answers may vary.
c. a = -2 , b = 4 , and c = 1
d. $y = -2x^2 + 4x + 1$

After the **Activity 3**, discuss the presented examples (Refer to LM p. 42- 43) so that other students will clearly grasp the procedure in finding the equation of quadratic function from table of values or from 3 given points.

Give **Activity 4** and let the students observe the graph of the quadratic function. Let them identify the vertex and any point in the curve. In this activity, the students will be able to determine the equation of a quadratic function by using the form $y = a(x - h)^2 + k$.

Remind the students that if the vertex of the parabola cannot be exactly determined, they can get any three points from the graph and use the algebraic procedure in finding the equation of a quadratic function they learned in the previous activity.

Activity 4 Answer Key

1. The parabola opens upward. $a > 0$.
2. The vertex is (1, -3)
3. Point (0, –2) is on the graph.
4. $y = a(x - 1)^2 - 3$
   $-2 = a(0 - 1)^2 - 3$
5. a = 1
6. $y = (x - 1)^2 - 3$

Discuss the illustrative example if needed. (Refer to LM p. 44).

Let the students work on **Activity 5**. In this activity, the students will be able to think about the reverse process to get the equation of a quadratic function.

Guide for Activity 5

A. Solution

Since the zeros are $r_1 = 1$ and $r_2 = 2$

then the equation of the quadratic function is $f(x) = a(x - r_1)(x - r_2)$ where a is any nonzero constant.

It follows that $f(x) = a(x - 1)(x - 2)$

$f(x) = a(x^2 - 3x + 2)$

$f(x) = a(x^2 - 3x + 2)$

B. The equation of the quadratic function is $f(x) = a(x^2 - 4x - 1)$.

Discuss the illustrative example given at the end of the activity for them to have a better understanding on the procedure in finding the equation of a quadratic function given the zeros. (Refer to LM p. 45-46)

What to PROCESS

Let the students apply the concepts of finding the zeros of a quadratic function and their skills in finding the equation of a quadratic function to do the activities provided in this section. Allow the students to deal with the activities that will provide them the opportunity to develop further skills needed to perform some tasks ahead.

Allow the students to do **Activity 6.** In this activity, the students will show their skills in finding the zeros of the quadratic functions. Let the students reflect on the message they will get in the activity.

Activity 6 Answer Key

Hidden message**: GOD LOVES YOU.**

A skill in deriving the equation of the quadratic function is very important in solving real-life problems. Allow the students to perform **Activity 7**. In this activity, the students will be able to apply the procedure in finding the equation of a quadratic function given the table of values. The students will also be able to find patterns or relationships of two variables using different strategies.

Activity 7 Answer Key

A. $y = -x^2 - 4$ B. $y = 2(x+3)^2 + 5$ C. $y = \frac{1}{2}x^2 + \frac{1}{2}x$

Note: *For an alternative learning activity, please consider Activity 10" Hopping Rabbits" found on p. 44 of BEAM Module 3, Learning Guide 6.*

Let the students work in pairs on **Activity 8**. The students will apply what they have learned in deriving the equation of a quadratic function from the graph.

Activity 8 Answer Key

1) $y = (x-2)^2 + 3$ or $y = x^2 - 4x + 7$
2) $y = -2(x+3)^2 + 4$ or $y = -2x^2 - 12x - 14$
3) $y = 3(x-1)^2 - 3$ or $y = 3x^2 - 6x$
4) $y = -4(x+2)^2 - 3$ or $y = -4x^2 - 16x - 19$
5) $y = \frac{1}{3}(x-3)^2 - 2$ or $y = \frac{1}{3}x^2 - 2x + 1$

In **Activity 9,** let the students apply what they have learned in the transformation of the graph of quadratic function to derive the equation of a quadratic function described by the graph.

Activity 9 Answer Key

1) $f(x) = 3x^2 - 4$
2) $f(x) = 4(x+2)^2$
3) $f(x) = 3(x-2)^2 + 5$
4) $f(x) = -10(x+6)^2 - 2$

5) f(*x*) = $7\left(x+\frac{1}{2}\right)^2+\frac{1}{2}$

Note: *For alternative learning activities, please consider Activity 8A and 8B " Direct Thy Quadratic Paths" found on p. 40-41 of BEAM Module 3, Learning Guide 6.*

Let the students do the exercises in **Activity 10.** This is intended to provide practice for the students to master the skills in finding the equation of a quadratic function.

Activity 10 Answer Key

1. $y = x^2 – 5x + 6$
2. $y = 2x^2 – x - 10$
3. $y = x^2 – 2x - 2$
4. $y = x^2 – \frac{2}{3}x - \frac{1}{9}$
5. $y = 9x^2 – 121$

Note: The answer in this activity is not unique. Let the students give at least one solution for each item.

Problems in real-life can be modeled using a quadratic function. Allow the students to do **Activity 11.** In this activity, let the students use their understanding and skills in quadratic function to solve the given problem using different methods.

Activity 11 Answer Key

The equation is f(x) = $ax(400 – x)$ and a is obtained from the given, f(300) = 20.

The function should be:

$$f(x) = \frac{1}{1500}x(400-x)$$

or $$f(x) = \frac{4}{15}x-\frac{1}{1500}x^2$$

What to REFLECT and UNDERSTAND

Before you start this section, make a simple "checking of understanding" activity. Tell the students that the activities in this section are provided for them to deepen their understanding on the concept of finding the equation of the quadratic function.

Let the students perform **Activity12**. In this activity, the students will be able to experience a mathematical investigation. Let the students apply the concepts they learned to find the rule that represents the quadratic relationship in the problem.

Activity 12 Answer Key

$$C(n) = \frac{1}{2}n^2 - \frac{1}{2}n \qquad n > 0$$

n is the number of points on the circle

C is maximum number of chords that can be drawn
Allow the students to explain how they arrived at the correct answer.

Skills and knowledge in analyzing graphs are essential in solving math problems. Let the students work in pairs for **Activity 13**.This activity provides an opportunity for the students to analyze the graph of a quadratic function derived from the given real-life situation. Let them answer the question and determine the equation of a quadratic function.

Activity 13 Answer Key

a) parabola that opens downward
b) (2, 1280). This represents a maximum profit of Php 1280 after 2 weeks
c) 2 weeks
d) P = $-20(w - 2)^2 + 1280$

Give **Activity 14** to the students and let them work in groups of three members each. In this activity, the students will be able to develop their mathematical thinking skills.

Guide for Activity 14

1. A. What are the equations of the quadratic functions?
   a) $y = x^2 - x - 6$ b) $y = x^2 - 6x + 6$
   B. If we double the zeros, then the new equations are:
   a) $y = x^2 - 2x - 24$ b) $y = x^2 - 12x + 24$
   C. If the zeros are reciprocal of the given zeros, then the new equations are:
   a) $y = x^2 + \frac{1}{6}x - \frac{1}{6}$ b) $y = x^2 - x + \frac{1}{6}$
   D. If we square the zeros, then the new equations are:
   a) $y = x^2 - 13x + 36$ b) $y = x^2 - 24x + 36$
2. a) $y = x^2 - 19x + 25$ b) $y = x^2 + \frac{1}{6}x - \frac{1}{6}$ c) $y = 3x^2 + 8x - 20$

Let the students summarize what they have learned by doing the **Activity 15**, “Principle Pattern Organizer.

What to TRANSFER

Let the students work in groups (5 to 6 members) to do performance task in **Activity 16**. Use this rubric to assess the students’ group output.

Rubrics on Equations of Famous Parabolic Bridges

	4	3	2	1

Parabolic Bridges	Presented three parabolic bridges with complete data and information needed in the mathematical solution of the research.	Presented three parabolic bridges with complete data and information needed in the mathematical solution of the research.	Presented only two parabolic bridges with complete data and information needed in the mathematical solution of the research.	Only one parabolic bridge with complete data and information needed in the mathematical solution of the research.
Mathematical Concepts	Solution shows complete understanding of the mathematical concepts and used them in the research project.	Solution shows substantial understanding of the mathematical concepts and used them in the research project.	Shows some understanding of the mathematical concepts needed and used them in the research project.	Shows very limited understanding of the concepts and used them in the research project.
Research Organization	Research paper is complete and well organized	Research paper is complete and organized	Lapses in focus and/or coherence	Random or weak organization
Sources	All sources (information and graphics) are accurately documented in the desired format.	All sources (information and graphics) are accurately documented, but a few are not in the desired format.	All sources (information and graphics) are accurately documented, but many are not in the desired format.	Some sources are not accurately documented.

Lesson 4 Applications of Quadratic Functions

What to KNOW:

Introduce this lesson by telling the students that applications of quadratic functions can be seen in many different fields like physics, industry, business and in variety of mathematical problems. Emphasize to them that familiarity with quadratic functions, their zeros and their properties is very important in solving real-life problems.

You may start this lesson by giving **Activity 1**. In this activity, the students will be dealing with a geometry problem that requires concepts of quadratic functions to find the solution. This activity provides students with the opportunity to get exposed to strategy of solving problem involving quadratic function. Guide the students in doing the activity.

Guide for Activity 1

a. Table of values

width (*w*)	5	10	15	20	25	30	35	40	45
Length (l)	45	40	35	30	25	20	15	10	4
Area (*A*)	225	400	525	600	625	600	525	409	225

b. 625 m^2
c. The width is 25 m and the length is 25 m
d. $2l + 2w = 100$
e. $l = 50 – w$
f. $A = 50w – w^2$
g. The function is quadratic.
h. In standard form the area is $A = -(w – 25)^2 + 625$. The vertex is (25, 625)
i. Let the students draw the graph
j. The coordinates of the vertex is related to the width and the largest area.

Guide the students in formulating the solution to the problem and give emphasis on the mathematical concepts used to solve the problem. Discuss the illustrative example. (Refer to LM p. 56).

Another important application of a quadratic function is solving problems related to free falling bodies. Let the students perform **Activity 2**. This activity provides the students opportunity to solve problem in physics.

Guide for Activity 2

1. The function is quadratic.
2. $H(t) = -4.9(t – 1)^2 + 6.9$
3. The vertex is (1, 6.9).
4. 6.9 meters
5. 1 second
6. $– 4.9t^2 + 9.8t -2 = 0$
7. 1.77 seconds and 0.23 second

Guide the students in answering the guide questions.

Teacher's Notes and Reminders

Free falling objects can be modeled by a quadratic function $h(t) = –4.9t^2 + V_0t + h_0$, where $h(t)$ is the height of an object at *t* seconds, when it is thrown with an initial velocity of V_0 m/s and an initial height of h_0 meters. If units are in feet, then the function is $h(t) = –16t^2 + V_0t + h_0$

Discuss the illustrative example and give emphasis to the time that the object reached the maximum height and the maximum height is represented by the vertex of the function. **(Refer to LM p. 57-58)**

Let the students solve the problem in **Activity 3**. This activity provides students an opportunity to solve a problem involving maximizing profit. Guide the students in doing the activity.

Guide for Activity 1

a. Table of values

No. of weeks of waiting (w)	0	1	2	3	4	5	6	7	8	9	10
No. of crates	40	45	50	55	60	65	70	75	80	85	90
Profit per crates (P)	100	90	80	70	60	50	40	30	20	10	0
Total profit (T)	4000	4050	4000	3850	3600	3250	2800	2250	1600	850	0

b. Let the students draw the graph.
c. Number of crates times profit per crate
d. $P = (40 + 5x)(100 – 10x)$
$P = - 50x^2 + 100x + 4000$
d. 1 week

For the students to understand better the strategies on how to solve the problem, discuss the illustrative example. (Refer to LM p. 59)

Teacher's Notes and Reminders

Discuss the concepts of the revenue function.

Suppose *x* denotes the number of units a company plans to produce or sell. The revenue function *R*(*x*) is defined as: *R*(*x*)= (price per unit) x (number of units produced or sold).

What to PROCESS:

In this section, let the students apply what they have learned in the previous activities and discussion. Tell them to use these mathematical ideas and the skills they learned to answer the activities in this section.

Ask the students to perform **Activity 4**. In this activity, the students will be given an opportunity to deal with problems involving maximizing profit. Assist the students in answering the guide questions.

Guide for Activity 4

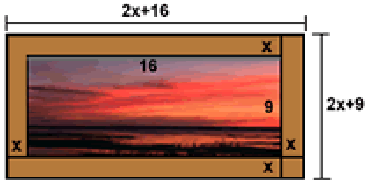
Problem A
a. Php 400 000.00
b. Php 640 000.00
c. Equation $R(x) = (200+50x)(2000-100x)$
d. Php 1 200.00
e. Php 720 000.00

Problem B.
a. Php 25. 00
b. Php 6 250.00

Activity 5 involves a geometry problem. Let the students perform the activity. Guide the students in formulating the equations to solve the problem. Let the students explain the mathematical concepts they applied to solve the problem.

Guide for Activity 1

Illustration



Area of Photograph and Frame - Area of Photograph = Area of Frame

$$(2x+16)(2x+9) \quad - \quad (16)(9) \quad = \quad 84$$

$$(2x + 16)(2x + 9) – (16)(9) = 84$$

$$(4x^2 + 18 + 32x + 144 – 144 = 84$$

$$4x^2 + 50x = 84$$

$$4x^2 + 50x – 84 = 0$$

$$x = -14 \text{ or } 1.5$$

Since x represents the width of a frame, clearly we cannot accept -14 as the width of the frame. Thus x = 1.5.

Let the students work in pairs to solve the problem in **Activity 6.** This activity will allow the students to apply the mathematical concepts they learned like the zeros of the function. They will also have the opportunity of solve the handshakes problem.

Guide for Activity 6

Problem A

a. 243 ft b. 2 seconds c. $19\frac{1}{5}$ seconds

Problem B

a.

Number of Persons (n)	1	2	3	4	5	6	7
Number of Handshakes (H)	0	1	3	6	10	15	21

b. Equation $H(n) = \frac{1}{2}n^2 - \frac{1}{2}n$

d. 4950 handshakes

What to REFLECT and UNDERSTAND

Let the students perform the activities in this section. The activities provide the students an opportunity to extend their understanding and skill in the use of the quadratic function to real-life problems.

Let the students work in group for **Activity 7**. This activity allows the students to solve number problems and geometry problems involving quadratic functions. Remind the students to apply the mathematical concepts they learned throughout the module to answer the problems.

Activity 7 Answer Key

1. 15 m by 15 m
2. 8281 m^2
3. 18 and 18
4. 14 and 14
5. 20m by 40 m Area = 800 m^2
6. 50m by 58 m
7. 12 and 12

Let the students work in group for **Activity 8**. This activity allows the students to solve problems on free falling objects which involve quadratic functions. Remind the students to apply the mathematical concepts they learned in the module to answer the problems.

Activity 8 Answer Key

1. a. 40 m b. $\frac{10}{7}$ seconds c. 4.28 seconds
2. 144 ft
3. 136ft maximum height 1.5 seconds
4. 2.5 seconds
5. 80 m
6. 40.8 ft/s

Let the students work in groups for **Activity 9**. This activity allows the students to solve problems on maximizing profit involving quadratic functions. Remind the students to apply the mathematical concepts they learned in the module to answer the problems.

Activity 9 Answer Key

1. Php 3 625
2. a. R(x) = (500+100x)(160-20x)
   b.Php 130
   c. P 84500
3. Php 363.00

Let the students do a Math investigation in **Activity 10**. Guide the students to do the activity.

Guide for Activity 10

a. The relationship represents a quadratic function.
b. The number of angles A(r) can be determined using the equation

$$A(r) = \frac{1}{2}r^2 - \frac{1}{2}r \text{ where } r \text{ is the number of rays}$$

c. Answers may vary.

- By deriving the equation of quadratic function given 3 points.
- By counting and observing the pattern.

What to TRANSFER

Let the student work in groups of 5-6 members to perform **Activity 11** and to show the extent of what they have learned in this lesson. You may use this rubric to assess their product/performance.

Rubrics on Maximizing Profit

	4	3	2	1
Identifying Problems	Anticipated different kinds of variables in complicated projects and thought of ways to manipulate them.	Anticipated variables in complicated projects and thought of ways to manipulate them.	Anticipated variables in complicated projects, but I usually do not think of ways to manipulate them.	Have a hard time anticipating variables.
Identifying Relevant Information	Important information needed to solve problem/do the task are clearly identified.	Important information from unimportant information are analyzed.	Sometimes need help in identifying important information in the problem.	Missed the important information in the problem.
Analyzing Problems	All the characteristics of a problem are carefully analyzed.	Some of the characteristics of a problem are carefully analyzed.	Not all the characteristics of a problem are analyzed.	Some of the characteristics of a problem not analyzed.
Content /Key Points	Relationships between variables are clearly presented. Information/ Data needed in the proposed plan are well organized.	Relationships between variables are clearly presented. Information/ Data needed are determined.	Relationships between variables hardly recognized Information/ Data needed are not enough.	Relationships between variables cannot be recognized Lack of data/Information.
Proposed Plan	Proposed plan is well organized. Information and data needed in the plan are complete and with varied explanations.	Proposed plan is organized. Information and data needed in the plan are complete.	Proposed plan is organized. Information and data needed in the plan presented but others are missing.	Proposed plan organized. Lack of Information and data.

After finishing the four lessons on quadratic function, let the students take the summative test as well as accomplish the performance task.

Summative Assessment

Part I. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items.

1. Which of the following table of values represents a quadratic function?

a

x	-2	-1	0	1	2	3
y	1	2	3	4	5	6

b.

x	-3	-2	-1	0	1	2
y	9	4	1	1	1	4

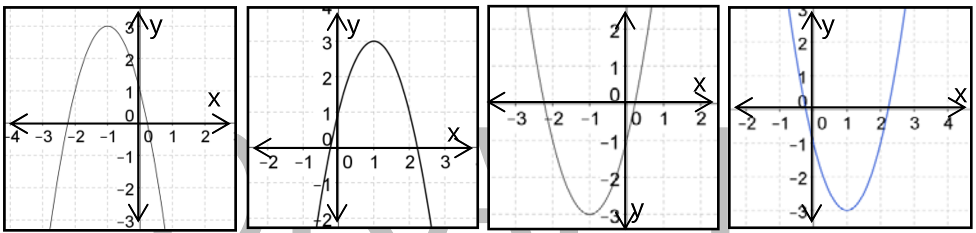
c.

x	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3

d.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7

2. Which of the following shows the graph of $f(x) = 2(x-1)^2 - 3$



a b. c. d.

3. The maximum point of the quadratic function $f(x) = -3x^2 + 6x + 6$ is

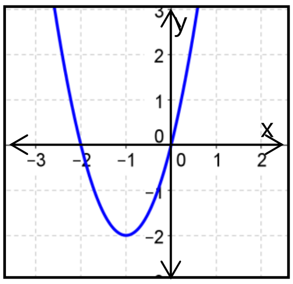
a. (-3, 6) b. (3, -6) c. (1, -9) d. (1, 9)

4. What is $f(x) = 2(x - 3)^2 + 4$ when written in the form $f(x) = ax^2 + bx + c$?

a. $f(x) = 2x^2 -12 x +22$ b. $f(x) = 2x^2 -12 x +10$
c. $f(x) = 2x^2 +12 x +22$ d. $f(x) = 2x^2 -12 x -10$

5. The equation of the function represented by the graph at the right is

a. $f(x) = x^2 + 2x +-2$
b. $f(x) = 2x^2 -4 x + 4$
c. $f(x) = -2x^2 +4x$
d. $f(x) = 2x^2 +4x$



6. The vertex of the quadratic equation $f(x) = (x + 1)^2 - 2$ is

a. (1, -2) b. (-1, 2) c. (1, -2) d. (-1, -2)

7. Consider the quadratic function $f(x) = (x + 3)^2 + 2$, the axis of symmetry of the function is

a. x = 3 b. x = -3 c. x = 2 d. x = -2

8. The product of the zeros of quadratic function $y = -3x^2 - 2x + 5$ is
a. 5/3 b. -5/3 c. 3/5 d. -3/5

9. The sum of the zeros of quadratic function $y = -3x^2 - 2x + 5$ is
a. 2/3 b. -2/3 c. 3/2 d. -3/2

10. The graph of $f(x) = (x + 1)^2 - 2$ is obtained by sliding the graph of $y = x^2$
a. 1 unit to the right and 2 units downward
b. I unit to the left and 2 units downward
c. 1 unit to the right and 2 units upward
d. 1 unit to the left and 2 units upward

11. Which of the following equation of the quadratic function whose zeros are the squares of the zeros of $y = 2x^2 + 5x -3$?
a. $f(x) = 4x^2 -27 x +9$ b. $f(x) = 4x^2 -37 x +9$
c. $f(x) = 4x^2 - x -1$ d. $f(x) = -4x^2 -27 x - 2$

12. Carl Allan hit the volleyball at 3 ft above the ground with an initial velocity of 32 ft/sec. The path of the ball is given by the function $S(t) = -16t^2 + 32t+3$, where t is the time in seconds and S is the height. What is the maximum height reached by the ball?
a. 4 ft b. 8 ft c. 16 ft d. 19 ft

13. A projectile is fired straight up with a velocity of 64 ft/s. Its altitude (height) *h* after *t* seconds is given by $h(t) = -16t^2 + 64t$. When will the projectile's height be half of its maximum height?
a. $-4 \pm \sqrt{2}$ b. $4 \pm \sqrt{2}$ c. $2 \pm \sqrt{2}$ d. $-2 \pm \sqrt{2}$

14. Find the maximum rectangular area that can be enclosed by a fence that is 364 meters long.
a. 8279 m^2 b. 8280 m^2 c. 8281 m^2 d. 8282 m^2

Part II Performance Task
Directions: Work in groups of 5 - 6 members.
Task: Bridge Project

a. Begin the activity by having online bridge simulation.
b. Create a bridge design.
c. Construct a scale model.
d. Test the strength of the model.
e. Showcase your output to the class and make presentation using the following guide questions/topics
   - How are the key concepts of mathematics especially a quadratic functions are being used in designing your bridge? (eg. the intersection of the driving lane and the arch, vertical pole and the arch..)
   - What are the properties of the graph of the quadratic functions that help strengthen your bridge?
   - .What is the equation of the quadratic function that modeled your design?

- How does changing the length of the main span affect the equation?

f. Submit the write up of your project and your presentation.

Summative Test

Answer Key

1. D   2. D   3. D   4. A   5. D   6. D   7. B   8. B
9. B   10. B   11. B   12. D   13. C   14. C

Performance task of the students might be assessed using the suggested rubric below.

Rubric on Bridge Design and Scale Model			
CATEGORY	**5**	**3**	**1**
Content	Presentation covers all elements relating to the bridge design in depth with details and examples. Our subject knowledge is excellent.	Presentation includes essential knowledge about the bridge design. Details are presented.	Presentation includes essential information about the bridge design but lacks of details
Online Simulation	Knowledge gained from bridge simulation is shown in the design. Forces applied to a brigde model are clearly seen.	Knowledge gained from bridge simulation is shown.	Knowledge gained from bridge simulation is not fully applied.
Designs	Design is complex, detailed and shows imagination and creativity. Numerous alternate concepts and solutions are included.	Design is simple and shows some imagination and creativity. Few concepts and solutions are included.	Design is limited and shows repetition of single ideas. Few concepts and solutions are included.
Understanding	Effectively demonstrates a thorough understanding of the importance of the concepts of quadratic functions in designing and constructing a scale model.	Demonstrates understanding of the importance of the concepts of quadratic functions in designing and constructing a scale model.	Concepts of quadratic functions are slightly applied in designing and constructing a scale model.
Organization of Presentation (Group)	The presentation is well organized. The application of the concepts of quadratic function in the design and construction is clearly presented and explained.	The application of the concepts of quadratic function in the design and construction is clearly presented and explained	The application of the concepts of quadratic function in the design and construction are presented.
Reflection Note	Reflection note provides clear and convincing evidence for our choice of design, accurate and detailed blueprints of our model bridge, and other supporting documentation, including a concept web, graphs, charts, and photos.	Reflection note provides evidence for our choice of design, accurate blueprints of our model bridge, and some supporting documentation, such as a concept web, graphs, charts, and photos.	Reflection note provides minimal evidence for our choice of design, blueprints of our model bridge with errors, and one other supporting document.

Summary/Synthesis/Feedback

This module was about concepts of quadratic functions. In this module, you were encouraged to discover by yourself the properties and characteristics of quadratic functions. The knowledge and skills gained in this module will help you solve real-life problems involving quadratic functions which would lead you to make better decisions in life and to perform practical tasks. Moreover, concepts you learned in this module will allow you to formulate real-life problems and solve them in variety of ways.

Glossary of Terms

axis of symmetry – the vertical line through the vertex that divides the parabola into two equal parts

direction of opening of a parabola – can be determined from the value of a in $f(x) = ax^2 + bx + c$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

domain of a quadratic function – the set of all possible values of x. thus, the domain is the set of all real numbers.

parabola – the graph of a quadratic function.

quadratic function- a second-degree function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. This is a function which describes a polynomial of degree 2.

R**ange of a quadratic function** - consists of all real numbers greater than or equal to the y-coordinate of the vertex if the parabola opens upward.

- consists of all y less than or equal to the y-coordinate of the vertex if the parabola opens downward.

revenue function - suppose *x* denotes the number of units a company plan to produce or sell, a revenue function $R(x)$ is defined as: $R(x)$= (price per unit) x (number of units produced or sold).

vertex – the turning point of the parabola or the lowest or highest point of the parabola. If the quadratic function is expressed in standard form $y = a(x – h)^2 + k$, the vertex is the point (h, k).

zeros of a quadratic function – is the value/s of x when y equals 0. The real zeros are the x-intercepts of the parabola.

REFERENCES

Basic Education Curriculum (2002)

Basic Education Assistance for Mindanao. (2008) *Module 3:Quadratic Functions and their Graphs (Learning Guide 6)*, p. 14-15, 34, 37, 40-41, 44

Catao, E. et al. PASMEP Teaching Resource Materials, Volume II

Cramer, K., (2001) Using Models to Build Middle-Grade Students' Understanding of Functions. *Mathematics Teaching in the Middle School.* 6 (5),

De Leon, Cecile, Bernabe, Julieta. (2002) *Elementary Algebra.* JTW Corporation, Quezon City, Philippines.

Gallos, F., Landrito, M. & Ulep, S. (2003) *Practical Work Approach in High School Mathematics (Sourcebook fro Teachers)*, National Institute for Science and Mathematics Education Development, Diliman, Quezon City.

Hayden, J., & Hall, B. (1995) *Trigonometry,* (Philippine Edition) Anvil Publishing Inc., Quezon City, Philippines.

Hernandez, D. et al. (1979) *Functions (Mathematics for Fourth Year High School),* Ministry of Education, Culture and Sports. Capitol Pub. House Inc., Diliman Quezon City.

INTEL, Assessment in the 21st Century Classroom E Learning Resources.

Joson, Lolita, Ymas Jr., Sergio. (2004) *College Algebra.* Ymas Publishing House, Manila, Philippines.

Lapinid, M., & Buzon, O. (2007) *Advanced Algebra, Trigonometry and Statistics,*Salesiana Books by Don Bosco Press, Inc., Makati City.

Leithold, L. (1992). *College Algebra and Trigonometry*. Philippines :National Book Store Inc.

Marasigan, J., Coronel, A. et. Al. *Advanced Algebra with Trigonometry and Statistics,* The Bookmark Inc., San Antonio Villagr, Makati City.

Ministry of Education, Culture and Sports. (1979) *Functions (Mathematics for Fourth Year High School),* Capitol Pub. House Inc., Diliman Quezon City.

Numidos, L. (1983) *Basic Algebra for Secondary Schools,* Phoenix Publishing House Inc., Quezon Avenue, Quezon City.

Orines, Fernando B., (2004). *Advanced Algebra with Trigonometry and Statistics:* Phoenix Publishing House, Inc.

WEBLINKS

Website links for Learning Activities

1. http://www.analyzemath.com/quadraticg/Problems1.html
2. http://www.learner.org/workshops/algebra/workshop4/
3. http://www.cehd.umn.edu/ci/rationalnumberproject/01_1.html

4. http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html
5. http://answers.yahoo.com/question/index?qid=20090531221500AAoXyOi
6. http://algebra2trigplt.wikispaces.com/file/view/cb_sb_math_miu_L6_U3_embedded_assessment3-1_SE_v2.pdf
7. http://www.youtube.com/watch?v=BYMd-7Ng9Y8
8. http://www.utdanacenter.org/mathtoolkit/downloads/scope/OLDalg2scope/ovadawatta.pdf
9. http://www.west-fargo.k12.nd.us/district/academic/images/MathProblemSolvingRubric.pdf
10. http://www.mathopenref.com/quadraticexplorer.html
11. http://mathitude.perso.sfr.fr/PDF/safety_kit_1.pdf
12. http://mathwithclayton.wikispaces.com/file/view/unit07.pdf/418858436/unit07.pdf
13. http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html
14. http://www.teacherweb.com/ny/arlington/algebraproject/U5L6ApplicationsofQuadraticFunctionsDay1.pdf
15. http://www.youtube.com/watch?v=5bKch8vitu0
16. **http://ts2.mm.bing.net/th?id=H.4577714805345361&pid=15.1**
17. http://www.mathopenref.com/quadraticexplorer.html
18. http://fit-ed.org/intelteachelements/assessment/index.php

Website links for Images

1. http://www.popularmechanics.com/cm/popularmechanics/images/y6/free-throw-0312-mdn.jpg
2. http://sites.davidson.edu/mathmovement/wp content/uploads/2011/11/trajectories.png
3. http://web.mnstate.edu/lindaas/phys160/lab/Sims/projectileMotion.gif -throw-0312-mdn.jpg
4. http://64.19.142.13/australia.gov.au/sites/default/files/agencies/culture/library/images/site_images/bridgesunset-web.jpg
5. http://ts1.mm.bing.net/th?id=H.5065653090977496&pid=15.1
6. http://ts2.mm.bing.net/th?id=H.4852742999509017&pid=15.1

TEACHING GUIDE

MODULE 3: VARIATIONS

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of variations.

Performance Standard:

The learner is able to formulate and solve accurately problems involving variations.

UNPACKING THE STANDARDS FOR UNDERSTANDING

SUBJECT: Math 9	LEARNING COMPETENCIES
QUARTER: Second **TOPIC: Variations, Direct Variations, Inverse Variations, Joint Variations, Combined Variations** **LESSONS:** **1. Direct Variations** **2. Inverse Variations** **3. Joint Variations** **4. Combined Variations** **SUBJECT: Math 9** **QUARTER: Second** **TOPIC: Variations, Direct Variations, Inverse Variations, Joint Variations, Combined Variations**	1. Illustrates situations that involve the following variations: (a) direct; (b) inverse; (c) joint; (d) combined. 2. Translates into variation statement a relationship between two quantities given by (a) table of values; (b) a mathematical equation; (c) a graph, and vice versa. 3. Solve problems involving variations.

<table>
<tr><td>LESSONS:
1. Direct Variations

2. Inverse Variations

3. Joint Variations

4. Combined Variations</td><td colspan="2"></td></tr>
<tr><td>Writer: Sonia E. Javier</td><td>ESSENTIAL UNDERSTANDING:

Students will understand that variations are useful tools in solving real-life problems and in making decisions given certain constraints.</td><td>ESSENTIAL QUESTION:

How do variations facilitate finding solutions to real-life problems and making decisions?</td></tr>
<tr><td></td><td colspan="2">TRANSFER GOAL:

Students will be able to apply the key concepts of variations in formulating and solving real-life problems and in making decisions.</td></tr>
</table>

Product/Performance

The following are products and performances that students are expected to come up with in this module.

1. Journal writing and portfolio of real-life situations/pictures where concepts of variation are applied.
   a. Illustration of situations that involve variations
   b. Table of values and graphs representing direct and inverse variations
   c. Mathematical equations representing variation statements
   d. Solution of problems involving variations
2. Scenario of task in paragraph form incorporating GRASPS: Goal, Role, Audience, Situation, Product/Performance, Standards. e.g. Design of a plans on how to market a particular product considering the number of product sold, cost of the product, and the budget for advertising.

Assessment Map

TYPE	KNOWLEDGE	PROCESS/SKILL	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	**Pre-Test:** Identifying situations which illustrate direct, inverse, joint and combined variations	**Pre-Test:** Translating statements into mathematical sentences. Solving problems applying the concepts of direct, inverse, joint and combined variations	**Pre-Test:** Solving real-life problems involving direct, inverse, joint and combined variations	**Pre-Test:** Products and performances related to direct, inverse, joint and combined variations
Formative	**Quiz: Lesson 1** Identifying situations that illustrate direct variations Illustrates situations that involve direct variation Describes the relationship between quantities in a direct variation. **Quiz: Lesson 2** Identifying situations that illustrate inverse variations	**Quiz: Lesson 1** Representing situations by mathematical sentences Translates into variation statement a relationship between two quantities given by (a) table of values; (b) a mathematical equation; (c) a graph, and vice versa. Solving problems applying the concepts of direct variation. **Quiz: Lesson 2** Representing situations by mathematical sentences	**Quiz: Lesson 1** Formulating a mathematical equation that represents a given situation. Solving real-life problems involving direct variations. **Quiz: Lesson 2** Formulating a mathematical equation that represents a	

	Illustrates situations that involve inverse variation. Describes the relationship between quantities in an inverse variation.	Translates into variation statement a relationship between two quantities given by (a) table of values; (b) a mathematical equation; (c) a graph, and vice versa. Solving problems applying the concepts of inverse variation.	given situation. Solving real-life problems involving inverse variation.	
	Quiz: Lesson 3 Identifying situations that illustrate joint variation. Illustrates situations that involve joint variation. Describes the relationship between quantities in a joint variation.	**Quiz: Lesson 3** Representing situations by mathematical sentences Translates into a variation statement a relationship between two quantities given by an equation and vice versa. Solving problems applying the concepts of joint variation.	**Quiz: Lesson 3** Formulating a mathematical equation that represents a given situation. Solving real-life problems involving joint variation.	
	Quiz: Lesson 4 Identifying situations that illustrate combined	**Quiz: Lesson 4** Representing situations by mathematical sentences	**Quiz: Lesson 4** Formulating mathematical equation that represents a	

	variation. Illustrates situations that involve combined variation Describes the relationship between quantities in a combined variation.	Translates into a variation statement a relationship between two quantities given by an equation and vice versa. Solving problems applying the concepts of combined variation.	given situation. Solving real-life problems involving combined variation.	
Summative	**Post Test** Identifying situations that illustrate direct, inverse, joint, and combined variations	**Post Test** Representing situations by mathematical sentences. Translates into variation statement a relationship between two quantities given by (a) a table of values; (b) a mathematical equation; (c) a graph, and vice versa. Solving problems applying the concepts of variations.	**Post Test** Formulating mathematical equation that represents a given situation. Solving real-life problems involving variation.	**Post Test** Products and performances related to variation.
Self-Assessment	Journal writing and portfolio of real-life situations/pictures where concepts of variation are applied. Scenario of task in paragraph form incorporating GRASPS: Goal, Role, Audience, Situation, Product/Performance, Standards.			

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	The learner demonstrates understanding of key concepts of variation. Identifying situations that illustrate direct, inverse, joint, and combined variations Describing relationship between quantities. Illustrating situations that are direct, inverse, joint and combined variations. Solving problems applying the concept of variations. Solving real-life problems involving variations.	**Paper and Pencil Test** Items 1, 2, 3, and 4	1 point for every correct response
Process/Skills 25%		Items 5, 6, 7, 9, 11, 16, 19, and 20	1 point for every correct response
Understanding 30%		Items 8, 10, 12, 13, 14, 15, 17, and 18	1 point for every correct response
Product/ Performance 30%			Portfolio and Journal Writing Design of Plan GRASP Form Criteria: 1. Clarity of Presentation 2. Accuracy 3. Justification

C. Planning for Teaching-Learning

This module covers key concepts of variations. It is divided into four lessons namely: Direct Variation; Inverse Variation; Joint Variation and Combined Variation.

In Lessons 1 and 2 of this module, the students will illustrate situations that involve direct and inverse variations; translate into variation statement a relationship involving direct and inverse variations between two quantities given by a table of values, a mathematical equation, and a graph, and vice versa; and solve problems involving direct variations.

In Lessons 3 and 4, the students will illustrate situations that involve joint and combined variations; translate into variation statement a relationship involving joint and combined variations between two quantities given by a a mathematical equation, and solve problems involving direct variations.

In all the lessons, the students are given the opportunity to use their prior knowledge and skills in learning variations. They are also given varied activities to process the knowledge and skills learned and further deepen and transfer their understanding of the different lessons.

As an introduction to the main lesson, a situation where the concept of variation is applied will be provided the students. The students should be able to answer the questions that follow.

INTRODUCTION AND FOCUS QUESTIONS:

Do you know that an increasing demand for paper contributes to the destruction of trees from which paper is made?

http://massrealestatelawblog.com/wp-content/uploads/sites/9/2012/07/tree-cutting.jpg

If waste papers were recycled regularly, it would help prevent the cutting down of trees, global warming and other adverse effects that would destroy the environment. Paper recycling does not only save the earth but also contributes to the economy of the country and to the income to some individuals.

This is one situation where questions, such as "Will a decrease in the production of papers contribute to the decrease in the number of trees being cut?" can be answered using the concepts of variations.

There are several relationships of quantities which we will encounter from this situation. You will learn how a change in one quantity could correspond to a predictable change on the other.

Explain to the students the importance of studying variations to be able to answer the following questions:

- How can I make use of the representations and descriptions of a given set of data?
- What are the benefits in studying variation help solve problems in real life?

Objectives:

After the learners have gone through the lessons contained in this module, they are expected to:

a. identify and illustrate practical situations that involve variations.

b. translate variation statements into mathematical statements.

c. translate into variation statement a relationship involving variation between two quantities given by a table of values, a mathematical equation, and a graph, and vice versa.

c. solve problems involving variations.

To do well in this module, they will need to remember and do the following:

1. Study each part of the module carefully.
2. Take note of all the concepts discussed in each lesson.

3. **PRE - ASSESSMENT**

Check students' prior knowledge, skills, and understanding of mathematics concepts related to direct, inverse, joint and combined variations. Assessing these will facilitate teaching and the students' understanding of the lessons in this module.

Answer Key

1. a	11. d
2. b	12. c
3. c	13. d
4. a	14. d
5. a	15. b
6. a	16. d
7. c	17. d
8. c	18. b
9. a	19. b
10. a	20. c

LEARNING GOALS AND TARGETS:

Students are expected to demonstrate an understanding of key concepts of variations, to formulate real-life problems involving these concepts, and to solve these using a variety of strategies. They are also expected to investigate mathematical relationships in various situations involving variations.

Allow the students to begin with exploring situations that will introduce them to the basic concepts of variations and how they are applied in real life.

Activity No.1: Before Lesson Response

Answer Key

1. I	6. D
2. N	7. N
3. I	8. N
4. D	9. D
5. I	10. D

Lesson 1: DIRECT VARIATIONS

What to KNOW:

Assess students' knowledge of the concepts of variations by providing practical situations. This will facilitate teaching and students' understanding of variations. Tell them that as they go through this lesson, they have to think of this important question: *"How are concepts of variations used in solving real-life problems and in making decisions?"*

Allow the students to start with Activity 1. This will reveal their background knowledge on variations

Activity 2: What's the Story Behind!

Answer Key

Allow students to react to the situation. Accept possible answers. Let them recall an instance where they experienced a situation similar to that in the activity. These will serve as a springboard for the discussion of the relationship of quantities involved like time, distance and rate.

Go to the next activity to introduce to them those situations involving direct variations. Students' responses in Activity 3 may be based on students' analysis of the situation. Their skills in recognizing mathematical patterns may help them in answering items 1 and 2. Answers to items 3 and 4 will vary with the students' responses. Encourage discussions to allow them to recognize relationships among quantities.

Activity 3: Let's Recycle!

Answer Key

1. The number of points doubled or tripled as the number of kilos of papers doubled or tripled.
2. 100 kilograms; P = 5n
3. and 4. Answers will vary.

Activity 4 is another situation to reinforce the background knowledge you have established about the concept on direct variations. This time, we will deal with the mathematical solutions in answering the items in the activity.

Activity 4: How Steep is Enough!

Answer Key

1. The distance covered by the cyclist gets larger as the time in travelling gets larger.

2. 85 kilometers

3. A mathematical equation is required. The equation is d = 10t, where *d* is the distance in kilometers (km) and *t* is the time in hours (hr).

4. To get the constant number which is very evident in the values of the distance, the distance is divided by the time.

What to PROCESS:

In this section, discuss with the students the concept behind the activities they have just performed.

Teacher's Notes

There is *direct variation* whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"

"y is directly proportional to x" and

"y is proportional to x"

Is translated mathematically as $y = kx$, where k is the constant of variation.

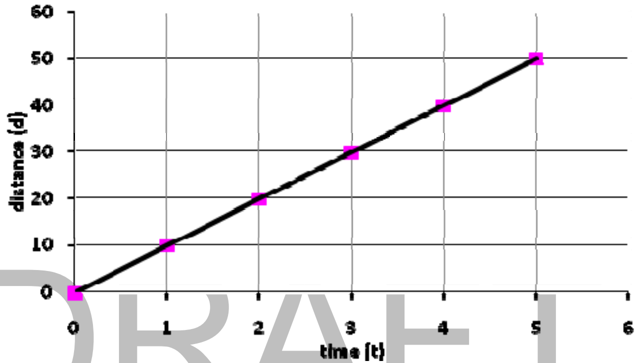
For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Teach them how to transform a statement into a mathematical sentence, and how to determine the constant of variation.

Activity 5: Watch This!

In this activity, allow the students to participate in the discussion. Allow them to apply the concepts to the previous lessons presented as in Activity 4. Let them determine the relationship of quantities from tables and graphs.

Show them the graph of a direct variation in the form y = kx. Tell them that the graph illustrated is that of d = 10t, where d is the distance and t is the time. Ask them if they noticed that the graph utilizes only the positive side. Explain that in practical situations, only quadrant 1 is used.



For emphasis, provide statements which is not of the form y = kx such as $y = 2x + 3$, $y = 3^x$ and $y = x^2 - 4$.

Explain that there are other direct variations of the form y = kx, as in the case of $y = kx^2$. The graph which is not a line but a parabola.

Provide examples for detailed solutions to problems involving direct variation.

Teacher's Notes

Example:

1. If *y* varies directly as *x* and *y* = 24 when *x* = 6, find the variation constant and the equation of variation.

   Solution:

   a. Express the statement "*y varies directly as x*", as $y = kx$.

   b. Solve for *k* by substituting the given values in the equation.

   $$y = kx$$
   $$24 = 6k$$
   $$k = \frac{24}{6}$$
   $$k = 4$$

   Therefore, the constant of variation is 4.

   c. Form the equation of the variation by substituting 4 in the statement *y = kx*. Thus, *y = 4x*.

Let the students determine the mathematical statement using a pair of values from the table in Example 2.

Teacher's Notes

2. The table below shows that the distance *d* varies directly as the time *t*. Find the constant of variation and the equation which describes the relation.

Time (hr)	1	2	3	4	5
Distance (km)	10	20	30	40	50

Solution:

Since the distance *d* varies directly as the time *t*, then

$$d = kt$$

Using one of the pairs of values, (2, 20), from the table, substitute the values of d and t in $d = kt$ and solve for k.

$$d = kt$$
$$20 = 2k$$
$$k = \frac{20}{2}$$
$$k = 10$$

Therefore, the constant of variation is 10.

Form the mathematical equation of the variation by substituting 10 in the statement $d = kt$.

$$d = 10t$$

Provide them with the next example where more than one pair of quantities is required to solve the problem. Show them both solutions to the problem.

Teacher's Notes

3. If x varies directly as y and $x = 35$ when $y = 7$, what is the value of y when $x = 25$?

Solution 1.

Since x varies directly as y, then the equation of variation is in the form $x = ky$.

Substitute the given values of y and x to solve for k.

$$35 = k(7)$$
$$k = \frac{35}{7}$$
$$k = 5$$

Hence, the equation of variation is $x = 5y$.

Solve for y when $x = 25$,

$$25 = 5y$$
$$y = \frac{25}{5}$$
$$y = 5$$

Solution 2:

Since $\frac{x}{y}$ is a constant, we can write k = $\frac{x}{y}$. From here we can establish a proportion such that, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$, where $x_1 = 35$, $y_1 = 7$ and $x_2 = 25$.

Substituting the values, we get

$$\frac{35}{7} = \frac{25}{y_2}$$
$$5 = \frac{25}{y_2}$$
$$y_2 = \frac{25}{5}$$
$$y_2 = 5$$

Therefore, y = 5 when $x = 25$.

Tell the students to use the mathematical ideas and the examples presented in the preceding section to answer the exercises in Activity 6. You may use some of these items to assess students understanding.

Activity 6: It's Your Turn!

Answer Key

A.
1. $F = kd$
2. $C = kw$
3. $S = kd$
4. $A = ks^2$
5. $D = ks$
6. $L = kh$
7. $C = kl$
8. $V = kh$
9. $W = km$
10. $A = kh$

B.
1. $k = -3$; $y = -3x$
2. $k = \frac{3}{7}$; $y = \frac{3}{7}x$
3. $k = \frac{1}{5}$; $y = \frac{1}{5}x$
4. not direct variation
5. $k = \frac{3}{4}$; $y = \frac{3}{4}x$
6. not direct variation
7. $k = 10$; $y = 10x$
8. not direct variation
9. $k = 5$; $y = 5x$
10. $k = \frac{1}{2}$; $y = \frac{1}{2}x^2$

C.
1. $y = 4x$
2. $y = \frac{15}{4}x$
3. $y = \frac{7}{4}x$
4. $y = \frac{8}{5}x$
5. $y = 16x$
6. $y = \frac{7}{9}x$
7. $y = \frac{2}{3}x$
8. $y = \frac{1}{2}x$
9. $y = 8x$
10. $y = \frac{5}{12}x$

D.
1. $y = 36$
2. $y = -63$
3. $x = \frac{8}{3}$
4. $x = 4$
5. $y = 7.5$

What to REFLECT and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of direct variations. Provide them with opportunities to think deeper and test further their understanding of the lesson by doing Activity 7.

Activity 7: Cans Anyone!

Answer Key

1. a. c = kn
   b. k = 15; c = 15n
   c. c is doubled when n is doubled
      c is tripled when n is tripled
   d. provide graph to students
   e. PhP 400.00
   f. answers of students vary

2. 10π cm, 15π cm, 18π cm, 20π cm
   a. c = kd
   b. π ; c = πd
   c.

d	7	10	15	18	20
c	7π	10π	15π	18π	20π

3. PhP 4,550.00
4. 5 gallons
5. PhP 30,000.00
6. 1107
7. 70
8. 12 ½
9. 281.25 Pascal
10. $5\frac{1}{3}$ m

You may provide this example as an enrichment exercise.

If y varies directly as the square of x, how is y changed if x is increased by 20%?

Solution:

The equation is $y = kx^2$
If x is increased by 20%, the equation becomes $y = k(x + .2x)^2$
$= k(1.2x)^2 = 1.44kx^2$
Then y is increased by 44%

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of direct variation by doing a practical task. Let them perform Activity 8. They can do this in groups. Explain to them that this is one of their group's outputs for the second quarter.

Summary/Synthesis/Generalization:

This lesson was about variations and how they are illustrated in real life. The lesson provided the students with opportunities to describe variations using practical situations and their mathematical representations. Let the students do Activity 9.

Their understanding of the lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next lesson on Inverse Variations.

Lesson 2: INVERSE VARIATIONS

What to KNOW:

Assess students' knowledge of the concepts discussed previously. These will facilitate teaching and students' understanding of variations.

The activities on direct variation showed them the behavior of the quantities involved. In one of the activities, an increase in time travelled by a car causes an increase in the distance travelled. Ask the students how an increase in speed affect the time in travelling? Let them find out in one of the activities.

Activity 10: Who's Increasing or Decreasing!

Answer Key

1. a. As the speed of the car increases, the time in travelling decreases.

   b. s = $\frac{k}{t}$, where, s is the speed in kph and t is the time in hours.

   c. Yes. Multiplying the values of the speed and time gives us the constant.

Teacher's Notes

The situation in the problem shows that "an increase in speed produces a decrease in time in travelling." The situation produces pairs of numbers, whose product is constant. Here, the time t varies inversely as the speed s such that

$$st = 40 \text{ (a constant)}$$

In this situation, "the speed s is inversely proportional to the time t," and is written as $s = \frac{k}{t}$, where k is the proportionality constant or constant of variation.

Hence, the equation represented in the table and graph is $s = \frac{40}{t}$, where, $k = 40$.

Answer Key

2. a. The seesaw tends to balance as one of the kids moves closer or farther from the fulcrum.

   b. The heavier kid should move closer to the fulcrum in order to balance the lighter kid on the other side of the seesaw.

   c. The weight is inversely proportional to the distance from the fulcrum.

   d. Yes, as one quantity increases, the other quantity decreases.

Reinforce further their understanding in both the previous and the present lessons by doing the next activity.

Activity 11: Observe and Compare

Answer Key

1. The values in both tables follow a certain pattern.
2. In Table A, the value of y increases as the value of x increases. In Table B, the value of y increases as the value of x decreases.
3. In Table A, the value of y is doubled as the value of x is doubled and y is tripled as x is tripled. In Table B, the value of y is halved as the value of x is doubled and y is divided by 3 when x is tripled.
4. The relationships illustrated in tables A and B are two different types of relationships.
5. Table A: y = 2x; Table B: $y = \frac{480}{x}$
6. Direct variation occurs whenever a situation produces pairs of numbers whose ratio is a constant. On the other hand, inverse variation occurs whenever a situation produces pairs of numbers whose product is a constant.

What to PROCESS:

In this section, let the students apply the key concepts of inverse variations. Tell them to use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

Teacher's Notes

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

The statement, "y varies inversely as x," translates to $y = \frac{k}{x}$, where k is the constant of variation.

For two quantities x and y, an increase in x causes a decrease in y or vice versa.

Provide the students with the necessary examples to guide them in performing the activities that follow

Teacher's Notes

Examples:

1. Find the equation and solve for k: y varies inversely as x and $y = 6$ when $x = 18$.

Solution: The relation y varies inversely as x translates to $y = \frac{k}{x}$. Substitute the values to find k:

$$y = \frac{k}{x}$$

$$6 = \frac{k}{18}$$

$$k = (6)(18)$$

$$k = 108$$

The equation of variation is $y = \frac{108}{x}$

Teacher's Notes

2. If y varies inversely as x and y = 10 when x = 2, find y when x = 10.

This concerns two pairs of values of x and y which may be solved in two ways.

Solution 1: First, set the relation, and then find the constant of variation k.

$$xy = k$$

$$(2)(10) = k$$

$$k = 20$$

The equation of variation is $y = \frac{20}{x}$

Next, find y when x = 10 by substituting the value of x in the equation

$$y = \frac{20}{x}$$

$$y = \frac{20}{10}$$

$$y = 2$$

Solution 2: Since $k = xy$, then for any pairs of x and y, $x_1y_1 = x_2y_2$.

If we let $x_1 = 2$, $y_1 = 10$, and $x_2 = 10$, find y_2.

By substitution:

$$x_1y_1 = x_2y_2$$

$$2(10) = 10(y_2)$$

$$20 = 10y_2$$

$$y_2 = \frac{20}{10}$$

$$y_2 = 2$$

Hence, y = 2 when x = 10.

Ask the students to perform Activity 12. In this activity, the students will identify which relationships are inverse variations and which are not. Ask them to explain important concepts.

Activity 12: It's Your Turn!

Answer Key

A. 1. p = $\frac{k}{n}$

2. n = $\frac{k}{s}$

3. n = $\frac{k}{d}$

4. r = $\frac{k}{t}$

5. c = $\frac{k}{n}$

6. l = $\frac{k}{w}$

7. d = $\frac{k}{v}$

8. a = $\frac{k}{m}$

9. b = $\frac{k}{h}$

10. m = $\frac{k}{g}$

B. 1. k = 2, y = $\frac{2}{x}$

2. k = 72, y = $\frac{72}{x}$

3. k = 6, y = $\frac{6}{x}$

4. k = 5, y = $\frac{5}{x}$

5. k = 12, y = $\frac{12}{x}$

6. k = 4, y = $\frac{4}{x}$

7. k = 60, y = $\frac{60}{x}$

8. k = 54, y = $\frac{54}{x}$

9. k = 4, y = $\frac{4}{x}$

10. k = 8, y = $\frac{8}{x}$

C. 1. y = 2

2. r = 60

3. p = $\frac{3}{16}$

4. x = 8

5. w = 1

6. m = 2

7. y = $\frac{50}{7}$

8. a = 16

9. w = 2

10. y = $\frac{10}{3}$

What to REFLECT and FURTHER UNDERSTAND:

Ask the students to have a closer look at some aspects of inverse variations. Provide them with opportunities to think deeper and test further their understanding of the lesson by doing Activity 13.

Activity 13: Think Deeper!

Answer Key

1. 5 men
2. 2 more students
3. 7 ½ hours
4. PhP 500.00
5. 84 km/hr

Activity 14: After Lesson Response

Provide the students with Activity 14 to check if their answers are the same as their answers in Activity 1. This will rate how well they understood the discussions on direct and inverse variations.

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of inverse variations through culminating activities that reflect meaningful and relevant problems/situations. Let them perform Activity 15. Ask the students to work in groups.

Activity No.15: Demonstrate Your Understanding!

Create a scenario of the task in paragraph form incorporating GRASP: Goal, Role, Audience, Situation, Product/Performance, Standards

Summary/Synthesis/Generalization:

Activity 16: WRAP IT UP!

Tell the students to summarize what they have learned from this lesson. Provide real-life examples. Illustrate using tables, graphs and mathematical equations showing the relationships of quantities.

Lesson 3: Joint Variation

What to KNOW:

This lesson deals with another concept of variation, the joint variation.

Tell the students that the situations that they have studied involved only two quantities. What if the situation requires the use of more than two quantities?

Physical relationships, such as area or volume, may involve three or more variables simultaneously.

What to PROCESS:

Tell the students that the concept of joint variation will help them deal with problems involving more than two variables or quantities.

Teacher's Notes

The statement *a* varies jointly as *b* and *c* means $a = kbc$, or $k = \frac{a}{bc}$, where *k* is the constant of variation.

At this phase, provide students with examples that will lead them in solving problems on joint variation. Test the students' understanding by asking them to:

a. translate statements into mathematical sentences.

b. find the constant of variation.

c. solve for the values of missing variables.

d. formulate the equation for the relationship of variables or quantities.

Teacher's Notes

Examples:

1. Find the equation of variation where *a* varies jointly as *b* and *c*, and $a = 36$ when $b = 3$ and $c = 4$.

Solution: $a = kbc$

$$36 = k(3)(4)$$

$$k = \frac{36}{12}$$

$$k = 3$$

Therefore, the required equation of variation is $a = 3bc$

2. z varies jointly as x and y. If $z = 16$ when $x = 4$ and $y = 6$, find the constant of variation and the equation of the relation.

Solution: $z = kxy$

$$16 = k(4)(6)$$

$$k = \frac{16}{24}$$

$$k = \frac{2}{3}$$

The equation of variation is $z = \frac{2}{3}xy$

Activity No.17: What is Joint Together?

Answer Key

A.
1. $P = kqr$
2. $V = klwh$
3. $A = kbh$
4. $V = khr^2$
5. $H = kI^2R$
6. $F = kma$
7. $V = kBh$
8. $A = kbh$
9. $S = kwd$
10. $V = kIR$

B.
1. a. $z = 84$
   b. $x = 9$
   c. $y = 10$
2. $z = \frac{18}{5}$
3. $x = \frac{3}{2}$
4. $g = \frac{16}{9}$
5. $q = .08$
6. $d = .04$
7. $x = 54$
8. $y = 8$
9. a. $W = \frac{1}{9}ld^2$
   b. $l = 22.5$ cm
   c. $W = 5.6$ kg
10. 44 cm^2

What to REFLECT and FURTHER UNDERSTAND:

Tell the students that after having developed their understanding of the concepts applied in the different activities, their goal now is to apply these concepts to various real-life situations. Provide them with opportunities to think deeper and test further their understanding of the lesson by doing Activity 18.

Activity 18: Who is He?

Answer Key

<u>H</u>	<u>U</u>	<u>E</u>	<u>N</u>	<u>D</u>	<u>A</u>
1	2	3	4	5	6

Activity 19: Think Deeper!

Having developed your knowledge of the concepts in the previous activities, your goal now is to apply these concepts to various real-life situations.

Answer Key

1. $38\frac{2}{5}$ liters
2. 35 cm^3
3. 2816 cm^3
4. 150 grams
5. $18\frac{2}{3}$ kg

What to TRANSFER:

Give the students opportunities to demonstrate their understanding of variations through journal writing and portfolio making of real-life situations where concepts of joint variation are applied.

Summary / Synthesis / Generalization:

Activity 20: WRAP IT UP!

On a sheet of paper, let them summarize what they have learned from this lesson. Ask them to provide real-life examples. Tell them to illustrate situations using variation statements and mathematical equations to show the relationship of quantities.

Lesson 4: Combined Variation

What to KNOW:

Tell the students that combined variation is another physical relationship among variables. This is the kind of variation that involves both the direct and inverse variations.

What to PROCESS

Teacher's Notes

The statement "*z* varies directly as *x* and inversely as *y*" means $z = \frac{kx}{y}$, or $k = \frac{zy}{x}$, where *k* is the constant of variation.

This relationship among variables will be well illustrated in the following examples. Use these examples to guide the learners in solving the activities that follow.

Teacher's Notes

Examples:

1. Translating statements into mathematical equations using *k* as the constant of variation.

   a. T varies directly as *a* and inversely as b.

   $$T = \frac{ka}{b}$$

   b. Y varies directly as x and inversely as the square of z.

   $$Y = \frac{kx}{z^2}$$

The following is an example of combined variation where one of the terms is unknown.

2. If z varies directly as x and inversely as y, and z = 9 when x = 6 and y = 2, find z when x = 8 and y = 12.

Solution:

The equation is $z = \frac{kx}{y}$

Solve for k by substituting the first set of values of z, x and y in the equation

$$z = \frac{kx}{y}$$

$$9 = \frac{6k}{2}$$

$$k = \frac{9}{3}$$

$$k = 3$$

Solve for *z* when *x* = 8 and *y* = 12. Using the equation $z = \frac{3x}{y}$,

$$z = \frac{(3)(8)}{12}$$

$$z = 2$$

Activity No.21: DV and IV Combined!

Answer Key

A. 1. W = $\frac{ka^2c}{b}$

2. P = $\frac{kx^2}{s}$

3. R = $\frac{kl}{d^2}$

4. A = $\frac{kd}{t^2}$

5. P = $\frac{kt}{v}$

B. 1. a. $\frac{4}{3}$

b. 36

c. 4

B. 2. a. $p = \frac{kqr^2}{s}$

b. k = 3

c. P = 96

d. s = 6

3. $533\frac{1}{3}$

4. b = 48

5. x = 108

What to REFLECT and FURTHER UNDERSTAND

Having developed their knowledge on the concepts in the previous activities, their goal now is to apply these concepts to various real-life situations.

Let the students solve problems in Activity 22.

Activity No.22: How Well do you Understand?

Answer Key

1. I = 30 amperes
2. F = $\frac{25}{9}$ Newtons
3. 100 meters
4. 1 meter/second^2
5. $3067\frac{7}{8}$ kg

What to TRANSFER

Design a plan for how to market a particular product considering the number of units of the product sold, cost of the product, and the budget for advertising. The number of units sold varies directly with the advertising budget and inversely as the price of each product. Incorporate GRASP.

Summary/Synthesis/Generalization:

Activity 23: WRAP IT UP!

On a sheet of paper, ask the students to summarize what they have learned from this lesson. Provide real-life examples. Illustrate using variation statements and mathematical equations showing the relation of quantities.

POST ASSESSMENT

Find out how much the students have learned about these lessons.

Answer Key

1. a	11. d
2. b	12. c
3. c	13. d
4. a	14. d
5. b	15. b
6. a	16. a
7. c	17. d
8. c	18. b
9. a	19. b
10. a	20. c

GLOSSARY OF TERMS

Direct Variation

There is *direct variation* whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"

"y is directly proportional to x" and

"y is proportional to x"

are translated mathematically as $y = kx$, where k is the constant of variation.

For two quantities x and y, an increase/decrease in x causes an increase/decrease in y as well.

Inverse Variation

Inverse variation occurs whenever a situation produces pairs of numbers, whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or $y = \frac{k}{x}$.

Joint Variation

The statement *a* varies jointly as *b* and *c* means $a = kbc$, or $k = \frac{a}{bc}$, where *k* is the constant of variation.

Combined Variation

The statement *t* varies directly as *x* and inversely as *y* means $t = \frac{kx}{y}$, or $k = \frac{ty}{x}$, where *k* is the constant of variation.

REFERENCES

DepEd Learning Materials that can be used as learning resources for the lesson on Variation

a. EASE Modules Year II
b. Distance Learning Modules Year II

Dionio, Janet D., Flordelita G. Male, and Sonia E. Javier, Mathematics II, St. Jude Thaddeus Publications, Philippines, 2011

Barberan, Richard G., et. Al., Hyper8 Math II, Hyper8 Publications, Philippines 2010

Atela. Margarita E., Portia Y. Dimabuyu, and Sonia E. Javier, Intermediate Algebra, Neo Asia Publishing Inc., Philippines 2008

TEACHING GUIDE

Module 4: Zero, Negative, Rational Exponents and Radicals

A. Learning Outcomes

1. Grade Level Standard

The learner demonstrates understanding of key concepts and principles of algebra, geometry, probability and statistics as applied, using appropriate technology, in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.

2. Content and Performance Standards

Content Standards:

The learner demonstrates understanding of key concepts of radicals.

Performance Standards:

The learner is able to formulate and solve accurately problems involving radicals.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="3">SUBJECT:
Grade 9 Mathematics
QUARTER:
Second Quarter
STRAND:
Algebra
TOPIC
Patterns and Algebra
LESSONS:
1. Zero, Negative Integral and Rational Exponent
2. Radicals
3. Radical Equations and its Application</td><td colspan="2">LEARNING COMPETENCIES</td></tr>
<tr><td colspan="2">a. applies the laws involving positive integral exponents to zero and negative integral exponents.
b. illustrates expressions with rational exponents.
c. simplifies expressions with rational exponents.
d. writes expressions with rational exponents as radicals and vice versa.
e. derives the laws of radicals from the laws of rational exponents
f. simplifies radical expressions using the laws of radicals.
g. performs operations on radical expressions.
h. solves equations involving radical expressions.
i. solves problems involving radicals.</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:
Students will understand that the concept of radicals can be applied in formulating and solving real-life problems.</td><td>ESSENTIAL QUESTION:
How can you apply the concepts of radicals to real life?</td></tr>
<tr><td></td><td colspan="2">TRANSFER GOAL:
Apply the concepts of radicals in formulating and solving real-life situations and related problems.</td></tr>
</table>

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to come up with in this module.

a. Correctly answered activities.
b. Derive laws of radicals using specified activities.
c. Real life problems involving radicals solved.
d. Create features/write-ups and design/proposal that demonstrate students' understanding of rational exponent and radicals.

ASSESSMENT MAP

<table>
<tr><th>TYPE</th><th>KNOWLEDGE</th><th>PROCESS/SKILLS</th><th>UNDERSTANDING</th><th>PERFORMANCE</th></tr>
<tr>
<td>Pre-assessment/ Diagnostic</td>
<td>Pre-Test: Part I
Applies the laws involving positive integral exponents to zero and negative integral exponents.
Pre-Test: Part I
Illustrates expressions with rational exponents.
Pre-Test: Part I
Writes expressions with rational exponents as radicals and vice versa.</td>
<td>Pre-Test: Part I
Simplifies radical expressions using the laws of radicals.
Pre-Test: Part I
Performs operations on radical expressions.
Pre-Test: Part I
Solves equations involving radical expressions.</td>
<td>Pre-Test: Part I
Solves problems involving radicals.</td>
<td>Pre-Test: Part II
Formulates and accurately solves problems involving radicals.</td>
</tr>
<tr>
<td rowspan="2">Formative</td>
<td colspan="4">Lesson 1:
• applies the laws involving positive integral exponents to zero and negative integral exponents.
• illustrates expressions with rational exponents.
• simplifies expressions with rational exponents.</td>
</tr>
<tr>
<td></td>
<td>Activity 7:
Applies the laws involving positive integral exponents to zero, negative integral exponents and rational exponents.
Activity 8:
Applies the laws involving positive integral exponents to zero and negative integral exponents.</td>
<td>Activity 10:
Applies the laws involving positive integral exponents to zero, negative integral and rational exponents.
Activity 11:
Applies the laws involving positive integral exponents to negative integral exponents in solving real-life related problems.</td>
<td>Activity 21:
Applies the laws involving positive integral exponents to zero and negative integral exponents through formulating and solving real-life problems.</td>
</tr>
</table>

		Activity 9: Applies the laws involving positive integral exponents to zero and negative integral exponents. **Activity 12:** Applies the laws involving positive integral exponents to rational exponents. **Activity 13:** Applies the laws involving positive integral exponents to rational exponents. **Activity 14:** Applies the laws involving positive integral exponents to rational exponents. **Activity 15:** Applies the laws involving positive integral exponents to rational exponents.	**Activity 16:** Applies the laws involving positive integral exponents to zero, negative integral exponents and rational exponents. **Activity 17:** Applies the laws involving positive integral exponents to zero and negative integral exponents through formulating and solving real-life problems. **Activity 18:** Applies the laws involving positive integral exponents to zero and negative integral exponents through formulating and solving real-life problems. **Activity 19:** Anticipation-Reaction Guide.	
	Lesson 2: • writes expressions with rational exponents as radicals and vice versa. • derives the laws of radicals from the laws of rational exponents • simplifies radical expressions using the laws of radicals. • performs operations on radical expressions			

		Activity 4: Writes expressions with rational exponents as radicals and vice versa. **Activity 5:** Writes expressions with rational exponents as radicals and vice versa. **Activity 6 to 7** Simplifying radicals. **Activity 9 to 16** Performs operations on radical expressions.	**Activity 8** Simplifying radicals. **Activity 17:** Writes expressions with rational exponents as radicals and vice versa, followed by questions. **Activity 18:** Deriving the laws of radicals. **Activity 19:** Applies the concepts of radicals in solving real-life problems. **Activity 20 to 21:** Applies the concepts of rational exponents and radicals through formulating and solving real-life problems.	**Activity 23:** Applies the concepts of rational exponents and radicals through formulating and solving real-life problems.
	Lesson 3: • solves equations involving radical expressions • solves problems involving radicals.			
		Activity 4: Solving Radical equations **Activity 5:** Solving radical equations with reasons.	**Activity 6 to 8:** Applies the concepts of radical equations in solving real-life problems.	**Activity 11:** Applies the concepts of rational exponents and radicals through formulating and solving real-life problems.

Summative	**Post-Test: Part I** Illustrates expressions with rational exponents. **Post-Test: Part I** Writes expressions with rational exponents as radicals and vice versa.	**Post-Test: Part I** Simplifies radical expressions using the laws of radicals. **Post-Test: Part I** performs operations on radical expressions. **Post-Test: Part I** Solves equations involving radical expressions.	**Post-Test: Part I** Solves problems involving radicals.	**Post-Test: Part II** Formulates and accurately solves problems involving radicals.
Self-assessment	**Lesson 1: Activity 20:** 3-2-1 CHART **Lesson 1: Activity 22:** Synthesis Journal **Lesson 2: Activity 22 :** IRF SHEET (revise) **Lesson 2: Activity 24:** IRF SHEET (finalization) **Lesson 2: Activity 25:** Synthesis Journal **Lesson 3: Activity 9:** K-W-L CHART **Lesson 3: Activity 10:** Synthesis Journal **Lesson 3: Activity 12:** SUMMARY (Lesson Closure)			

ASSESSMENT MATRIX (Summative Test)

Levels of assessment.	What will I assess?	How will I assess?	How will I score?
Knowledge	Identifying radical equations illustrates expressions with rational exponents. Simplifying radical expressions.	**Part I:** Item 1 **Part I:** Item 2 **Pat I:** Item 3	1 point for every correct answer
Process/Skills	Simplifies radical expressions using the laws of radicals. Performs operations on radical expressions. Simplifies radical expressions.	**Part I:** Item 4 **Part I:** Items 5, 6, 7, **Part I:** Item 8	1 point for every correct answer
Understanding	Solves problems involving radicals.	**Part I:** Items 9, 10, 11, 12,13, 14,	1 point for every correct answer

Product/ Performance	Formulate and accurately solve problems involving radicals.	**Part II:** Items: 15, 16, 17, 18, 19, 20 **GRASPS Form** It is recommended that a ramp has a 14.5 degrees inclination for buildings to be accessible to handicapped persons. The city's engineering department is planning to construct ramps on identified parts of the city. As part of the department you were required to develop a proposal regarding the ramp's dimensions and present this to the Board. The Board ouldl like to see the concept used, practicality and accuracy of computation.	**Rubric:** Mathematical Concept Used **SATISFACTORY(2pts)** Demonstrate a satisfactory understanding of the concepts and uses it to solve the problem. **DEVELOPING (1 pt)** Demonstrate incomplete understanding and has some misconceptions. Accuracy of Computations **SATISFACTORY(2pts)** The computations are correct. **DEVELOPING (1 pt)** Generally, most of the computations are not correct. Practicality **SATISFACTORY(2pts)** The output is suited to the needs of the client and can be executed easily. **DEVELOPING (1 pt)** The output is not suited to the needs of the client and cannot be executed easily

C. Planning for Teaching-Learning

Introduction:

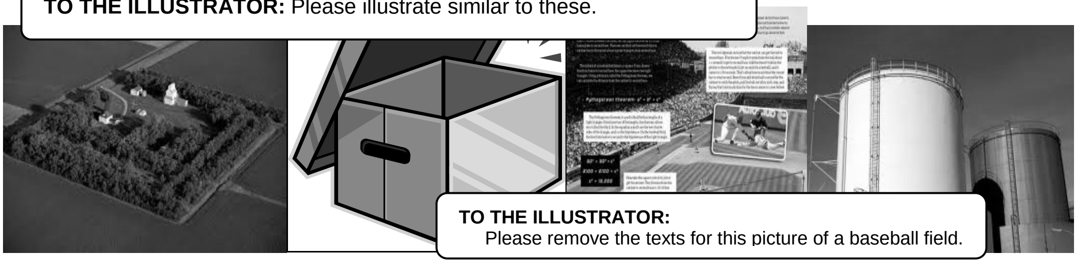
This module covers key concepts of rational exponents and radicals. It is divided into three lessons namely: Zero, Negative Integral and Rational Exponents, Radicals and Radical Equations. In Lesson 1, students will be skilled in simplifying expressions with zero, negative integral and rational exponents. The students will also be given the opportunity to apply these skills in solving real-life problems that involve zero, negative integral and rational exponents. In Lesson 2, students will be capable of rewriting expressions with rational exponents to radicals and vice versa, simplify radicals by reducing the order of the radical and through rationalization; students will also be skilled at simplifying radicals through the fundamental operations. Still, they will be provided with the chance to apply their understanding to solve real-life problems. Lesson 3 will deal basically with the applications of what they learned from Lesson 1 to Lesson 2. They will be able to simplify radical equations and use this skill in solving real-life problems. In each lesson, students are required to accomplish a transfer task that will evaluate their learning of the particular lesson.

In all lessons, students are given the opportunity to use their prior knowledge and skills on laws of exponents. They are also given varied activities to process the knowledge and skills learned and deepen and transfer their understanding of the different lessons.

As an introduction to the main lesson, ask the students the following questions:

Have you ever wondered how to identify the side lengths of a square box or the dimensions of a square lot if you know its area? Have you tried solving the length of any side of a right triangle? Has it come to your mind how you can find the radius of a cylindrical water tank?

TO THE ILLUSTRATOR: Please illustrate similar to these.



TO THE ILLUSTRATOR:
Please remove the texts for this picture of a baseball field.

Motivate the students to find out the answers to these questions and to determine the vast applications of radicals through this module.

Objectives:

After the learners have gone through the lessons found in this module, they are expected to:

a. apply the laws involving positive integral exponents to zero and negative integral exponents.
b. illustrate expressions with rational exponents.
c. simplify expressions with rational exponents.
d. write expressions with rational exponents as radicals and vice versa.
e. derive the laws of radicals from the laws of rational exponents.
f. simplify radical expressions using the laws of radicals.
g. perform operations on radical expressions.
h. solve equations involving radical expressions.
i. solve problems involving radicals.

Check students' prior knowledge, skills, and understanding of mathematics concepts related to Zero, Negative Integral and Rational Exponents and Radicals.

Assessing these will facilitate teaching and students' understanding of the lessons in this module.

III. PRE - ASSESSMENT

1) What is the simplified form of $4^0 5^{-2} 6^{-1} 100^0 \dfrac{1}{2^{-1}}$?

a. 1

b $\dfrac{1}{75}$

c. $\dfrac{1}{150}$

d. $\dfrac{1}{6000}$

2) Which of the following is true?

a. $5^{\frac{1}{2}} + 5^{\frac{1}{3}} = 5^5$

b. $\dfrac{2^{\frac{2}{3}}}{2^{\frac{1}{3}}} = 2^{\frac{2}{9}}$

c. $\left(3^{\frac{1}{3}}\right)^2 = 3^{\frac{3}{2}}$

d. $4^{\frac{-2}{3}} = \dfrac{1}{4^{\frac{2}{3}}}$

3) What is the equivalent of $\sqrt[3]{4} + \sqrt[3]{2}$ using exponential notation?

a. $4^{\frac{1}{3}} + 2^{\frac{1}{3}}$

b. $4^3 + 2^5$

c. 6^3

d. $6^{\frac{1}{3}}$

4) Which of the following radical equations will have x = 6 as a solution?

a. $\sqrt{x} - 2x + 7 = 0$

b. $\sqrt{2x-3} = x - 3$

c. $\sqrt{x} = 9$

d. $3\sqrt{x} = 5$

5) What is the result after simplifying $2\sqrt{3} + 4\sqrt{3} - 5\sqrt{3}$?

a. $-\sqrt{3}$

b. $\sqrt{3}$

c. $11\sqrt{3}$

d. $21\sqrt{3}$

6) If we simplify $\left(2\sqrt{8} + 3\sqrt{5}\right)\left(6\sqrt{8} + 7\sqrt{5}\right)$, the result is equal to _____.

a. $12\sqrt{64} + 14\sqrt{40} + 18\sqrt{40} + 21\sqrt{25}$

b. $12\sqrt{8} + 32\sqrt{40} + 21\sqrt{5}$

c. 201 + $64\sqrt{10}$

d. 195 $\sqrt{10}$

7) What is the result when we simplify $\dfrac{6-\sqrt{2}}{4-3\sqrt{2}}$?

a. 5

b. - 2 $\sqrt{2}$

c. 5 - $\sqrt{2}$

d. -9 - 7 $\sqrt{2}$

8) What is the simplified form of $\dfrac{\sqrt{3}}{\sqrt[4]{3}}$?

a. $\sqrt{3}$

b. $\sqrt[4]{3}$

c. $\sqrt{27}$

d. $\sqrt[4]{27}$

9) Luis walks 5 kilometers due east and 8 kilometers due north. How far is he from the starting point?

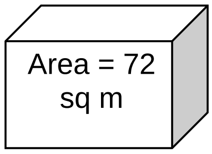
a. $\sqrt{89}$

b. $\sqrt{64}$

c. $\sqrt{39}$

d. $\sqrt{25}$

10) Find the length of an edge of the cube given.



a. $6\sqrt{2}$ meters
b. $6\sqrt{12}$ meters
c. $6\sqrt{2}$ meters
d. $\sqrt{2}$ meters

11) A newborn baby chicken weighs 3^{-2} pounds. If an adult chicken can weigh up to 3^4 times more than a newborn chicken. How much does an adult chicken weigh?

a. 9 pounds
b. 10 pounds
c. 64 pounds
d. $\frac{144}{9}$ pounds

12) A giant swing completes a period in about 15 seconds. Approximately how long is the pendulum's arm using the formula $t = 2\pi\sqrt{\frac{l}{32}}$, where l is the length of the pendulum in feet and t is the amount of time? (use: $\pi \approx 3.14$)

a. 573.25 feet
b. 182.56 feet
c. 16.65 feet
d. 4.31 feet

13) A taut rope starting from the top of a flag pole and tied to the ground is 15 meters long. If the pole is 7 meters high, how far is the rope from the base of the flag pole?

a. 2.83 meters
b. 4.69 meters
c. 13.27 meters
d. 16.55 meters

14) The volume (*V*) of a cylinder is represented by $V = \pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cylinder. If the volume of a cylinder is 120 cubic meters and the height is 5 meters, what is the radius of its base?

a. 2.76 meters
b. 8.68 meters
c. 13.82 meters
d. 43.41 meters

Part II:

for nos. 15-20:

Formulate and solve a problem based on the given situation below. Your output shall be evaluated according to the given rubric below.

You are an architect in a well-known establishment. You were tasked by the CEO to give a proposal for the diameter of the establishment's water tank design. The tank should hold a minimum of 800 cm^3. You were required to have a proposal presented to the Board. The Board would like to assess the concept used, practicality, accuracy of computation and organization of report.

RUBRIC

CATEGORIES	2 SATISFACTORY	1 DEVELOPING
Mathematical Concept	Demonstrates a satisfactory understanding of the concept and uses it to simply the problem.	Demonstrates incomplete understanding and has some misconceptions.
Accuracy of Computation	The computations are correct.	Generally, most of the computations are not correct.
Practicality	The output is suited to the needs of the client and can be executed easily.	The output is suited to the needs of the client and cannot be executed easily.

Pre-assessment: (Answer Key)

1) B	6) C	11) A	15-20) Product/Performance
2) D	7) D	12) B	*Answer to this subtest*
3) A	8) B	13) C	*depends on the students. Just be*
4) B	9) A	14) A	*guided by the rubric on how to*
5) B	10) C		*score their out-put.*

IV: Learning Goals and Targets

After going through this module, learners should be able to demonstrate an understanding of key concepts of rational exponents, radicals, formulate real-life problems involving these concepts, and solve these with utmost accuracy using a variety of strategies.

Lesson 1: **ZERO, NEGATIVE INTEGRAL and RATIONAL EXPONENTS**

WHAT TO KNOW:

Start Lesson 1 of this module by assessing students' knowledge of laws of exponents. As your class goes through this lesson, frequently revisit the following important question: "***How do we simplify expressions with zero, negative integral and rational exponents?" How can we apply what we learn in solving real-life problems?*** Inform the class that to find the answer to these questions they must perform each activity and if they find any difficulty in answering the exercises, seek your assistance or their peers' help or refer to the modules they have gone over earlier.

Provide Activity 1 that will require students' understanding of the laws of exponent. Be sure that students do well in this activity since this is just a recall of what they learned in Grade 7.

Activity 1: **Remember Me this Way!**

Answer Key:

A. 1) b^8 2) $\frac{r^6}{s^{12}}$ 3) -8 4) $\frac{5}{m^4}$ 5) m^{15}

B. 1.5×10^{11} meters is the distance between the sun and the Earth.

Be sure to recall the laws of exponents through processing the second question in the follow-up questions.

After eliciting students' understanding of the laws of exponents, let them answer the next activity to measure their prior knowledge of the lesson. Be reminded that their initial response to this activity is non-graded.

Activity 2: **Agree or Disagree!**

Read each statement under the column STATEMENT then write **A** if you agree with the statement; otherwise, write **D**. Write your answer on the "Response-Before-the-Discussion" column".

Anticipation-Reaction Guide

Response-Before-the-Discussion	STATEMENT	Response-After-the-Discussion
D	Any number raised to zero is equal to one (1).	DO NOT ANSWER THIS PART YET!
D	An expression with a negative exponent CANNOT be transformed into an expression with a positive exponent.	
A	2^{-3} is equal to $\frac{1}{8}$.	
A	Laws of exponents may be used in simplifying expressions with rational exponents.	

In the previous activity, you just identified how much students know about the lesson. Require the students to answer the "Response-Before-the-Discussion" column'. Let them know that the previous activity is non-graded since it is just meant to elicit their prior knowledge.

Introduce the next activity by stating that the lesson on zero, negative integral and rational exponents have applications in the real world. Let them compare their answer with the answers of their classmates, maybe two to three members in a group, but instruct them to come up with one final solution to the problem.

Activity 3: **PLAY WITH NEGATIVE!**

Answer Key:

There are 64, 000, 000 grains of rice in the box.

Do not expect students to arrive at the correct answer for this activity. Process students' answers to the follow-up questions. Make them realize that negative integral exponents have applications in real life.

Provide activity 4 that will require them to simplify expressions with zero and negative integral exponents.

Activity 4: **YOU COMPLETE ME!**

Answer Key:

1) $\frac{h^5}{h^5}$ → h^{5-5} → h^0 → 1

2) $\frac{4^8}{4^8}$ → 4^{8-8} → 4^0 → $\underline{1}$

Process students' answers to the follow-up questions then discuss how to simplify expressions with zero as the exponent. It is important to inform the class that operations with exponents must follow directly the laws already established for positive integral exponents, such as the expression $\frac{3^2}{3^2} = \frac{9}{9} = 1$. Extending the law $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$, to the case where $m = n$, then $\frac{3^2}{3^2} = 3^{2-2} = 3^0$. But $\frac{3^2}{3^2} = 1$. This suggests that we can define 3^0 to be 1. In general, for $a \neq 0$, a^0 is defined by $\boldsymbol{a^0 = 1}$. Provide additional and varied examples if necessary.

The next set of exercises will introduce how to simplify expressions with negative exponents.

6) 3^{-2} → $\frac{1}{3^2}$ → $\frac{1}{9}$

7) 4^{-3} → $\frac{1}{4^7}$ → $\frac{1}{64}$

Process students' answers to the follow-up questions then discuss how to simplify expressions with a negative integer as the exponent. It is important to inform the class that operations with exponents must follow directly the laws already established for positive integral exponents, such as the expression $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, where $n<m$, then $\frac{3^5}{3^8} = \frac{1}{3^{8-5}} = \frac{1}{3^3} = \frac{1}{27}$. On the other hand if $\frac{a^m}{a^n} = a^{m-n}$ is to hold even when *m>n, for example* $\frac{3^5}{3^8} = 3^{5-8} = 3^{-3}$, then 3^{-3} must be defined as $\frac{1}{3^3}$. In general, a^{-m} is defined by $a^{-m} = \frac{1}{a^m}, where\ a \neq 0$. Provide additional and varied examples if necessary.

The next set of exercises will introduce how to simplify expressions with rational exponents. Discuss that the given illustrative examples (see learners' material) are expressions with rational exponents in the form of $b^{1/n}$, $n \neq 0$. Recall the definition of rational then provide the necessary concepts:

Let n be a positive integer. Then $b^{1/n}$ is defined as the *principal nth root of b*. This means that:

1) If b is positive, then $b^{1/n}$ is the unique *positive number a such that* $a^n = b$. If $b = 0$, then $b^{1/n}$ is 0.
2) If b is negative and n is odd, then $b^{1/n}$ is the unique *real number a such that* $a^n=b$.
3) If b is negative and n is even, then $b^{1/n}$ is not defined.

Let m and n be positive integers. Then $b^{m/n}$ and $b^{-m/n}$ are defined as follows.

1) $b^{m/n} = (b^{1/n})^m$, provided that $b^{1/n}$ is defined.
2) $b^{-m/n} = \frac{1}{b^{m/n}}$

During the discussion, provide Activities 5 and 6 to further enhance students' understanding.

Activity 5: **A NEW KIND OF EXPONENT**

Answer Key:

Column A $b^{1/n}$	**Column B** $(b^{1/n})^n$	**Column C** Value(s) of $b^{1/n}$ that satisfy the equation in Column B
$25^{1/2}$	$(25^{1/2})^2 = 25$	5 and –5
$64^{1/3}$	$(64^{1/3})^3 = 64$	4
$(-8)^{1/3}$	$((-8)^{1/3})^3 = -8$	–2
$(-1)^{1/2}$	$((-1)^{1/2})^2 = -1$	No possible value

QUESTIONS:

1) When n is odd
2) When b is negative and n is even
3)? When b is positive and n is even
4) They are additive inverse of each other.

Activity 6: **EXTEND YOUR UNDERSTANDING!**

Directions: In this activity, you will learn the definition of $b^{m/n}$. If we assume that the rules for integer exponents can be applied to rational exponents, how will the following expressions be simplified? One example is worked out for you.

Answer Key:

2) $(2^{1/3})\ (2^{1/3})\ (2^{1/3})\ (2^{1/3})\ (2^{1/3})\ (2^{1/3})\ (2^{1/3}) = 2^{7/3}$
3) $(10^{1/2})\ (10^{1/2})\ (10^{1/2})\ (10^{1/2}) = 10^{4/2} = 10^2 = 100$
4) $(-4)^{1/7}(-4)^{1/7}(-4)^{1/7} = (-4)^{3/7}$
5) $13^{-1/4}13^{-1/4}13^{-1/4}13^{-1/4}13^{-1/4}13^{-1/4} = 13^{-6/4} = \dfrac{1}{13^{\frac{6}{4}}} = \dfrac{1}{13^{\frac{3}{2}}}$

QUESTIONS:

1) Ans: $b^{m/n}$ 2) Ans: $b^{-m/n}$

At the end of this section, students are expected to be knowledgeable in simplifying expressions with zero and negative integral exponents. Moreover, the concept of expressions with rational exponents was already introduced.

WHAT TO PROCESS:

In this section, students will learn how to simplify expressions with zero, negative integral and rational exponents by completing the provided activities, most importantly, through answering the follow-up questions after each activity.

The next activity will enable them to formulate the pattern in simplifying zero and negative integral exponents.

Activity 7: **WHAT'S HAPENNING?**

Directions: Complete the table below and observe the pattern.

Answer Key:

Column A	Column B	Column C	Column D	Column E	Column F	Column G	Column H
4^0	1	4^{-1}	$\frac{1}{4}$	4^{-2}	$\frac{1}{16}$	4^{-3}	$\frac{1}{64}$
3^0	1	3^{-1}	$\frac{1}{3}$	3^{-2}	$\frac{1}{9}$	3^{-3}	$\frac{1}{27}$
2^0	1	2^{-1}	$\frac{1}{2}$	2^{-2}	$\frac{1}{4}$	2^{-3}	$\frac{1}{8}$
1^0	1	1^{-1}	1	1^{-2}	1	1^{-3}	1
$\left(\frac{1}{2}\right)^0$	1	$\left(\frac{1}{2}\right)^{-1}$	2	$\left(\frac{1}{2}\right)^{-2}$	4	$\left(\frac{1}{2}\right)^{-3}$	8

Make sure to process students' answers in the follow-up questions because this will serve as your discussion of the lesson. Questions no. 2, 3, 5, 6, 7, 8, 9 and 10 will deal with how to simplify expressions with zero and negative integral exponents.

IMPORTANT: Zero raised to the power zero is undefined (0^0 = undefined).

Be sure that before leaving this session, the learners already have a clear understanding of the topic because the preceding activities will deal with applying the learned skill.

Activity 8: **I'LL GET MY REWARD!**

Directions: You can get the treasures of the chest if you will be able to correctly rewrite each expression without zero or negative integral exponent.

Answer Key:

$\frac{1}{9x^2y^4}$ | 1 | $-\frac{1}{144}$ | $\frac{8x^2}{z}$ | 25 | $64m^3$ | $\frac{-3n}{m^2p^4}$ | $\frac{b^{11}c^5}{a^3}$ | $\frac{1}{d^8}$

In the previous activity, the students were able to simplify expressions with zero and negative integral exponents. Simplifying these expressions would mean that it is free of zero and negative integral exponent. Make sure to process students' answers in the follow-up questions because this will serve as your springboard for discussion. Give more attention to questions no. 3 and 5 for these will deepen students' understanding of the topic. Do not forget to deal with question no. 4 for self-reflection.

The next activity will require the students to use multiple operations in simplifying the given expressions, so make sure that they were able to sharpen their skill in the previous activity.

Activity 9: **I CHALLENGE YOU!** **Answer Key:**

$\frac{10,125}{206}$ 2 $\frac{9x^{12}}{y^6z^{14}}$ $-\frac{5m^{12}n^{15}q^{32}}{7p^{24}}$ $\frac{c^5e^8}{2d^6}$

During this activity, students are expected to have mastered their skill in simplifying zero and negative integral exponents.

Provide the next activity that will test their understanding of the topic.

Activity 10: **AM I RIGHT?** **Answer Key:**

Des and Richard are both correct in simplifying the given expression with negative exponent. Des used the concept of negative exponent that $a^{-n} = \frac{1}{a^n}, where\, a \neq 0$ then followed the rule on dividing fraction that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$, $where\ b \neq 0,\ c \neq 0\ and\ d \neq 0$ while Richard used the law of exponent that $\frac{a^m}{a^n} = a^{m-n}, if\ m > n$. Both solutions are acceptable since they both follow mathematical properties. That is why they arrived at the same correct answer.

In the previous activity, students were able to analyze two different processes in simplifying expressions with negative integral exponents. Make sure to process students' answers to correct misconceptions. Provide the next activity that deals with the application of negative integral exponents to real-life problems.

***Activity 11:* HOW MANY..?**
Directions: Analyze and solve the problem below.

Answer Key: A larvae can reach 144 grams during its life cycle.

Be sure to process students' answers in the follow-up questions. Give emphasis to questions no. 2 and 3 for they deal with their understanding of the topic. Let the students try to answer question no. 4 then process their responses

Activity 12 will require the class to apply their knowledge that $b^{1/n}$ is defined as the *principal nth root of b.*

***Activity 12:* TWO SIDES OF THE SAME COIN**
Directions:. Simplify the following expressions. If the expression is undefined, write "undefined."

Answer Key:

1) $49^{1/2} = 7$ 3) $1000^{1/3} = 1$ 5) $(-64)^{1/3} = -4$ 6) $(-4)^{1/2}$ = undefined
2) $125^{1/3} = 5$ 4) $(-32)^{1/5} = -2$ 6) $(-100)^{1/2}$ = undefined 7) $-81^{1/4} = -3$

***Activity 13:* FOLLOW ME!**
Directions: Fill in the missing parts of the solution in simplifying expressions with rational exponents. Then answer the process questions below

Answer Key:

1) $\left(m^{\frac{2}{3}}\right)\left(m^{\frac{4}{3}}\right)=m^{\frac{2+4}{3}}=m^{\frac{6}{3}}=m^{2}$

2) $\left(k^{\frac{1}{4}}\right)\left(k^{\frac{2}{3}}\right)=k^{\frac{3}{12}+\frac{8}{12}}=k^{\frac{11}{12}}$

3) $\dfrac{a^{\frac{5}{7}}}{a^{\frac{3}{2}}}=a^{\frac{10}{14}-\frac{21}{14}}=a^{-\frac{11}{14}}=\dfrac{1}{a^{\frac{11}{14}}}$

4) $\left(\dfrac{y^{\frac{2}{3}}}{y^{\frac{1}{2}}}\right)^{2}=\dfrac{y^{\frac{2}{3}(2)}}{y^{\frac{1}{2}(2)}}=\dfrac{y^{\frac{4}{3}}}{y^{\frac{2}{2}}}=y^{\frac{4-3}{3}}=y^{\frac{1}{3}}$

The previous activity should enable them to realize that laws of exponents for integral exponents may be used in simplifying expressions with rational exponents.

Provide the following to the class: Let ***m* and *n* be rational numbers** and ***a* and *b* be real numbers**.

$a^{m}\bullet a^{n}=a^{m+n}$ $\left(a^{m}\right)^{n}=a^{mn}$ $(ab)^{m}=a^{m}b^{m}$

$\left(\dfrac{a}{b}\right)^{m}=\dfrac{a^{m}}{b^{m}}, b\neq 0$ $\dfrac{a^{m}}{a^{n}}=a^{m-n}, if\ m>n$ $\dfrac{a^{m}}{a^{n}}=\dfrac{1}{a^{m-n}}, if\ m<n$

Note: Some real numbers raised to a rational exponent are not real numbers such as $(-1)^{\frac{1}{2}}$*, and for such cases, these laws do not hold.*

Aside from the laws of exponents, students were also required to use their understanding of addition and subtraction of similar and dissimilar fractions.

Provide the next activity to strengthen students' skill of simplifying expressions with rational exponents.

Activity 14: **FILL-ME-IN! (by dyad / triad)**
Directions: Simplify the following expressions with rational exponents by filling in the boxes with solutions. Then answer the process questions below.

Answer Key:

$\left(c^{\frac{2}{9}}\right)\left(c^{\frac{4}{3}}\right)$ →	$c^{\frac{2}{9}+\frac{4}{3}}$ →	$c^{\frac{2+12}{9}}$ →	$c^{\frac{14}{9}}$
$\left(x^{\frac{2}{5}}y^{\frac{4}{10}}\right)^{-5}$ →	$x^{\frac{-10}{5}}y^{\frac{-20}{10}}$ →	$x^{-2}y^{-2}$ →	$\frac{1}{x^2y^2}$
$\frac{a^{\frac{1}{2}}}{a^{\frac{3}{10}}}$ →	$a^{\frac{1}{2}-\frac{3}{10}}$ →	$a^{\frac{5-3}{10}}$ →	$a^{\frac{1}{5}}$

Process students' understanding of the topic through the answers they have in the follow-up questions. This is also an opportunity to correct their misconceptions. Emphasize that in simplifying rational exponents, follow the laws of exponent. After this activity, make sure they already understand how to simplify rational exponents.

Activity 15: **MAKE ME SIMPLE!**
Directions: Using your knowledge of rational expressions, simplify the following.

Answer Key:

FINAL ANSWER		
1) $k^{\frac{44}{35}}$	2) $x^4y^5z^2$	4) $\frac{m^{\frac{9}{20}}}{n^{\frac{3}{7}}}$

In this section, students are expected to learn all the needed skills in simplifying zero, negative integral and rational exponents. The next section will require the students to apply their understanding in analyzing and solving problems.

WHAT TO UNDERSTAND:

In this section, let the students apply the key concepts of zero, negative integral and rational exponents. Tell them to use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.
Provide the next activity and let them answer the follow-up questions.

Activity 16: **TKE-IT-2-D-NXT-LVL**
Directions: Solve the given problem then answer the process questions.

Answer Key:

1) $\frac{11,121}{1000}$ and $\frac{201}{100}$
2) Simplify first those number with zero and negative integral exponents then perform the indicated operation/s.
3) Strictly follow the laws of exponents and other necessary mathematical skills in simplifying the expressions.

In the previous activity, we let our students apply their skill in simplifying the given expressions. Process students' answers to the follow-up questions. This may be an opportunity to further deepen their understanding of the topic. Give emphasis to question no. 3. Also, tackle questions no. 4 and 5 for self-evaluation.

Activity 17: **HOW MANY…?**
Directions: Solve the following problems.

Answer Key:

The dandelion is 15^6 heavier than its seeds.

Make sure to process students' answers to the follow-up questions. *Pose the question:* ***How well did you answer real-life problems on negative integral exponents?*** In this sense, students will realize that they can definitely encounter real-life related problems with negative integral exponents. Then provide the next activity that will require them to formulate and solve problems with negative integral exponents. Be guided by the rubrics in evaluating students' output. You may provide the class with the rubrics for them to be guided by the criteria.

Activity 18: **CREATE A PROBLEM FOR ME.**
Directions: Formulate and solve a problem based on the illustration below. Your work shall be evaluated according to the rubric.

Answer Key:

Formulated problems with solutions:
Formulated problems are subjective. Nevertheless, it is expected that students will write the values in scientific notation though exponents may be positive or negative. Check if the values given are realistic.

Pose the question: ***How did you apply your understandings in accomplishing this activity?***
Process students' answers to the follow-up questions. Give emphasis on question no. 3. Provide the next activity. This time, answers to the "Response-After-the-Discussion" column will be graded.

Activity 19: **Agree or Disagree! (revisited)**
Directions: Read each statement under the column <u>STATEMENT</u> then write **A** if you agree with the statement; otherwise, write **D**. Write your answer on the "Response-After-the-Discussion" column"

Answer Key:

Anticipation-Reaction Guide

Response-Before-the-Discussion	STATEMENT	Response-After-the-Discussion
	Any number raised to zero is equal to one (1).	**A**
	An expression with a negative exponent CANNOT be written into an expression with a positive exponent.	**D**
Have answered already.	2^{-3} is equal to $\frac{1}{8}$.	**A**
	Laws of exponents may be used in simplifying expressions with rational exponents.	**A**
	$\left(\frac{1}{3}\right)^{-2} = 9$	**D**
	$3^0 4^{-2} = 16$.	**D**
	$\frac{1}{(32x^3y^5)^{-2}}$ may be written as $(32x^3y^5)^2$ where $x \neq 0$ and $y \neq 0$	**A**
	$(-16)^{\frac{2}{3}} = -16$	**D**
	The exponential expression is $\frac{1}{(x+10)^{-\frac{1}{2}}}$ equivalent to $x^{\frac{1}{2}} + 10^{\frac{1}{2}}$.	**D**
	$3^2 \cdot 4^0 + 1^{\frac{1}{2}} \cdot 5^0 = 11$	**D**

Pose the questions: ***Were you able to answer the preceding activities correctly? Which activity interests you the most? What activity did you find difficult to answer? How did you overcome these difficulties?***

Process students' answers to these questions, then provide the next activity for self-assessment.

Activity 20: **3-2-1 CHART**
Directions: Fill-in the chart below.

3 things I learned	**SUBJECTIVE ANSWER**
2 things that interest me	**SUBJECTIVE ANSWER**
1 application of what I learned	**SUBJECTIVE ANSWER**

Ask some volunteer students to share their answers to the previous activity. Give emphasis to interesting and important answers specially those that deal with real-life applications.

WHAT TO TRANSFER:

Give the students opportunities to demonstrate their understanding of zero, negative integral and rational exponents by doing a practical task. Let them perform Activity 17. You can ask the students to work individually or in groups. In this activity, the students will create a magazine feature that deals with the application of zero, negative integral and rational exponents in a real-life setting. Be guided by the rubrics in evaluating students' output.

***Activity 21:* WRITE ABOUT ME!**

A math magazine is looking for new and original articles for their edition on the topic: Zero, Negative and Rational Exponents Around Us. As a freelance researcher/writer you will join the said competition by submitting your own article/feature. The output will be evaluated by the chief editor, feature editor and other writers of the said magazine. They will base their judgment on accuracy, creativity, mathematical reasoning and organization of the report.

RUBRICS FOR THE PERFORMANCE TASK

CATEGORIES	4 EXCELLENT	3 SATISFACTORY	2 DEVELOPING	1 BEGINNING
Mathematical Concept	Demonstrates a thorough understanding of the topic and uses it appropriately to solve the problem.	Demonstrates a satisfactory understanding of the concepts and uses it to simply the problem.	Demonstrates incomplete understanding and has some misconceptions.	Shows lack of understanding and has severe misconceptions.
Accuracy	The computations are accurate and show a wise use of the key concepts of zero, negative and rational exponents.	The computations are accurate and show the use of key concepts of zero, negative and rational exponents.	The computations are erroneous and show some use of the key concepts of zero, negative and rational exponents.	The computations are erroneous and do not show the use of key concepts of zero, negative and rational exponents.
Creativity	The design is comprehensive and displays the aesthetic aspects of the mathematical concepts learned.	The design is presentable and makes use of the mathematical concepts learned.	The design makes use of the mathematical concepts learned but is not presentable.	The design doesn't use mathematical concepts learned and is not presentable.
Organization of Report	Highly organized. Flows smoothly. Observes logical connections of points.	Satisfactorily organized. Sentence flow is generally smooth and logical.	Somewhat cluttered. Flow is not consistently smooth. Appears disjointed.	Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.

After the transfer task, provide the next activity for self evaluation.

***Activity 22:* Synthesis Journal**

Complete the table below by answering the questions.

How do I find the performance task?	What are the values I learned from the performance task?	How do I learn them? What made our task successful?	How will I use these learning/insights in my daily life?
SUBJECTIVE ANSWER	SUBJECTIVE ANSWER	SUBJECTIVE ANSWER	SUBJECTIVE ANSWER

This is the end of Lesson 1: Zero, Negative Integral and Rational Exponents of Module 4: Radicals. Inform the class to remember their learning on this lesson for they will use it to successfully complete the next lesson on radicals.

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about zero, negative integral and rational exponents. The lesson provided opportunities to simplify expressions with zero, negative integral and rational exponents. The students learned that any number, except 0, when raised to 0 will always result in 1, while expressions with negative integral exponents can be written with a positive integral exponent by getting the reciprocal of the base. They were also given the chance to apply their understanding of the laws of exponents to simplify expressions with rational exponents. Students identified and described the process of simplifying these expressions. Moreover, they were given the chance to demonstrate their understanding of the lesson by doing a practical task. Students' understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next lesson on radicals.

Lesson 2: **RADICALS**

Start the lesson by posing these questions: *Why do we need to know how to simplify radicals? Are radicals really needed in life outside math studies?* ***How can you simplify radical expressions? How do you add, subtract, multiply and divide radicals? How can the knowledge of radicals help us solve problems in daily life?***

Inform the class that in this lesson we will address these questions and look at some important real-life applications of radicals.

Provide the first activity the will require the students to recall their understanding on simplifying zero, negative and rational exponents.

Activity 1: **LET'S RECALL**

Directions: Simplify the following expressions.

Answer Key

1) $\left(5^{\frac{2}{7}}\right)\left(5^{-\frac{5}{6}}\right) = \dfrac{1}{5^{\frac{23}{42}}}$ 2) $\left(x^{16}y^{0}z^{8}\right)^{\frac{1}{4}} = x^{4}z^{2}$ 3) $\left(\dfrac{s^{\frac{1}{4}}}{t^{\frac{1}{8}}}\right)^{24} = \dfrac{s^{6}}{t^{3}}$ 4) $\dfrac{m^{\frac{1}{5}}n^{-\frac{1}{7}}}{m^{-\frac{1}{4}}n^{0}} = \dfrac{m^{\frac{9}{20}}}{n^{\frac{1}{7}}}$

Process students' answers to the follow-up questions. Give emphasis on the second question that may be used as a springboard for recalling the important concepts that are necessary for this lesson.

Provide the next activity that will elicit students' initial knowledge regarding writing expressions with rational exponents to radicals, vice versa and operations on radicals. This activity is non-graded.

Activity 2 **: IRF SHEET**

Directions: Below is an IRF Sheet. It will help check your understanding of the topics in this lesson. You will be asked to fill in the information in different sections of this lesson. For now you are supposed to complete the first column with what you know about the topic.

INITIAL *What are your initial ideas about radicals?*	**REVISE**	**FINAL**
	DO NOT ANSWER THIS PART YET	**DO NOT ANSWER THIS PART YET**

Process students' answer on the previous activity, cite some ideas that deal with the application of the lesson. This may also be an opportunity to motivate students.

Pose these questions to the class. Give the class some time to think and answer the questions. *How did you find the preceding activities? Are you ready to learn about simplifying and operations on radicals? How are radicals used in solving real-life problems?*

WHAT TO PROCESS:

In this section, students will learn how to write expressions with rational exponents to radicals and vice versa and simplify radicals, most importantly; students will develop further understanding of the topic through answering the follow-up questions after each activity.

The next activity will enable the learners to write expressions with rational exponents to radicals and vice versa.

Activity 3: **FILL-ME-IN**

Directions: Carefully analyze the first exercise below then fill in the rest of the exercises with correct answer.

Answer Key

$\left(\frac{3}{2p^2}\right)^{\frac{2}{3}}$	$\rightarrow$	$\sqrt[3]{\frac{3^2}{(2p^2)^2}}$	$\rightarrow$	$\sqrt[3]{\frac{9}{4n^4}}$
$\frac{(x^2+3)^{\frac{1}{3}}}{(x^2-3)^{-\frac{1}{3}}}$	$\rightarrow$	$(x^2+3)^{\frac{1}{3}}(x^2-3)^{\frac{1}{3}}$	$\rightarrow$	$\sqrt[3]{x^4-9}$
$\sqrt{3}$	$\rightarrow$	$3^{\frac{1}{2}}$		
$\sqrt[4]{8n^6}$	$\rightarrow$	$\sqrt[4]{2^3(n^2)^3}$	$\rightarrow$	$(2n^2)^{\frac{3}{4}}$

This activity may serve as a spring board for the discussion of writing expressions with rational exponents as radicals and vice versa. Discussion may be guided through processing students' answers to the conclusion table.

In the previous lesson, your student learned that the symbol $\sqrt[n]{a^m}$ is called radical. A ***radical expression or radical*** is an expression containing the symbol $\sqrt{\ }$ which called ***radical sign.*** In the symbol $\sqrt[n]{a^m}$, ***n*** is called the ***index or order*** which indicates the degree of the radical such as square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$) and fourth root ($\sqrt[4]{\ }$), $\boldsymbol{a^m}$ is called the ***radicand*** which is a number or expression inside the radical symbol and ***m*** is the power or exponent of the radicand. Furthermore, if $\frac{m}{n}$ is a rational number and *a* is a positive real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ provided that $\sqrt[n]{a}$ is a real number**.** The form $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ is called the *principal nth root of a*^{m}. Through this, we can write expression with rational exponents to radical. Give emphasis that we need to impose the condition that $a > 0$ in the definition of $\sqrt[n]{a^m}$ for an even *n* because it will NOT hold true if $a < 0$. If *a* is a negative real number and *n* is an even positive integer, then *a* is has NO real *nth root.* If *a* is a positive or negative real number and *n* is an odd positive integer, then there exists exactly one real *nth root of a,* the sign of the root being the same as the sign of the number.

Provide the next activity that will test students' skill in writing expression with rational exponents to radicals and vice versa.

Activity 4: TRANSFORMERS I

Directions: Transform the given radical form to exponential form and exponential form to radical form. Assume that all the letters represent positive real numbers.

Answer Key

RADICAL FORM	EXPONENTIAL FORM
$\sqrt[3]{11x^2}$	$\left(11x^2\right)^{\frac{1}{3}}$
$\sqrt[3]{25a^6b^4}$	$\left(5a^3b^2\right)^{\frac{2}{3}}$
$-29\sqrt{\frac{4n}{m^5p}}$	$-29\left(\frac{4n}{m^5p}\right)^{\frac{1}{2}}$
$\sqrt[5]{16r^4s^6t^8}$	$\left(4r^2s^3t^4\right)^{\frac{2}{5}}$
$\frac{3}{\sqrt[7]{4b^4}}$	$3\left(2b^2\right)^{-\frac{2}{7}}$
$-\frac{1}{\sqrt[7]{9k^6}}$	$-\left(3k^3\right)^{-\frac{2}{7}}$

For sharpening of the learned skill in this topic, a discussion may follow when processing students' answers to the follow-up questions. Let the students answer question no. 4 to personally create a step-by-step process on how to write this expression into radicals and vice versa. The teacher may also deal with question no. 5 for self-assessment and to identify learners' difficulties and eventually deal with them.

Days or week before executing this activity, inform the class to prepare the needed material. Group the students then instruct them to produce pairs of cards such as a radical expression with its equivalent expression with a radical exponent.

Check the cards for repetitions or incorrect expressions. When all is set, provide the next activity that will require learners' understanding of writing expressions with rational exponents to radicals and vice versa.

Activity 5: THE PAIR CARDS (Group Activity)

Mechanics of the Game

1. You will be playing "The Pair Cards" game similar to a well known card game, "Unggoyay".
2. Every group shall be given cards. Select a ***dealer***, who is at the same time a player, to facilitate the distribution of cards. There must be at most 10 cards in every player. (*Note: There should be an even number of cards in every group*.)
3. After receiving the cards, pair the expressions. *A pair consists of a radical expression and its equivalent expression with a rational exponent.* Then, place and reveal the paired cards in front.

4. If there will be no paired cards left with each player, the dealer will have the privilege to be the first to pick a card from the player next to him following a clockwise direction. He/she will then do step 3. This process will be done by the next players one at a time.
5. The game continues until all the cards are paired.
6. The group who will finish the game ahead of others will be declared the "WINNER!!!"

Source: Beam Learning Guide, Second Year – Mathematics, Module 10: Radical Expressions in General, pages 31-33

End this topic by processing students' answers in the follow-up questions.

Since students are now capable of writing expressions with rational exponents as radicals and vice versa, provide a discussion that deals with simplifying radicals begin with the **laws on radicals;** assume that when *n* is even, a > 0.

a) $\left(\sqrt[n]{a}\right)^n = a$

b) $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$

c) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad b > 0$

d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Simplifying Radicals:

a) Removing Perfect *nth* Powers

Break down the radicand into perfect and nonperfect *nth powers* and apply the property $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$.

example: $\sqrt{8x^5y^6z^{13}} = \sqrt{2^2(x^2)^2(y^3)^2(z^6)^2} \bullet \sqrt{2xz} = 2x^2y^3z^6\sqrt{2xz}$

b) Reducing the index to the lowest possible order

Express the radical into an expression with rational exponent then simplify the exponent or apply the property $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$.

examples: $\sqrt[20]{32m^{15}n^5} = \sqrt[4]{\sqrt[5]{2^5\left(m^3\right)^5n^5}} = \sqrt[4]{2m^3n}$ *or*

$$\sqrt[20]{32m^{15}n^5} = \left(2^5m^{15}n^5\right)^{\frac{1}{20}} = 2^{\frac{5}{20}}m^{\frac{15}{20}}n^{\frac{5}{20}} = 2^{\frac{1}{4}}m^{\frac{3}{4}}n^{\frac{1}{4}} = \left(2m^3n\right)^{\frac{1}{4}} = \sqrt[4]{2m^3n}$$

c) Rationalizing the denominator of the radicand

Rationalization is the process of removing the radical sign in the denominator.

examples: $\sqrt[3]{\dfrac{3}{4k}} = \sqrt[3]{\dfrac{3}{2^2k} \bullet \dfrac{2k^2}{2k^2}} = \sqrt[3]{\dfrac{6k^2}{2^3k^3}} = \dfrac{\sqrt[3]{6k^2}}{\sqrt[3]{2^3k^3}} = \dfrac{\sqrt[3]{6k^2}}{2k}$

$$\sqrt[4]{\frac{1}{72}} = \sqrt[4]{\frac{1}{36 \bullet 2}} = \sqrt[4]{\frac{1}{6^2 \bullet 2} \bullet \frac{6^2 \bullet 2^3}{6^2 \bullet 2^3}} = \frac{\sqrt[4]{288}}{\sqrt[4]{6^4 \bullet 2^4}} = \frac{\sqrt[4]{18 \bullet 16}}{\sqrt[4]{6^4 \bullet 2^4}} = \frac{\sqrt[4]{18 \bullet 2^4}}{\sqrt[4]{6^4 \bullet 2^4}} = \frac{2\sqrt[4]{18}}{6 \bullet 2} = \frac{\sqrt[4]{18}}{6}$$

The **simplified form of a radical expression** would require;

- NO prime factor of a radicand that has an exponent equal to or greater than the index.
- NO radicand contains a fraction
- NO denominator contains a radical sign.

The succeeding activities will deal with simplifying radicals.

Activity 6: **WHY AM I TRUE/ WHY AM I FALSE?**
Directions Given below are examples of how to divide radicals. Identify if the given process below is TRUE or FALSE then state your reason. For those you identified as false, make it true by writing the correct part of the solution.

Answer Key

	TRUE or FALSE	WHY?	IF FALSE, WRITE THE CORRECT PART OF THE SOLUTION
Simplify $\sqrt[3]{16}$			
$\sqrt[3]{16} = \sqrt[3]{8} \bullet \sqrt[3]{2}$	TRUE	Identifying the perfect cube factor of 16.	
$= \sqrt[3]{2^3} \bullet \sqrt[3]{2}$	TRUE	Using the law of radical $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ and separating the radicand into perfect and nonperfect nth power	
$= 2 \bullet \sqrt[3]{2}$	TRUE	Extracting a perfect nth root	
$\sqrt[3]{16} = 2\sqrt[3]{2}$	TRUE	Multiplying the coefficient of $\sqrt[3]{2}$ which is 1 and the integer 2.	

The follow-up questions after this activity may serve as a springboard for discussion. Process students' answers to the follow-up questions. Focus on simplifying radicals by removing perfect nth powers, reducing the index to the lowest possible order and rationalizing the denominator of the radical.
Introduce the process of rationalization and reducing the order of the radical in simplifying radicals. Then provide the next activity that will test students' understanding of simplifying radicals.

Activity 7: **WHO AM I?**
Directions: Using your knowledge of rational exponents, decode the following.

Answer Key

The First Man to Orbit the Earth

Y U R I G A G A R I N
(12)(13)(-7)(6) (5)(27)(5)(81)(-7)(6)($-\frac{8}{27}$)

Source: EASE Modules, Year 2 – Module 2 Radical Expressions, pages 9 – 10

Process students' answers to the follow-up questions, this may serve as an opportunity to further correct any misunderstanding of the topic. Let the students answer the next activity.

Activity 8: **GENERALIZATION**
Directions: Write your generalization on the space provided regarding simplifying radicals.

Answer Key

We can simplify radicals through…
- *the laws of radicals*
- *writing radicals to exponential form*
- *simplifying the radicand by removing perfect nth powers, reducing the index to the lowest possible order and rationalizing the denominator of the radical*

End this topic by citing some students' generalization. Provide the next activity that deals with addition and subtraction of radicals. Let the students analyze the given illustrative examples.

Answer Key

CONCLUSION TABLE	
Questions	Answer
How do you think the given expressions were added? What processes have you observed?	Coefficients of radicals with same order and radicand are added/subtracted following the rule of addition and subtraction of integers
What concepts/skills are necessary to simplify the given expression?	addition and subtraction of integers simplifying radicals rationalization of radicals
Based on the given illustrative examples, how do we add radicals? How do we subtract radicals? What can be your conclusion on how to subtract and/add radicals?	Only same radicals can be added/subtracted Add/subtract the coefficient then copy the radical
What are your bases for arriving at your conclusion?	Addition and subtraction of real numbers and simplifying radicals.

Students' answers to the conclusion table may serve as a springboard for discussion. Explain that;
- <u>like radicals or similar radicals</u> are radicals of the same order(index) and having the same radicand
- only like/similar radicals can be added or subtracted
- add/subtract the coefficients then copy the common radical

Provide the next activity that will ask the students to add/subtract radicals.

Activity 9: **PUZZLE-MATH**

Directions: Perform the indicated operation/s as you complete the puzzle below.

Answer Key

$3\sqrt{2}$	+	$5\sqrt{2}$	=	$8\sqrt{2}$		$6\sqrt[4]{5}$
-		+		-		-
$13\sqrt{2}-5\sqrt[3]{6}$	+	$10\sqrt[3]{6}$	-	$6\sqrt{2}$	=	$7\sqrt{2}+15\sqrt[3]{6}$
=		=		+		-
$-10\sqrt{2}+5\sqrt[3]{6}$	+	$5\sqrt{2}+10\sqrt[3]{6}$	=	$-5\sqrt{2}+15\sqrt[3]{6}$		$14\sqrt{2}-20\sqrt[3]{6}$
				=		=
$-24\sqrt{2}$	-	$-20\sqrt[3]{6}$	-	$-3\sqrt{2}+15\sqrt[3]{6}$	=	$6\sqrt[4]{5}-21\sqrt{2}+5\sqrt[3]{6}$

Processing students' answers to the follow-up questions may deepen their understanding of the topic. Make sure that students already master the skill of adding and subtracting radicals before going to the next activity.

Activity 10: **FILL-IN-THE-BLANKS.**

Directions: Provided below is the process of multiplying radicals where x > 0 and y > 0. Carefully analyze the given example then provide the solution for the rest of the problems.

Answer Key

$$2\sqrt{3}\left(3\sqrt{3}+4\sqrt{3}\right)=\left(2\sqrt{3}\right)\left(3\sqrt{3}\right)+\left(2\sqrt{3}\right)\left(4\sqrt{3}\right)$$
$$=6\sqrt{9}+8\sqrt{9}$$
$$=6\bullet 3+8\bullet 3$$
$$2\sqrt{3}\left(3\sqrt{3}+4\sqrt{3}\right)=42$$

$$\left(\sqrt[3]{2x}\right)\left(\sqrt{3x}\right)=(2x)^{\frac{1}{3}}(3x)^{\frac{1}{2}}$$
$$=(2x)^{\frac{2}{6}}(3x)^{\frac{3}{6}}$$
$$=\left(\sqrt[6]{4x^2}\right)\left(\sqrt[6]{27x^3}\right)$$
$$\left(\sqrt[3]{2x}\right)\left(\sqrt{3x}\right)=\sqrt[6]{108x^5}$$

Discussion will begin as the teacher processes students' answers to the conclusion table. In multiplying radicals;

a) To multiply radicals of the same order, use the property $\sqrt[n]{ab}=\sqrt[n]{a}\bullet\sqrt[n]{b}$. Then simplify by removing the perfect nth powers from the radicand.

b) To multiply binomials involving radicals, use the property for *product of two binomials* $(a\pm b)(c\pm d)=ac\pm(ad\pm bc)\pm bd$. Then simplify by removing perfect nth powers from the radicand or by combining similar radicals.

c) To multiply radicals of different orders, express them as radicals of the same order then simplify.

Provide the next activity that will deal with multiplying radicals.

Activity 11: **WHAT'S THE MESSAGE?**

Directions: *Do you feel down even with people around you? Don't feel low*. Decode the message by performing the following radical operations. Write the words corresponding to the obtained value in the box provided.

Answer Key

<u>Do not</u> <u>consider yourself</u> <u>more/less</u> <u>not even equal</u> <u>to others</u> <u>for people</u>

<u>are not</u> <u>of identical quality.</u> <u>Each one</u> <u>is unique</u> <u>and irreplaceable</u>

Source (Modified): EASE Modules, Year 2-Module 5 Radical Expressions page 10

The teacher may ask the students to explain how they answered this activity. You may let them write their solution on the board.

The next section deals with rationalization and division of radicals.

When simplifying radicals, there should be NO radicals in the denominator. In this section, students will learn techniques to deal with radicals in the denominator. Introduce the process of rationalization through answering the next exercises. Let them complete the process of simplifying the given expressions.

$$\frac{10}{\sqrt{6}} \Rightarrow \frac{10}{\sqrt{6}}\bullet\frac{\sqrt{6}}{\sqrt{6}} \Rightarrow \frac{10\sqrt{6}}{\sqrt{36}} \text{ or } \frac{5\sqrt{6}}{3}$$

$$\frac{\sqrt{5}}{\sqrt{7}} \Rightarrow \frac{\sqrt{5}}{\sqrt{7}}\bullet\frac{\sqrt{7}}{\sqrt{7}} \Rightarrow \frac{\sqrt{35}}{\sqrt{49}} \text{ or } \frac{\sqrt{35}}{7}$$

$$\frac{a}{\sqrt[3]{5}} \Rightarrow \frac{a}{\sqrt[3]{5}}\bullet\frac{\sqrt[3]{25}}{\sqrt[3]{25}} \Rightarrow \frac{a\sqrt[3]{25}}{\sqrt[3]{125}} \text{ or } \frac{a\sqrt[3]{25}}{5}$$

In the previous exercises, students were able to simplify the radical by rationalizing the denominator**. Rationalization** is a process where you simplify the expression by making the denominator free from radicals. Remove the radical by making it a perfect *nth* root. This skill is necessary in the **division of radicals**

a) To divide radicals of the same order, use the property $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ then rationalize the denominator.

examples: $\frac{\sqrt[3]{5}}{\sqrt[3]{2}} = \sqrt[3]{\frac{5}{2}\bullet\frac{2^2}{2^2}} = \sqrt[3]{\frac{5\bullet 4}{2^3}} = \frac{\sqrt[3]{20}}{2}$ $\qquad \frac{\sqrt[4]{1}}{\sqrt[4]{ab^3}} = \sqrt[4]{\frac{1}{ab^3}\bullet\frac{a^3b}{a^3b}} = \frac{\sqrt[4]{a^3b}}{\sqrt[4]{a^4b^4}} = \frac{\sqrt[4]{a^3b}}{ab}$

b) To divide radicals of a different order, it is necessary to express them as radicals of the same order then rationalize the denominator.

examples: $\frac{\sqrt[3]{3}}{\sqrt{3}} = \frac{3^{\frac{1}{3}}}{3^{\frac{1}{2}}} = \frac{3^{\frac{2}{6}}}{3^{\frac{3}{6}}} = \frac{\sqrt[6]{3^2}}{\sqrt[6]{3^3}} = \sqrt[6]{\frac{3^2}{3^3}\bullet\frac{3^3}{3^3}} = \sqrt[6]{\frac{3^5}{3^6}} = \frac{\sqrt[6]{243}}{3}$

This time, let the students simplify expressions with two terms in the denominator. Let them answer the exercises below that deal with determining the conjugate. (NOTE: Do not define "conjugate pair" let the students discover its characteristics through this activity.)

$(\sqrt{2}+3)$	$(\sqrt{2}+3)(\sqrt{2}-3)$	$4-9$	-5
$(\sqrt{6}-5)$	$(\sqrt{6}-5)(\sqrt{6}+5)$	$6-25$	-19
$(\sqrt{2}+\sqrt{3})$	$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$	2 - 3	-1
$(3\sqrt{5}-2\sqrt{6})$	$(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})$	45 - 24	21

Process students' learning in the previous activity. Pose the questions: *How do you determine conjugate pairs? What happens to the expression when we multiply conjugate pairs?*

Let them use this technique to write the following expressions without radicals in the denominator.

$\frac{2}{2+\sqrt{7}}$	$\frac{2}{2+\sqrt{7}} \bullet \frac{2-\sqrt{7}}{2-\sqrt{7}}$	$\frac{-4+2\sqrt{7}}{3}$
$\frac{3+\sqrt{5}}{\sqrt{7}-\sqrt{10}}$	$\frac{3+\sqrt{5}}{\sqrt{7}-\sqrt{10}} \bullet \frac{\sqrt{7}+\sqrt{10}}{\sqrt{7}+\sqrt{10}}$	$-\sqrt{7}-\sqrt{10}-\frac{\sqrt{35}-\sqrt{50}}{3}$
$\frac{-7\sqrt{2}+4\sqrt{3}}{3\sqrt{2}-5\sqrt{3}}$	$\frac{-7\sqrt{2}+4\sqrt{3}}{3\sqrt{2}-5\sqrt{3}} \bullet \frac{3\sqrt{2}+5\sqrt{3}}{3\sqrt{2}+5\sqrt{3}}$	$\frac{18-23\sqrt{6}}{57}$

Process students' answers to the follow-up questions.

c) To divide radicals with a denominator consisting of at least two terms, rationalize the denominator using its conjugate.

examples:

$$\frac{3}{\sqrt{3}-\sqrt{2}}=\frac{3}{\sqrt{3}-\sqrt{2}}\bullet\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{3\sqrt{3}+3\sqrt{2}}{\sqrt{9}-\sqrt{4}}=\frac{3\sqrt{3}+3\sqrt{2}}{3-2}=3\sqrt{3}+3\sqrt{2}$$

$$\frac{\sqrt{5}+3}{\sqrt{5}-3}=\frac{\sqrt{5}+3}{\sqrt{5}-3}\bullet\frac{\sqrt{5}+3}{\sqrt{5}+3}=\frac{\sqrt{25}+3\sqrt{5}+3\sqrt{5}+9}{\sqrt{25}-9}=\frac{5+3\sqrt{5}+3\sqrt{5}+9}{5-9}=\frac{14+6\sqrt{5}}{-4}=\frac{-7-3\sqrt{5}}{2}$$

Let the students cite the following important concepts/processes on dividing radicals.

- the *nth* root of a radical divided by the *nth* root of another radical is equal to the *nth* root of the quotient of their radicands. In symbols: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}, b\neq 0$, where a and b are positive if n is even
- if radicals have different orders, transform them into radicals with equal indices then divide
- if there is a radical in the denominator, rationalize the denominator of the radical.
- if radicals are binomial in form, determine the conjugate of the denominator then use the distributive property or the special products of polynomial expressions.

After this lesson, the students should have learned how to divide radical expressions. Provide the succeeding activities to sharpen this skill.

***Activity 12:* "DIVIDE-DIVIDE"**

Directions: Perform division of radicals and simplify the following expressions.

Answer Key

1) $\dfrac{\sqrt[6]{81,000}}{3}$ 2) $\dfrac{\sqrt[6]{243}}{3}$ 3) $\dfrac{\sqrt[12]{177,147}}{3}$ 4) $\dfrac{\sqrt[12]{279,936}}{6}$ 5) $\sqrt[12]{7,776}$

Source (Adopted): EASE Modules, Year 2-Module 5 Radical Expressions page 17

After this activity, make sure that majority of the students have already mastered division of radicals, for the next activity will require them to reason-out the process of dividing radical expressions.

***Activity 13:* JUSTIFY YOUR ANSWER**

Directions: Identify if the given process below is TRUE or FALSE based on the division of radicals then state your reason. For those you identified as false, make them true by writing the correct part of the solution.

Answer Key

	TRUE or FALSE	WHY?	IF FALSE, WRITE THE CORRECT PART OF THE SOLUTION
Simplify $\dfrac{\sqrt{3xy^2}}{\sqrt[4]{2x^2y}}$, where $x>0$ and $y>0$			
$\dfrac{\sqrt{3xy^2}}{\sqrt[4]{2x^2y}} = \dfrac{(3xy^2)^{\frac{1}{2}}}{(2x^2y)^{\frac{1}{4}}}$	TRUE	Write radicals in exponential form	
$= \dfrac{(3xy^2)^{\frac{2}{4}}}{(2x^2y)^{\frac{1}{4}}}$	TRUE	Transform the exponents into exponents with similar denominator.	
$= \sqrt[4]{\dfrac{9x^2y^4}{2x^2y}}$	TRUE	Write exponential form to radicals.	
$= \sqrt[4]{\dfrac{9y^3}{2}}$	TRUE	Simplify the radicand.	
$= \sqrt[4]{\dfrac{9y^3}{2} \bullet \dfrac{8}{8}}$	TRUE	Rationalize the denominator.	
$= \sqrt[4]{\dfrac{72y^3}{16}}$	TRUE	Multiply.	
$\dfrac{\sqrt{3xy^2}}{\sqrt[4]{2x^2y}} = \dfrac{\sqrt[4]{72y^3}}{2}$	TRUE	Simplify radicals.	

Process students' answers to the follow-up questions. This is an opportunity to deepen students understanding. Let the students answer the next activity that will require them to develop their generalization in dividing radicals

Activity 14: **GENERALIZATION**
Directions: Write your generalization on the space provided regarding simplifying radicals.

Answer Key

In division of radicals…

- with the same order, use the property $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ then rationalize the denominator.
- of a different order, it is necessary to express them as radicals of the same order then rationalize the denominator.
- with denominator consisting of at least two terms, rationalize the denominator using its conjugate.

Provide the next activity that will further test and sharpen learners' skill in dividing radicals.

Activity 15: **A NOISY GAME!**
Directions: Perform the indicated operations. Then fill up the second table with the letter that corresponds to the correct answer.

Answer Key

Why is tennis a noisy game?

$\frac{\sqrt{6}}{3}$	$\frac{\sqrt{x}}{x}$	$7\sqrt{6}-7\sqrt{5}$	4	$5\sqrt{2}$	$\frac{\sqrt[3]{12}}{3}$	$\sqrt[6]{2}$	$\frac{5}{2}$	$\frac{\sqrt[3]{15\ bxy}}{5\ y}$	$\frac{1}{5}$	$\frac{\sqrt{5}}{5}$
E	**V**	**E**	**R**	**Y**	**P**	**L**	**A**	**Y**	**E**	**R**

R	**A**	**I**	**S**	**E**	**S**	**A**	**R**	**A**	**C**	**K**	**E**	**T**
$\sqrt[4]{6}$	$\frac{3\sqrt{3}+3}{2}$	$\frac{\sqrt[3]{18}}{3}$	$2\sqrt{2}-\sqrt{6}$	$2\sqrt{7}$	$\frac{\sqrt[3]{75}}{5}$	$4\sqrt{2}$	$-2+\sqrt{5}$	$2\sqrt{5}$	$\frac{\sqrt{3}}{3}$	$4\sqrt{2}-3\sqrt{7}$	$3\sqrt[3]{2}$	$\frac{25}{6}$

Source: EASE Modules, Year 2 – Module 5 Radical Expressions

End this section by processing students' answers to the follow-up questions. Let them cite some important concepts/processes needed in simplifying radicals through the fundamental operations. Make them identify the application of their understanding of radicals. This may be an opportunity to motivate them for the next section.

WHAT TO UNDERSTAND

In this section, let the students apply the key concepts of simplifying radicals. Tell them to use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

Provide the next activity and let them answer the follow-up questions.

Activity 16: **TRANSFORMERS III**

Directions: Transform and simplify each radical and exponential form to exponential and radical form respectively, then answer the follow-up questions.

Answer Key

$\sqrt[4]{\sqrt{\sqrt{7}}}$	$7^{\frac{1}{16}}$
$\sqrt[3]{\sqrt{x^6}}$	x
$\left\{\left[(y)^{\frac{1}{4}}\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}}$	$\sqrt[24]{y}$
$-2\left(m^3\right)^{-\frac{2}{3}}+4\left(n^4\right)^{\frac{4}{3}}$	$-\frac{2}{m^2}+4n^{\frac{16}{3}}$

Strictly process students' answers to the follow up questions. Let them justify their answer whenever possible. The previous activity must gauge how well students understand the topics and if they can apply this understanding to a new situation or problem.

Provide the next activity that will require them to develop the laws of simplifying radicals.

Activity 17: **THEREFORE I CONCLUDE THAT….!**

Directions: Answer the given activity by writing the concept/process/law used to simply the given expression where b, x and y are positive real numbers.

Answer Key

1) $4\sqrt[3]{4}+5\sqrt[3]{4}$	WHY?
$(4+5)\sqrt[3]{4}$	Combine the coefficients of similar radicals.
$9\sqrt[3]{4}$	Perform the indicated operation then copy the common radical.
2) $3\sqrt{b}+4\sqrt{b}$	WHY?
$(3+4)\sqrt{b}$	Combine the coefficients of similar radicals.
$7\sqrt{b}$	Perform the indicated operation then copy the common radical.
3) $a\sqrt[n]{b}+c\sqrt[n]{b}$	WHY?
$(a+c)\sqrt[n]{b}$	Combine the coefficients of similar radicals. Perform the indicated operation then copy the common radical.
My conclusion: Only similar radicals can be added through combining the coefficients, performing the indicated operation then copying the common radical.	

Check students' answers in this activity. Instruct the students to develop the pattern for the other remaining operations on radicals. For the sake of discussion, the teacher may ask the class to think of any counterexample for each pattern/conclusion. Obviously there will be none. The teacher may then say that the pattern is valid. End this activity by processing students' answers to question no. 2.

Provide the next activity that will require the learners to apply their understanding to real-life problems. ***It will also be helpful if you can recall to the class how to approximate the square root of nonperfect square numbers that will facilitate realistic answers to the problems even without using the calculator.***

Activity 18: **TRY TO ANSWER MY QUESTIONS!**

Answer Key

Directions: Read carefully the given problem then answer the questions that follow.

If each side of a square garden is increased by 4m, its area becomes 144 m^2 .

1) The side of the square garden is 12m after increasing it.
2) The length of the sides of the original garden is 8m.
3) Supposing the area of a square garden is 192 m^2 , the length of its side is $8\sqrt{3}m$ or approximately 8 m and 16 m.

A square stock room is extended at the back in order to accommodate exactly the cartons of canned goods with a total volume of 588 m^3. If the room extension can exactly accommodate a 245 m^3 stocks, then find the original length of the stock room.

1) The dimensions of the new stock room are 12m x 7m x 7m.
2) Assuming that the floor area of a square stock room is 588 m^2, the length of its side is $14\sqrt{2}$ m or approximately 24.25m.
3) Approximately, we can find this length between 14m and 28m.

A farmer is tilling a square field with an area of 900 m^2. After 3 hrs, he accomplished $\frac{2}{3}$ of the given area.

1) The side of the square field is 30m.
2) The dimensions of the mowed portion are 30m x 20m.
3) If the area of the square field measures 180 m^2, the length of its side is $6\sqrt{5}m$.
4) Approximately, we can find this length between 13 m and 14 m.

A square swimming pool having an area of 25 m^2 can be fully filled with water for about 125 m ^{3}.

1) The dimensions of the pool are 5m x 5m x 6m.
2) If $\frac{3}{4}$ of the swimming pool is filled with water, then it will be 3.75 m deep.
3) Suppose the area of the square pool is 36 m^2, the length of its side is 6m.

Source: Beam Learning Guide, Year 2– Mathematics, Module 10: Radicals Expressions in General, Mathematics 8 Radical Expressions, pages 41-44

Activity 19: **BASED IT ON ME!**

Answer Key

Directions: Formulate a problem based on the given illustration then answer the questions that follow.

1) Solving for the distance a person can see the horizon.
2) Varied answers, depending on the students.
3) Varied answers, depending on the problem formulated by the student.
4) Subjective answer.

1) Solving for the time it takes a body to fall with the effect of gravity over a given distance.
2) Varied answers, depending on the students.
3) Varied answers, depending on the problem formulated by the student.
4) Subjective answer.

Pose the following questions after Activity no.20: *How did you come up with your own problem based on the illustration? Have you formulated and solved it correctly?* Process students' answers to these questions. Inform the class that their knowledge of simplifying and operating radicals has its application in a real-life setting.

For the next activity, introduce the formula $c^2 = a^2 + b^2$, then inform the class when the given formula is applicable to use. The teacher may also give some important characteristics of the formula. It is the Pythagorean Relation which will be discussed in the next module.

Activity 20: **WHAT IS MY PROBLEM?**

Answer Key

Directions: Develop a problem based on the given illustration below.

1) Solving for the time it takes a body to fall with the effect of gravity over a given distance.
2) Varied answers, depending on the students.
3) Varied answers, depending on the problem formulated by the student.
4) Subjective answer.

After Activity no.21, pose the question: *How do you feel when you can formulate and solve problems that involve radicals?* Solicit answers from the class. Once again, give emphasis to the application of radicals a in real-life setting.

Instruct the class to fill-out the second column of the IRF sheet for their revise ideas. (This will be graded.) Cite students' answers by comparing their initial and revise ideas.

Activity 21: **IRF SHEET (revisited)**

INITIAL *What are your initial ideas about radicals?*	REVISE ***What are your new ideas?***	FINAL
WITH ANSWER ALREADY		**DO NOT ANSWER THIS PART YET**

Say: *Now that you well know how to simplify radicals, let us now solve real-life problems.* Provide the transfer task and rubrics.

WHAT TO TRANSFER:

Give the students opportunities to demonstrate their understanding of zero, negative and rational exponents by doing a practical task. Let them perform Activity 23. You can ask the students to work individually or in groups. In this activity, the students will create a magazine feature that deals with the application of zero, negative integral and rational exponents to real-life setting. Be guided by the rubrics in evaluating students' output.

Activity 22: **TRANSFER TASK**

You are an architect in a well-known establishment. You were tasked by the CEO to give a proposal on the diameter of the establishment's water tank design. The tank should hold a minimum of 950 m^3. You were required to have a proposal presented to the Board. The Board would like to assess the concept used, practicality, accuracy of computation and organization of report

RUBRICS FOR THE TASK

CATEGORIES	4 EXCELLENT	3 SATISFACTORY	2 DEVELOPING	1 BEGINNING
Mathematical Concept	Demonstrates a thorough understanding of the topic and uses it appropriately to solve the problem.	Demonstrates a satisfactory understanding of the concepts and uses it to simplify the problem.	Demonstrates incomplete understanding and has some misconceptions.	Shows lack of understanding and has severe misconceptions.
Accuracy of Computation	All computations are correct and are logically presented.	The computations are correct.	Generally, most of the computations are not correct.	Errors in computations are severe.
Practicality	The output is suited to the needs of the client and can be executed easily. Ideas presented are appropriate to solve the problem.	The output is suited to the needs of the client and can be executed easily.	The output is suited to the needs of the client and cannot be executed easily.	The output is not suited to the needs of the client and cannot be executed easily.
Organization of Report	Highly organized. Flows smoothly. Observes logical connections of points.	Satisfactorily organized. Sentence flow is generally smooth and logical.	Somewhat cluttered. Flow is not consistently smooth, appears disjointed.	Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.

After the transfer task, provide the following activities for self evaluation.

***Activity 23* : IRF SHEET**

Directions: Below is an IRF Sheet. It will help check your understanding of the topics in this lesson. You will be asked to fill in the information in different sections of this lesson. This time, kindly fill-in the third column that deals with your final ideas about the lesson.

INITIAL *What are your initial ideas about radicals?*	REVISE *What are your revised initial ideas?*	FINAL *What are your final ideas about the lesson?*
WITH ANSWER ALREADY	WITH ANSWER ALREADY	

***Activity 24:* Synthesis Journal**

Complete the table below by answering the questions.

How do I find the performance task?	What are the values I learned from the performance task?	How do I learn them? What made our task successful?	How will I use these learning/insights in my daily life?

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about simplifying and operating radicals. The lesson provided the learners with opportunities to operate and simplify radical expressions. They identified and described the process of simplifying these expressions. Moreover, students were given the chance to demonstrate their understanding of the lesson by doing a practical task. Learners' understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next lesson on radicals.

Lesson 3: **SOLVING RADICAL EQUATIONS**

WHAT TO KNOW:

Start the lesson by posing these questions: Why do we need to know how to solve radical equations? Are radicals really needed in life outside mathematics studies? ***How can you simplify radicals? How can the understanding of radical equations help us solve problems in daily life?***

Provide the next activity that will require the students to recall their understanding of simplifying radicals.

Activity 1: **LET'S RECALL!**
Directions: Solve the given problem below.

Answer Key

A man can see $\frac{\sqrt{30}}{2}$ meters, between 2.5 meters and 3 meters or approximately 0.0017 mile to the horizon when he is 5 meters above the ground.

The man is $\sqrt{97}$ meters or approximately 9.85 meters or between 9 meters and 10 meters from his house.

Process students' answers to the follow-up questions. Let them cite some important understanding they gained on simplifying and operating radicals.

Provide the next activity that will solicit students' initial knowledge of the topic. Instruct them to answer the first and second column of the KWL chart.

Activity 2: **K-W-L CHART**
Direction: Fill in the chart below by writing what you **K**now and what you **W**ant to know about the topic "solving radical equations."

What I **K**now	What I **W**ant to know	What I **L**earned
Subjective Answer	**Subjective Answer**	DO NOT ANSWER THIS PART YET

Cite some interesting ideas written by the students on the What They Know and What They Want to know columns. Then give the next activity that deals with the application of their understanding in solving radical equations.

The next activity will require the learners to identify mathematical concepts used to solve the problem. Inform the class that using correct mathematical concepts/facts/laws/processes will lead them to answer the problem correctly.

Activity 3: **JUST GIVE ME A REASON!**

Answer Key

Directions: Answer the given activity by writing the concept/process/law used to simply the given equation.

1) $\sqrt{x} = 8$, where x >0	WHY?
$x^{\frac{1}{2}} = 8$	Write radicals in exponential form.
$\left(x^{\frac{1}{2}}\right)^2 = (8)^2$	Square both sides of the equation to simplify rational exponents.
$x^{\frac{2}{2}} = 8^2$	Apply the law of exponent, power of a power rule.
$x = 8^2$	Simplify exponents.
x = 64	Simplify.

Conclusion:

Radical equations can be solved by writing the expression in exponential form and simplifying it using the laws of exponents.

· Process students' answers to the follow-up questions giving emphasis to questions no. 2, 3, 4, 5 and 6. Elicit answers from the students.

Pose these questions and elicit students' answers: *How did you find the preceding activities? Are you ready to learn about solving radical expressions and its applications? Why?*

Before going further in the lesson, provide the class with some important notes on the topic. Give enough and varied examples to develop the necessary understanding. Discuss the illustrative examples given in the learner's material (Lesson 3: Radical Equations, pages 6 to 12) and define the extraneous root. After developing students' understanding on how to solve radical equations and determine extraneous root, provide the given illustrative example on how to solve problems involving radicals, the problem is also indicated in the learner's material.

In this section, students are expected to develop the skill of solving radical equations. The next section will deal with activities that will strengthen this understanding.

WHAT TO PROCESS:

The goal of learners in this section is to apply their understanding on solving radical equations. Towards the end of this module, they will be encouraged to apply their understanding on radicals in solving real- life problems.

Provide the next activity that will require them to solve radical equations.

Activity 4: **SOLVE ME!**

Answer Key

Directions: Solve the following radical equations and box the final answer.

1) $x = 100$

2) $m = 128$

3) $b = 100$

4) $n = 79$

5) $s = 27$

6) $x = 10$

7) $x = 4$

8) $a = -2$

9) $m = \pm\frac{6\sqrt{5}}{5}$

10) $h = \frac{25}{3}$

A follow-up discussion may follow (if needed) through processing students' answers to the questions. Then provide the next activity. Say: *How did you do in the preceding activity? Did you do well? The previous activity dealt with solving radical equations. Try to solve the next activity that requires postulates, definitions and theorems that you learned from geometry.*

Activity 5: **THE REASONS BEHIND MY ACTIONS!**

Answer Key

Directions: Solve the radical equations. Write your solution and the property, definition or theorem that you used with respect to your solution .

RADICAL EQUATION	SOLUTION	REASON
$6\sqrt{8a^2-72}=5$	$6(8a^2-72)^{\frac{1}{2}}=5$	Write radicals to exponential form.
	$\left[6(8a^2-72)^{\frac{1}{2}}\right]^2=[5]^2$	Square both sides of the equation to simplify rational exponents.
	$6^2(8a^2-72)^{\frac{2}{2}}=5^2$	Apply the law of exponents, power of a product law
	$36(8a^2-72)=25$	Simplify exponents.
	$8a^2-72=\frac{25}{36}$	Apply division property of equality.
	$8a^2=\frac{25}{36}+72$	Apply addition property of equality.
	$a^2=\frac{2617}{288}$	Apply addition of dissimilar fractions.
	$(a^2)^{\frac{1}{2}}=\left(\frac{2617}{288}\right)^{\frac{1}{2}}$	Raise the equation to ½ that will simplify the exponent of the variable.
	$a=\sqrt{\frac{2617}{288}}$	Transform to radical expression.
	$a=\frac{\sqrt{5234}}{24}$	Rationalize the denominator.

Process students' answers to the follow-up questions. Give more emphasis to questions no.2,3 and 6. Students must realize that in solving radical equations, it is necessary to be guided by mathematical laws/facts/concepts to arrive at the correct answer.

End this section by eliciting from the class the important lessons they gained from solving radical equations

WHAT TO UNDERSTAND

In this section, let the students apply their understanding of solving radical equations to real-life problems. Tell them to use the mathematical ideas and the examples presented in the preceding section to answer the activities provided. Expectedly, the activities aim to intensify the application of the different concepts students have learned.

***Activity 6:* PROBLEM-SOLVED!!**

Answer Key

Directions: Solve the problems below by analyzing the given statements and answering the questions that follow.

A. Number problems.

1. The numbers are 2 and 17.
2. Numbers zero and 1 are equal to their own square roots.
3. The number is 0.
4. The number is 49.
5. The number is 12.

***B.* Approximately, the distance d in miles that a person can see to the horizon is represented by the equation $d = \sqrt{\frac{3h}{2}}$, where h is the height. (1 mile = 1, 609.3m)**

1. At a height of 8,000m, one can see $\sqrt{7.455}$ miles or approximately 2.73 miles or 4 393 meters to the horizon through an airplane window.
2. A sailor can see $\sqrt{0.01864}$ miles or approximately 0.1364 miles or 219.51 meters to the horizon from the top of a 20m mast.
3. A man can see $\sqrt{9.1344}$ miles or approximately 3.02 miles or 4 860.22 meters on the horizon through an airplane window at a height of 9800m?
4. A sailor sees $\sqrt{0.2237}$ mile or approximately 0. 1496 miles or 240.76 meters from the top of a 24-m mast.

C. The formula r = 2 $\sqrt{5L}$ can be used to approximate the speed r, in miles per hour, of a car that has left a skid mark of L, in feet.

1. At 50 mph, a car leaves a skid mark at 125ft. At 70 mph, a car leaves a skid mark at 245ft.
2. At 60 mph, a car leaves a skid mark at 180ft. At 100 mph, a car skid leaves a skid mark at 500ft.

D. Carpenters stabilize wall frames with a diagonal brace. The length of the brace is given by L = $\sqrt{H^2 + W^2}$ ***.***

1. If the bottom of the brace is attached 9 meters from the corner and the brace is 12 meters long, the corner post should be nailed at $3\sqrt{7}$ meters.

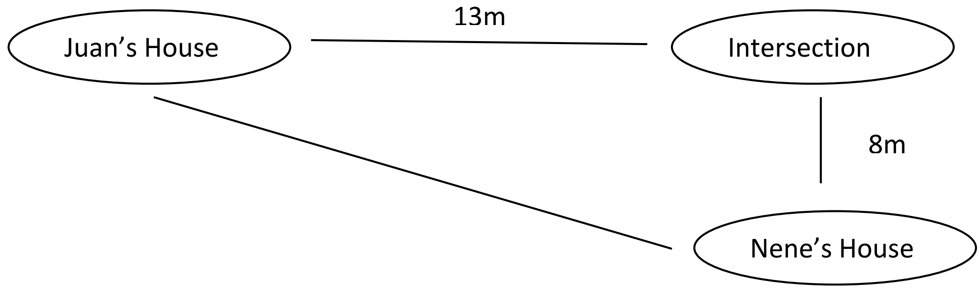
Source:EASE Modules, Year 2 – Module 6 Radical Expressions, pages 14-17

***Activity 7:* MORE PROBLEMS HERE!**

Answer Key

Directions: Solve the given problems then answer the questions that follow.

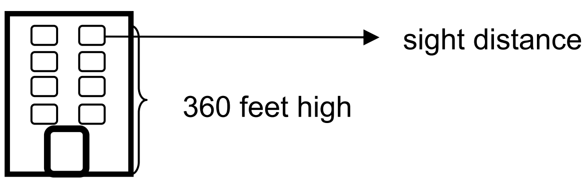
1)



2) Juan will travel $\sqrt{233}\ m$ or approximately 15.26 m along the shortcut.
3) Juan will save approximately 6m by taking the short cut rather than walking along the sidewalks.
4) If one of the distances increases/decreases, the distance of the shortcut will also increase/decrease respectively. Justify the answer by giving values.
5) Using the equation $c = \sqrt{a^2 + b^2}$ and the skill of simplifying radical equations.

1) The wire is $\sqrt{85}\ meters$ which is between 9 meters and 10 meters.
2) If the wire is farther/ nearer to the base, the length will be longer/shorter respectively. Justify the answer by giving values/examples.
3) Using the equation $c = \sqrt{a^2 + b^2}$ and the skill of simplifying radical equations.

1)



2) A person can see $\sqrt{165}\ meter$ or approximately between 12 meters and 13 meters far from a 110-meter high building on a clear day.
3) If the height of the building increases/decreases, the sight distance might go farther/nearer respectively. Justify the answer by giving values/examples.

In the previous activity, students were able to illustrate and solve real-life problems that involve radicals. Process students' learning in this activity.

Provide the next activity where they will be required to formulate and solve real-life problems.

Activity 8: **WHAT IS MY PROBLEM!**

Answer Key

Directions: Formulate a problem based on the given illustration then answer the questions that follow.

.

1) The illustration shows that a 63m lighthouse is 525 nautical miles away to the base of a boat on the sea.
2) From the top of the lighthouse, how far in meters is the base to the boat?
3) Using the equation $c = \sqrt{a^2 + b^2}$ and the skills needed in simplifying radical equations.
4) $525\ naut.miles \bullet \frac{1852\ m}{1 naut.mile} = 972{,}300\ m$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{(63m)^2 + (972300\ m)^2}$

$c \approx 972{,}300\ meters$

5) The lighthouse is approximately 972,300 meters away from the base of the boat on the sea.
6) If the height of the light house changed from 63 meters to 85 meters, there will be a little effect to the distance of the ship from the base of the light house, from 972,299.998 meters to 972,299.9963 meters.
7) Use the understanding on simplifying radical equations to solve real-life related problems. The skill of approximating radicals is also necessary.

$T = 2\pi\sqrt{\frac{L}{32}}$, the formula which gives the time (T) in seconds for a pendulum of length (L) to complete one full cycle.

1) A pendulum is 1.5 feet long.
2) How much time is needed for the pendulum to complete one full cycle.
3) Using the given formula and the skills needed to simplify radical equations.

$T = 2\pi\sqrt{\frac{L}{32}}$

4) $T = 2(3.14)\sqrt{\frac{1.5}{32}}$

$T = 1.36$

5) A 1.5 foot pendulum will take between 1 to 2 seconds to complete one full cycle.
6) A 0.811 foot pendulum take 1 second to complete one full cycle.
7) Use your understandings of simplifying radical equations to solve real-life related problems. The skill of approximating radicals is also necessary.

Pose the question: ***Were you able to answer the preceding activities? Which activity interests you the most? What activity did you find difficult to answer? How can you overcome these difficulties?***

Process students' answers to these questions then provide the next activity for self-assessment.

***Activity 9:* K-W-L CHART**

Direction: Fill in the chart below by writing what you have learned from the topic "solving radical equations."

What I **K**now	What I **W**ant to know	What I **L**earned
WITH ANSWER ALREADY	WITH ANSWER ALREADY	

***Activity 10:* Synthesis Journal**

Directions: Fill in the table below by answering the given question.

Synthesis Journal		
What interests me.	What I learned.	How can the knowledge of radical equations help us solve real life problems?

Ask some volunteer students to share their answers to the previous activity. Give emphasis on interesting and important answers, specially those that deal with real-life applications.

WHAT TO TRANSFER:

Give the students opportunities to demonstrate their understanding of solving radical equations. Let them perform Activity 17. You can ask the students to work individually or in groups. In this activity, the students will create a magazine feature that deals with the application of radical equations in a real-life setting. Be guided by the rubrics in evaluating students' output.

Activity 11: TRANSFER TASK

Hang time is defined as the time that you are in the air when you jump. It can be calculated using the formula $t = \sqrt{\frac{2h}{g}}$, where *h* is the height in feet, *t* is the time in seconds and *g* is the gravity given as $32\ {ft}/{sec^2}$.

Your school newspaper is to release its edition for this month. As a writer/researcher of the sports column, you were tasked to create a feature regarding the hang time of your school's basketball team members. Your output shall be presented to the newspaper adviser and chief editor and will be evaluated according to the mathematical concept used, organization of report, accuracy of computations and practicality of your suggested game plan based on the result of your research.

RUBRICS FOR THE TRANSFER TASK

CATEGORIES	4 EXCELLENT	3 SATISFACTORY	2 DEVELOPING	1 BEGINNING
Mathematical Concept	Demonstrates a thorough understanding of the topic and uses it appropriately to solve the problem.	Demonstrates a satisfactory understanding of the concepts and uses it to solve the problem.	Demonstrates incomplete understanding and has some misconceptions.	Shows lack of understanding and has severe misconceptions.
Accuracy of Computation	All computations are correct and are logically presented.	The computations are correct.	Generally, most of the computations are not correct.	Errors in computations are severe.
Practicality	The output is suited to the needs of the client and can be executed easily. Ideas presented are appropriate to solve the problem.	The output is suited to the needs of the client and can be executed easily.	The output is suited to the needs of the client but cannot be executed easily.	The output is not suited to the needs of the client and cannot be executed easily.
Organization of Report	Highly organized. Flows smoothly. Observes logical connections of points.	Satisfactorily organized. Sentence flow is generally smooth and logical.	Somewhat cluttered. Flow is not consistently smooth, appears disjointed.	Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.

After the transfer task, provide the next activity for self

Activity 12: **SUMMARY**
Directions: Complete the paragraph below.

LESSON CLOSURE

This lesson ____

____

One key idea was____
____
____

This is important because ____
____
____

Another key idea ____
____
____

This matters because ____
____
____

In sum, this lesson ____

____

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about solving radical equations. The lesson provided the learners with opportunities to solve radical equations. They identified and described the process of simplifying these expressions. Moreover, students were given the chance to demonstrate their understanding of the lesson by doing a practical task. Learner's understanding of this lesson and other previously learned mathematics concepts and principles will facilitate their learning of the next module.

SUMMATIVE ASSESSMENT

Part I:

Directions: Choose the letter that you think best answers the questions.

1) Which of the following is a radical equation in one variable?

a. $\sqrt{m} = 2$
b. $\sqrt{12} = 5$
c. $2x\sqrt{5} + 3y\sqrt{3} = 10$
d. $\sqrt{12} + \sqrt{30} = 7$

2) What is the exponential form of $2\sqrt{a} + 3\sqrt{b}$?

a. $2^{\frac{1}{2}}a + 3^{\frac{1}{2}}b$
b. $2a^{\frac{1}{2}} + 3b^{\frac{1}{2}}$
c. $(2a)^{\frac{1}{2}} + (3b)^{\frac{1}{2}}$
d. $(2a + 3b)^{\frac{1}{2}}$

3) Which of the following is an incorrect characteristic of a radical in simplest form?

a. No fraction as radicands.
b. No radicands with variable.
c. No radical appears in the denominator of a fraction.
d. No radicands have perfect square factors other than 1.

4) What is the sum of $2\sqrt{3}$, $-5\sqrt{2}$, $10\sqrt{3}$, $14\sqrt{2}$ and $-3\sqrt{2}$?

a. $34\sqrt{5}$
b. $18\sqrt{5}$
c. $6\sqrt{2} + 12\sqrt{3}$
d. $12\sqrt{2} + 6\sqrt{3}$

5) What value of *k* will make the equation $\sqrt[3]{k+4} = \sqrt[3]{2k+3}$ true?

a. 4
b. 3
c. 2
d. 1

6) Which of the following is the simplified form of $\left(2+\sqrt{5}\right)\left(5-\sqrt{7}\right)$?

a. $10 + 2\sqrt{35}$
b. $10 + 7\sqrt{5} - \sqrt{35}$
c. $10 + 3\sqrt{7} - \sqrt{35}$
d. $10 - 2\sqrt{7} + 5\sqrt{5} - \sqrt{35}$

7) Which of the following is true?

a. $10\sqrt{\frac{x}{5}} = 2\sqrt{5x}$
b. $20 + 2\sqrt{55} = 22\sqrt{55}$
c. $2\sqrt{3}\left(4\sqrt{5} + \sqrt{6}\right) = 6\sqrt{15} + 2\sqrt{18}$
d. $10 - 2\sqrt{5} + 5\sqrt{5} - 5 = 15 - 3\sqrt{5}$

8) What is the simplified form of $\dfrac{\sqrt{20x+5}}{\sqrt{5x}}$?

a. $\sqrt{4x^2 + x}$
b. $5x\sqrt{4x^4 + x}$
c. $\dfrac{5\sqrt{4x^2 + x}}{x}$
d. $\dfrac{\sqrt{4x^2 + x}}{x}$

9) What is the length of the hypotenuse of a right triangle if its sides measure 6 inches and 9 inches?

a. $\sqrt{45}$ inches
b. $\sqrt{117}$ inches
c. 45 inches
d. 117 inches

10) On Earth the acceleration g due to gravity is approximately 32 ft second per. Using the formula $v = \sqrt{2gd}$, what is the velocity of an object after falling 7 feet from the point where it was dropped?

a. 3.74 feet per second
b. 8 feet per second
c. 14.97 feet per second
d. 21.17 feet per second

11) The surface area of a basketball is 36 square inches. What is the radius of the basketball if the formula of the surface area of a sphere is $SA = 4\pi r^2$?

a. $\sqrt{\frac{36}{4\pi}}$ inches
b. $\sqrt{3\pi}$ inches
c. $3\sqrt{\pi}$ inches
d. $\frac{3\sqrt{\pi}}{\pi}$ inches

12) Using the formula $t = 2\pi\sqrt{\frac{l}{32}}$, approximately how long will it take for a 20 feet swing to complete one full cycle? (use: 1 inch $\approx$ 0.83 and $\pi \approx 3.14$)

a. 4 minutes and 31 seconds
b. 4 minutes and 30 seconds
c. 4 minutes and 21 seconds
d. 4 minutes and 20 seconds

13) In a flagpole, a 10 meter rope is attached to the top of the pole from a point on the field. If the rope is 8 meters away from the base, how high is the pole?

a. 1.41 meters
b. 4.24 meters
c. 6 meters
d. 12.81 meters

14) The volume *V* of a sphere is modelled by the formula $V = \frac{4}{3}\pi r^3$ where *r* is the radius. An Earth's miniature model has a volume of 123 m^3. Approximately, what must be its radius? (use π = 3.14)

a. 3.09 meters
b. 5.42 meters
c. 21.68 meters
d. 68.07 meters

PART II: (*for no.s 15 – 20)*

Formulate and solve a problem based on the given situation below. Your output shall be evaluated according to the given rubric below.

> It is recommended that a ramp has a 14.5 degrees inclination for buildings to be accessible to handicapped persons. The city's engineering department is planning to construct ramps on identified parts of the city. As part of the department you were required to develop a proposal regarding the ramp's dimensions and present this to the Board. The Board will assess the concept used, practicality and accuracy of computation.

RUBRIC

CATEGORIES	2 SATISFACTORY	1 DEVELOPING
Mathematical Concept	Demonstrates a satisfactory understanding of the concepts and uses it to simply the problem.	Demonstrates incomplete understanding and has some misconceptions.
Accuracy of Computation	The computations are correct.	Generally, most of the computations are not correct.
Practicality	The output is suited to the needs of the client and can be executed easily.	The output is not suited to the needs of the client and cannot be executed easily

Summative Assessment: (Key-Answer)

1) A	6) D	11) D	15-20) Product/Performance
2) B	7) A	12) A	*Answer to this subtest*
3) B	8) D	13) C	*depends on the students. Just be*
4) C	9) B	14) A	*guided by the rubric on how to*
5) D	10) D		*score their output.*

Glossary of Terms:

1) Conjugate Pair – two binomial radical expressions that have the same numbers but only differ in the sign that connects the binomials
2) Dissimilar Radicals – radicals with different order and having the same radicand or with same order and having different radicand
3) Exponent – a number that says how many times the base is to be multiplied to itself
4) Extraneous Solution – a solution that does not satisfy the given equation
5) Radical – an expression in the form of $\sqrt[n]{a}$ where *n* is a positive integer and *a* is an element of the real number system.
6) Radical equations – equations containing radicals with variables in the radicand
7) Rational Exponent – an exponent in the form of $\frac{m}{n}$ where *m* and *n* are integers and $n \neq 0$
8) Rationalization – simplifying a radical expression by making the denominator free of radicals
9) Similar Radicals – radicals with the same order and having the same radicand

REFERENCES AND WEBSITE LINKS USED IN THIS MODULE:

REFERENCES

1) Bautista, Leodegario S. Aurora C. Venegas, Asterio C. Latar, Algebra with Business and Economic Applications, GIC Enterprises and Co. Inc, 1992
2) Cabral, Josephine M., Julieta G. Bernabe and Efren L. Valencia, Ph.D., New Trends in Math Series, Algebra II, Functional Approach, Workbook, Vibal Publishing House Inc., 2005

3) Concepcion Jr., Benjamin T., Chastine T. Najjar, Prescilla S. Altares, Sergio E. Ymas Jr., College Algebra with Recreational Mathematics, 2008 Edition, YMAS Publishing House
4) Dignadice-Diongzon, Anne, Wizard Mathematics, Intermediate Algebra Worktext, Secondary II, Wizard Publishing Haws Inc., Tarlac City, 2006
5) Tizon, Lydia T. and Jisela Naz Ulpina, Math Builders, Second Year, JO-ES Publishing House, Inc., Valenzuela City, 2007

REFERENCES FOR LEARNER'S ACTIVITES:

1) Beam Learning Guide, Second Year – Mathematics, Module 10: Radicals Expressions in General, pages 31-33
2) Beam Learning Guide, Year 2– Mathematics, Module 10: Radicals Expressions in General, Mathematics 8 radical expressions, pages 41-44
3) EASE Modules, Year 2 – Module 2 Radical Expressions, pages 9 – 10
4) EASE Modules, Year 2 – Module 5 Radical Expressions, page 18
5) EASE Modules, Year 2 – Module 6 Radical Expressions, pages 14-17
6) Negative Exponents. Algebra-Classroom.com http://www.algebra-class.com/negative-exponents.html
7) (Negative Exponents) http://braingenie.ck12.org/skills/105553
8) (Rational Exponents) http://braingenie.ck12.org/skills/106294
9) (Rational Exponents and Radical Function)http://braingenie.ck12.org/skills/106294
10) Scientific Notation. khan Academy. Multiplication in radicals examples https://www.khanacademy.org/math/arithmetic/exponents-radicals/computing-scientific-notation/v/scientific-notation-3--new

WEBLINKS LINKS AS REFERENCES AND FOR LEARNER'S ACTIVITES:

1) Applications of surface area. braining camp.
http://www.brainingcamp.com/legacy/content/concepts/surface-area/problems.php
2)(Charge of electron) https://www.google.com.ph/#q=charge+of+electron
3)(Extraneous Solutions) http://www.mathwords.com/e/extraneous_solution.htm
4) Formula for hang time
http://www.algebra.com/algebra/homework/quadratic/Quadratic_Equations.faq.question.214935.html
5) (Formula for pendulum) http://hyperphysics.phy-astr.gsu.edu/hbase/pend.html
6) Gallon of Paint
http://answers.ask.com/reference/other/how_much_does_one_gallon_of_paint_cover
7) Gallon of paint
http://answers.reference.com/information/misc/how_much_paint_can_1_gallon_cover
8) Radical Equations
http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_04/add_lesson/radical_equations_alg1.pdf
9) Radical Equations in One Variable
http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_03/extra_examples/chapter11/lesson11_3.pdf
10) Radical Equations and Problems
http://www.palmbeachstate.edu/prepmathlw/Documents/0020.section.8.6.pdf
11) (Radio frequency)http://www.sengpielaudio.com/calculator-radiofrequency.htm
12) Small Number. Wikipediahttp://en.wikipedia.org/wiki/Small_number
13) Solving Radical Equations and Inequalities

http://www.glencoe.com/sec/math/algebra/algebra2/algebra2_04/add_lesson/solve_rad_eq_alg2.pdf
14) (Speed of Light) http://www.space.com/15830-light-speed.html
15) (Square meter to square ft)http://www.metric-conversions.org/area/square-feet-to-square-meters.htm
16)(Square meter to square feet)http://calculator-converter.com/converter_square_meters_to_square_feet_calculator.php
17) (Diameter of an atomic nucleus)
http://www.princeton.edu/~achaney/tmve/wiki100k/docs/Atomic_nucleus.html

TEACHER'S GUIDE

Module 5: QUADRILATERALS

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of quadrilaterals.

Performance Standard:

The learner is able to investigate, analyze, and solve problems involving quadrilaterals through appropriate and accurate representation.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr>
<td rowspan="2">SUBJECT: Math 9

QUARTER: Third Quarter

TOPIC: Quadrilaterals</td>
<td colspan="2">LEARNING COMPETENCIES</td>
</tr>
<tr>
<td colspan="2">The learner…
1. identifies quadrilaterals that are parallelograms.
2. determines the conditions that guarantee a quadrilateral a parallelogram.
3. uses properties to find measures of angles, sides and other quantities involving parallelograms.
4. proves theorems on the different kinds of parallelogram (rectangle, rhombus, square)
5. proves the Midline Theorem
6. proves theorems on trapezoids and kites
7. solves problems involving parallelograms, trapezoids and kites.</td>
</tr>
<tr>
<td>WRITERS:

JERRY DIMLA CRUZ
ROMMEL HERO A. SALADINO</td>
<td>ESSENTIAL UNDERSTANDING:

Students shall understand that quadrilaterals are very useful in dealing with real-life situations such as problem solving.</td>
<td>ESSENTIAL QUESTION:

How useful are the quadrilaterals in dealing with real-life situations such as problem solving?</td>
</tr>
</table>

B. Planning for Assessment

Product/Performance

The following are the products/performances that students are expected to come up with in this module.

a. Applying the different properties and theorems on the different kinds of quadrilateral to solve real-life problems and situations involving parallelograms, trapezoids and kites.
b. Solving with speed and accuracy on real-life problems and situations involving the kinds of quadrilateral.

ASSESSMENT MAP

TYPE	KNOWLEDGE	PROCESS / SKILLS	UNDERSTANDING	PRODUCTS / PERFORMANCE
PRE-ASSESSMENT / DIAGNOSTIC	**Pre-Test** 1. Identifying quadrilaterals that are parallelogram	**Pre-Test** 1. Solving problems involving parallelograms, trapezoids and kites 2. Proving theorems on the different kinds of parallelogram (rectangle, rhombus, square)	**Pre-Test** 1. Determining the conditions that guarantee a quadrilateral a parallelogram	Pre-Test 1. Finding the measures of angles, sides and other quantities involving properties of parallelograms.
FORMATIVE	**Quiz 1** 1. Identifying the information given in a problem involving parallelogram **Quiz 3** 1. Identifying the information given in a problem involving trapezoid	**Quiz 1** 1. Applying the properties of a parallelogram **Quiz 2** 1. Applying properties of a parallelogram **Quiz 3** 1. Applying the theorems on trapezoid	**Quiz 2** 1. Determining the conditions that guarantee a quadrilateral a parallelogram	**Quiz 1** 1. Finding the measures of angles, sides and other quantities involving properties of parallelograms.
SUMMATIVE	**Post-Test** 1. Identifying quadrilaterals that are parallelogram	**Post-Test** 1. Solving problems involving parallelograms, trapezoids and kites 2. Proving theorems on the different kinds of parallelogram (rectangle, rhombus, square)	**Post-Test** 1. Determining the conditions that guarantee a quadrilateral a parallelogram	**Post-Test** 1. Finding the measures of angles, sides and other quantities involving properties of parallelograms.

SELF ASSESSMENT	**Journal Writing:** Expressing understanding through appropriate use of the conditions which guarantee that a quadrilateral a parallelogram. Expressing understanding through accurate computations and wise use of the key concepts on the different quadrilaterals. Expressing understanding through valid explanations and justifiable reasoning of facts and principles related to quadrilaterals.

ASSESSMENT MATRIX (Summative Test)

LEVELS of ASSESSMENT	What will I assess?	How will I assess?	How will I score?
KNOWLEDGE 15%	The learner demonstrates understanding of key concepts of parallelograms, kites and trapezoids.	Paper and Pencil Test Part I: Item Nos. 1, 2, 10 and 13	1 point for every correct response
PROCESS and SKILLS 25%	Identifies quadrilaterals that are parallelograms Determines the conditions that guarantee a quadrilateral a parallelogram Solves problems involving parallelograms, trapezoids and kites Proves theorems on the different kinds of parallelogram (rectangle, rhombus, square	Part I: Item Nos. 3, 4, 5, 6, 7, 8, 9, 12, 14, 16	1 point for every correct response
UNDER-STANDING 30%	Finds the measures of angles, sides and other quantities involving properties of parallelograms.	Part I: Item Nos. 11, 15, 17, 18, 19 and 20	1 point for every correct response
PRODUCT and PERFORMANCE 30%	The learner is able to investigate thoroughly mathematical relationships in various real-life situations using a variety of strategies.	Part II: Item 1	**Rubric on Problem**

C. Planning for Teaching-Learning

INTRODUCTION

The module covers the key concepts on quadrilaterals. Students are given practical tasks to use their prior knowledge and skills in learning quadrilaterals in a deeper perspective. The students must have been informed always in advance the materials needed in performing activities given in the module. They shall undergo series of varied activities to process the knowledge and skills learned and reflect to further understand such concepts and be able to answer HOTS questions. In the end, they shall be able to transfer their understanding in dealing with real-life situations like problem solving.

Objectives:

After the learners have gone through the key concepts contained in this module, they are expected to:

a. identify quadrilaterals that are parallelograms;
b. determine the conditions that guarantee a quadrilateral a parallelogram;
c. use properties to find measures of angles, sides and other quantities involving parallelograms, trapezoids and kites;
d. apply the Midline Theorem and other theorems on the different kinds of parallelogram (rectangle, rhombus, square), trapezoids and kites;
e. solve problems involving parallelograms, trapezoids and kites; and
f. design and create a "quadrilatable" – a study table having parts showing the different quadrilaterals.

LEARNING GOALS AND TARGETS:

Students are expected to demonstrate understanding of key concepts of quadrilaterals. They are also expected to investigate, analyze, and solve problems involving quadrilaterals through appropriate and accurate representation and to justify how useful are the quadrilaterals in dealing with real-life situations.

Instructions to the Teacher

✓ **Answer Key to Pre-Assessment Test**

1. a	2. b	3. a	4. d	5. d	6. c	7. c	8. b	9. c	10. b
11. b	12. b	13. c	14. b	15. c	16. d	17. c	18. c	19. b	20. b

To formally start the module, the teacher shall ask the following questions:

a. Did you know that one of the world's largest domes is found in the Philippines?
b. Have you ever played billiards?
c. Have you joined a kite-flying festival in your barangay?
d. Have you seen a nipa hut made by Filipinos?

➢ **What to know?**

Before doing Activity 1, the teacher shall give the students an overview of the whole module using the module map and ask the question "How useful are the quadrilaterals in dealing with real-life situations like problem solving?" Through this, the students shall be able to know what they will learn as they go on with the module.

❖ **Activity 1. Four-Sided Everywhere!**

Present the illustrations of Activity 1 and let the students answer the questions presented. This entails guided discussion.

After doing Activity 1, the teacher shall lead the students in reviewing the definitions of the different kinds of quadrilateral through Activity 2.

❖ **Activity 2. Refresh Your Mind!**

Prepare copies of the table. Make this activity "Pair and Share". Give each pair a copy of the table and let them be guided by the given directions.

✓ **Answer Key**

1. *Quadrilateral is a polygon with 4 sides.*
2. *Trapezoid is a trapezoid is a quadrilateral with exactly one pair of opposite sides parallel.*
3. *Isosceles Trapezoid is a trapezoid whose non-parallel opposite sides are congruent.*
4. *Parallelogram is a quadrilateral where both pairs of opposite sides are parallel*
5. *Rectangle is a parallelogram with four right angles.*
6. *Rhombus is a parallelogram with four congruent sides.*
7. *Square is a parallelogram with four right angles and four congruent sides.*
8. *Kite is a quadrilateral with two pairs of adjacent, congruent sides.*

❖ Activity 3. Plot. Connect. Identify.

In this activity, see to it that all students are doing the right plotting and connecting of points. Ask them to answer the questions based on their drawings. They must be able to defend their answers using the definitions of parallelogram.

❖ Activity 4. Which is which?

Discuss with them well their answers to the questions presented. Make sure that they shall use the definitions as reasons why a given quadrilateral is or is not a parallelogram.

➢ What to process?

The teacher shall inform the students on what shall be done in learning the entire module. There will be different ***Check Your Guess*** and successive **Show Me!** activities which shall serve as their guide as they learn, discover and prove the key concepts on quadrilaterals. Let the students do ***Check Your Guess 1.*** Make sure that they are doing it correctly. Emphasize that they shall revisit the same table later on.

❖ Activity 5. Fantastic Four!

In this activity, the teacher shall divide the class into groups of four members each. Let them follow the procedures of the activity. Guide them as they go through each step. Make sure that they are doing the drawing and measuring with utmost accuracy. See to it that they are filling in the correct data in the table. After accomplishing all data needed, ask the questions presented and guide them as they give their answers based on their findings in the table.

The students must be able to have the same findings as follows:

a. Pairs of opposite sides have equal measures.
b. Pairs of opposite angles have equal measures.
c. Pairs of consecutive angles may have different measures but their sum is 180° thus they are supplementary.
d. Pairs of segments formed by intersecting diagonals have equal measures.

Since each diagonal divides a parallelogram into two congruent triangles, a conjecture that two triangles formed when a diagonal of a parallelogram is drawn are congruent. Explanation of the answer can be made by using SSS Congruence Postulate (Two pairs of opposite sides are congruent and a diagonal is congruent to itself by reflexivity, thus the two triangles are congruent).

The students must be able to say that their findings apply to all kinds of parallelogram because the answers to the given questions are the same in each kind.

Guide them as they fill up the correct words/phrases in the conditions that guarantee a quadrilateral as parallelogram.

✓ Answer Key to the Conditions that guarantee a Quadrilateral a Parallelogram

1. Opposite ; congruent
2. Opposite ; congruent
3. Consecutive ; supplementary
4. Diagonals
5. Diagonal
6. Congruent ; congruent triangles

❖ Activity 6.1 Draw Me!

Note: answers may vary.

❖ Activity 6.2 Defense! Defense!

This activity shall test the students' ability to reason out and defend why a given figure is a parallelogram. The students must be able to tell the condition/s that guarantee/s that the given figure is a parallelogram.

1. Two pairs of opposite sides are congruent because AD & BC are 7 and AB & DC are 6.
2. Two pairs of opposite angles are congruent or pairs of consecutive angles are supplementary as shown.
3. The diagonals bisect each other because of similar markings.
4. A diagonal of a parallelogram forms two congruent triangles because of SSS Congruence Postulate.

This time, instruct students to revisit ***Check Your Guess 1.*** If they answered R, they know the reason already. If they answered W, it means they were wrong as per guess made. Discuss with them their reason/s why they were wrong. In this way, the teacher can establish continuity of or strengthen the knowledge and skills they have learned.

Discussion.

The teacher shall present to the students the properties of a parallelogram. After which, instruct them to do ***Check Your Guess 2*** to determine their prior knowledge on the properties of parallelogram. The teacher shall guide the students in proving each property of parallelogram in the different **Show Me!** activities.

✓ **Answer Key to the Show Me! Activities on Properties of Parallelogram**

Parallelogram Property 1

Statements: 1. Parallelogram HOME ; 2. $\overline{HO} \parallel \overline{ME}$; $\overline{HE} \parallel \overline{MO}$; 4. $\angle$HOE $\cong \angle$MEO ; $\angle$HEO $\cong \angle$MOE ; 5. $\overline{EO} \cong \overline{OE}$
Reasons: 3. Line Postulate ; 6. ASA Congruence Postulate ; 7. CPCTC

Parallelogram Property 2

Statements: 1. Parallelogram JUST ; 3. $\overline{JT} \cong \overline{SU}$; $\overline{JU} \cong \overline{ST}$; 4. $\overline{TU} \cong \overline{TU}$; $\overline{JS} \cong \overline{JS}$
Reasons: 2. Line Postulate ; 5. SSS Congruence Postulate ; 6. CPCTC

Parallelogram Property 3

Statement: 1. Parallelogram LIVE ; 5. $\angle$E and $\angle$L are supplementary.
Reasons: 2. Definition of Parallelogram ; 3. Same side interior angles are supplementary. (SSIAS) ; 4. Parallelogram Property 2

Note: explain/remark that the other 3 pairs of consecutive angles in LIVE are also supplementary.

Parallelogram Property 4

Statements: 1. Parallelogram CURE with diagonals $\overline{CR}$ and $\overline{UE}$ that meet at point H ; 5. ΔCHU $\cong \Delta$RHE
Reasons: 2. Parallelogram Property 1. ; 3. Definition of parallelogram ; 4. AIAC ; 5. Vertical Angle Theorem (VAT) ; 7. CPCTC ; 8. Definition of Bisectors

Parallelogram Property 5

Statements: 1. Parallelogram AXIS with diagonal $\overline{AI}$; 4. $\overline{IA} \cong \overline{AI}$
Reasons: 2. Definition of Parallelogram ; 3. AIAC ; 5. AIAC ; 6. ASA

❖ **Activity 7. Yes You Can!**

✓ **Answer Key**

a. x = 6
   b. AB = 13 cm
   c. y = 10
   d. AD = 13 cm
   e. P = 52 cm

x and y were solved by applying parallelogram property 1. The lengths of AB and AD were determined through substitution.

2. a. a = 40
   b. m$\angle$BAD = 105
   c. m$\angle$CBA = 75

a was found by applying parallelogram property 2. $\angle$CBA was solved by applying parallelogram property 3.

3. a. BD = 16 cm
   b. AE = 6.5 cm

The lengths of $\overline{BD}$ and $\overline{AE}$ were solved by applying parallelogram property 4.

✓ **Answer Key to Quiz 1**

A.	1. $\overline{TH}$	2. ΔTHA	3. $\overline{TS}$	4. ΔMAT	5. $\angle$HMA
	6. 100	7. 105	8. 7	9. 6	10. 4.5
B.	1. 13	2. 84	3. m$\angle$OHE = 135 ; m$\angle$HER = 45		
	4. 35	5. 25			

This time, instruct students to revisit ***Check Your Guess 2***. If they answered R, they know the reason already. If they answered W, it means they were wrong as per guess made. Discuss with them their reason/s why they were wrong. In this way, the teacher can establish continuity of or strengthen the knowledge and skills students have learned.

Discussion.

The teacher shall ask the students the following questions:

1. What are the kinds of parallelogram?
    1. Are you aware of the different theorems that justify each kind?
    2. Do you want to know?

Answers may vary so just entertain each and tell later that as you go on with the different activities, they will be able to determine the different theorems that justify each kind. Let them do ***Check Your Guess 3*** to determine their prior knowledge about the relationships that exist between the different kinds of parallelogram.

❖ **Activity 8. I Wanna Know!**

Roam around to check if they are following the procedures as directed. Accurate drawings and measurements must be done. Ask the questions presented. They must be able to tell that $\angle$OHE and $\angle$PEH measure 90^{O} each and therefore, they are right angles. The diagonals must have the same lengths. Quadrilateral HOPE appears to be a parallelogram because opposite angles are congruent and consecutive angles are supplementary. Rectangle is the specific parallelogram that it represents.

Tell the students that Activity 8 helped them discover the following theorems related to rectangles:

<u>Theorem 1</u>. If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.

<u>Theorem 2</u>. The diagonals of a rectangle are congruent.

Just like what they did to the properties of parallelogram, the students are going to prove also the two theorems on rectangle. Guide them as they go on with the **Show Me!** activities that follow. (***<u>Note</u>**<u>: SN stands for Statement Number</u>*)

✓ **Answer Key to Theorem 1.**

Statements: 1. WINS is a parallelogram with $\angle$W a right angle ; 3. $\angle$W ≅ N & $\angle$I ≅ $\angle$S ; 8. 90 = 90 ; 10. m$\angle$S = 90 ; 12. WINS is a rectangle.
Reasons: 2. Definition of right angle ; 4. Definition of congruent angles ; 5. Substitution (SN 2 & 4) ; 6. Consecutive angles are supplementary. ; 7. Substitution (SN 2 & 6) ; 9. Subtraction Property (SN 7 & 8) ; 11. If the measure of an angle is 90, then it is a right angle.

✓ **Answer Key to Theorem 2.**

Statements: 1. WINS is a rectangle with diagonals $\overline{WN}$ and $\overline{SI}$. ; 4. $\angle$WSN $\angle$INS ; 6. ΔWSN ≅ ΔINS
Reasons: 2. Opposite sides of a parallelogram are congruent. ; 3. Theorem 1 ; 5. Reflexive Property ; 7. CPCTC

After proving Theorems 1 and 2 on rectangles, tell the students that rhombus has also theorems and they shall discover those by doing Activity 9.

❖ **Activity 9. I Wanna Know More!**

In this activity, they must answer the questions based on the table they filled up with the correct information.

1. m$\angle$NIE = $\frac{1}{2}m\angle NIC$, so $\overline{IE}$ bisects $\angle$NIC.
2. m$\angle$INC = $\frac{1}{2}m\angle INE$, so $\overline{NC}$ bisects $\angle$INE.
3. m$\angle$NRE = m$\angle$CRE. The angles form a linear pair. Since the linear pair are formed by two right angles, then $\overline{NC} \perp \overline{IE}$.

They've just found out the following theorems on rectangles:

<u>Theorem 3</u>. The diagonals of a rhombus are perpendicular.

<u>Theorem 4</u>. Each diagonal of a rhombus bisects opposite angles.

The students are now ready to prove the above theorems. Guide them as they go on with the **Show Me!** activities that follow.

✓ **Answer Key to Theorem 3.**

Statements: 1. Rhombus ROSE ; 3. $\overline{RS}$ and $\overline{EO}$ bisect each other. ; 5. $\overline{RH} \cong \overline{HS}$; 7. ΔRHO ≅ ΔSHO ; 10. $\overline{RS} \perp \overline{OE}$
Reasons: 2. Definition of rhombus ; 4. $\overline{EO}$ bisects $\overline{RS}$ at H. ; 6. Reflexive Property ; 8. CPCTC ; 9. $\angle$RHO and $\angle$SHO form a linear pair and are congruent.

- ✓ **Answer Key to Theorem 4.**

  Statements: 1. Rhombus VWXY ; 3. $\overline{WY} \cong \overline{YW}$
  Reasons: 2. Definition of rhombus ; 4. SSS Congruence Postulate ; 5. CPCTC

The teacher shall ask the students using the question "Do you want to know the most special among the kinds of parallelogram and why?" This question shall make the students curious about it. Thus, instruct them to do Activity 10.

- ❖ **Activity10. Especially for You.**

With utmost accuracy, the students must be able to answer the questions based on what they've discovered in this activity.

1. $\angle$GDL = 90^{O}. Square GOLD is a rectangle because of Theorem 1.
2. GL = DO. Square GOLD is a rectangle because of Theorem 2.
3. $\angle$GCO and $\angle$OCL are both 90^{O}. Square is a rhombus because of Theorem 3.
4. The m$\angle$GDO = m$\angle$ODL and the m$\angle$GOD = m$\angle$LOD. Square GOLD is a rhombus because of Theorem 4. (*To further prove Theorem 4, consider $\overline{OD}$ as another diagonal and let them find out if $\overline{OD}$ bisects opposite angles also.)*

Emphasize that theorems 1 to 4 are applicable to a square. Tell to the students that the theorems true to a rectangle and the theorems true to a rhombus are both true to a square.

- ✓ **Answer Key to Quiz 2**

A.	1. AT	2. ST	3. ST	4. NT	5. AT
	6. ST	7. ST	8. AT	9. AT	10. AT
B.	1. Rh, S	2. All	3. Rc, S	4. Rc, S	5. S
C.	1. Rh, S	2. All	3. All	4. All	5. Rc, S
	6. Rc, S	7. All	8. Rh, S	9. Rh, S	10. All

This time, instruct students to revisit ***Check Your Guess 3***. If they answered R, they know already the reason. If they answered W, it means they were wrong as per guess made. Discuss with them their reason/s why they were wrong. In this way, the teacher shall establish continuity of or strengthen the knowledge and skills their students have learned.

Discussion.

The teacher shall ask the students the following questions:
1. Can you still remember the different kinds of triangle?

2. Is it possible for a triangle to be cut to form a parallelogram?
3. Did you know that a parallelogram can actually be formed out of a given triangle?
4. Do you want to know how it is done?

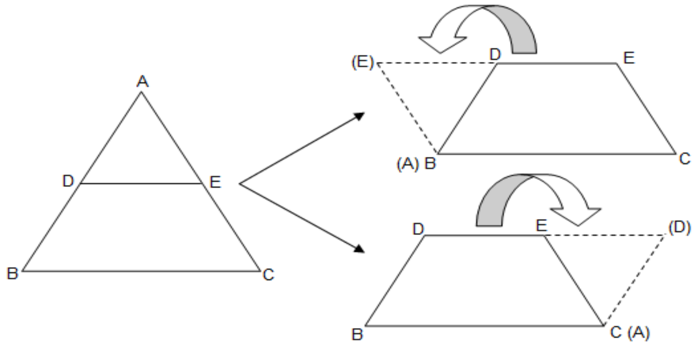
Answers may vary but they must recall correctly the different kinds of triangle. Let them do ***Check Your Guess 4*** to determine their prior knowledge about trapezoids and kite.

❖ Activity 11. It's Paperellogram!

This activity needs the teacher's guidance. Divide the class into groups of 4 members each. Let them follow the procedures of the activity. Make sure that they are doing the activity with utmost accuracy. As they go on with each step in the procedures (P_1 to P_5), ask the different questions presented.

In each of the triangle drawn (P_1), the segment joining the midpoints of any two sides is parallel (P_2) to the third side. The students must find out that the length of the segment drawn is one-half the length of the third side (P_3). After cutting the triangle along the segment drawn, the two figures formed must be a triangle and a trapezoid (P_4). Reconnect the triangle with the trapezoid in such a way that their common vertex was a midpoint and that congruent segments formed by a midpoint coincide.

There are two possible ways. See example below.



A conjecture can be made that a parallelogram is formed (P_5) when a segment joining the midpoints of any two sides is cut and reconnecting the figures formed in such a way that their common vertex was a midpoint and that congruent segments formed by a midpoint coincide. The students must be able to say that their findings are the same and such findings apply to all kinds of triangles because all figures formed are all parallelograms.

Their findings in Activity 11 helped them discover The Midline Theorem.

- **Answer Key to Theorem 5.**

  Reasons: 1. Given ; 3. Definition of Midpoint ; 4. VAT ; 5. SAS Congruence Postulate ; 6. CPCTC ; 7. If AIAC, then the lines are parallel. ; 8. Definition of Midpoint ; 9. CPCTC (SN 5) ; 10. Transitive Property ; 11. Definition of parallelogram. ; 12. $\overline{OE}$ is on the side of OT of HOTS ; 13. Segment Addition Postulate (SAP) ; 14. Substitution (SN 2) ; 15. Addition ; 16. Parallelogram Property 1. ; 17. Substitution ; 18. Substitution (SN 14 and 15)

After the **Show Me!** activity, the students are now ready to do Activity 12. They are going to apply what they have learned on the Midline Theorem.

- **Activity 12. Go For It!**

- **Answer Key**

1. MC = 21 by applying the Midline Theorem.
2. GI = 16 by definition of midpoint.
3. MG + GC = 30 applied definition of midpoint.
4. x = 3 by applying the Midline Theorem.
   AI + MC = 21 by addition.
5. y = 6 by definition of midpoint and of congruent segments.
   MG = 22; CG = 22 by definition of congruent segments.

Discussion.

Recall the definition of a trapezoid and its parts. They must be able to identify the bases, legs and base angles of a trapezoid. Then ask them to do Activity 13.

- **Activity 13. What a Trap!**

In this activity, $\overline{GO}$ must be parallel to the bases. After measuring $\overline{GO}$ and getting the sum of the bases, they must make a conjecture that the length of the segment joined by the midpoints of the legs is one-half the sum of the bases. As a result, they discovered Theorem 6 which is the Midsegment Theorem. Guide the students in doing **Show Me!** activity to prove the next theorem.

- **Answer Key to Theorem 6.**

  Statements: 1. Trapezoid MINS with median $\overline{TR}$; 4. PR = $\frac{1}{2}IN$ and $\overline{PR} \parallel \overline{IN}$; 9. TR = $\frac{1}{2}MS + \frac{1}{2}IN$

Reasons: 2. Line Postulate ; 3. Theorem 5 (Midline theorem), on ∆IMS ; 5. Definition of trapezoid ; 6. Definition of parallel, $\overline{TP} \parallel \overline{MS}$ and $\overline{MS} \parallel \overline{IN}$; 7. $\overline{TP}$ and $\overline{PR}$ are either parallel or the same line (definition of parallel). Since they contain a common point P, then $\overline{TP}$ and $\overline{PR}$ are contained in the same line. ; 8. SAP ; 10. Distributive Property of Equality

Discussion.

The teacher shall ask the students the following questions:

What if the legs of the trapezoid are congruent?
What must be true about its base angles and its diagonals?

❖ **Activity 14. Watch Out! Another Trap!**

Through the teacher's guidance, the students must be able to state that:

a. the angles in each pair of base angles have the same measure. They must make a conjecture that base angles in an isosceles trapezoid are congruent.
b. two opposite angles have a sum of 180^{O}. They must make a conjecture that opposite angles of an isosceles trapezoid are supplementary.
c. the diagonals have equal lengths. They must make a conjecture that the diagonals of an isosceles trapezoid are congruent.

From Activity 14, they've discovered three theorems related to isosceles trapezoid as follow:

Theorem 7. The base angles of an isosceles trapezoid are congruent.
Theorem 8. Opposite angles of an isosceles trapezoid are supplementary.
Theorem 9. The diagonals of an isosceles trapezoid are congruent.

✓ **Answer Key to Theorem 7.**

Statements: 1. Isosceles Trapezoid AMOR ; 4. MORE is a parallelogram. ; 7. $\overline{AM} \cong \overline{ME}$

Reasons: 2. Definition of Isosceles Trapezoid ; 3. Parallel Postulate ; 5. Parallelogram Property 1 ; 6. Symmetric Property ; 8. Definition of an Isosceles Triangle ; 9. Base angles of an isosceles triangle are congruent. ; 10. Corresponding angles are congruent. ; 11. Substitution ; 12. Symmetric Property ; 13. SSIAS ; 14. Supplements of congruent angles are congruent.

✓ **Answer Key to Theorem 8.**

Statements: 1. Isosceles Trapezoid ARTS ; 4. REST is a parallelogram. ; 6. $\overline{AR} \cong \overline{RE}$

Reasons: 2. Definition of Isosceles Trapezoid ; 3. Parallel Postulate ; 5. Parallelogram Property 1 ; 7. Definition of isosceles triangle ; 8. Isosceles Triangle Theorem ; 9. Interior Angle Sum Theorem on Triangle ; 10. AIAC ; 11. Theorem 7 ; 12. Substitution (SN 9, 10 & 11) ; 13. Angle Addition Postulate ; 14. Substitution ; 15. SSIAS ; 16. Substitution ; 17. Definition of Supplementary Angles

✓ **Answer Key to Theorem 9.**

Statements: 1. Isosceles Trapezoid ROMA ; 5. $\Delta ROM \cong \Delta AMO$

Reasons: 2. Definition of Isosceles Trapezoid ; 3. Theorem 7 ; 4. Reflexive Property ; 6. CPCTC

❖ **Activity 15. You Can Do It!**

✓ **Answer Key**

1. y = 3 by applying Theorem 6 ; MA = 7 and HT = 10
2. m$\angle$TAM = 115 by applying Theorem 7
3. x = 35 by applying Theorem 8 ; m$\angle$MHT = 115 and m$\angle$MAT = 65 through Theorem 8
4. y = 4 by applying Theorem 9 ; Each diagonal measures 13

Discussion.

The teacher shall lead the students in recalling the definition of kite. Let them do Activity 16 that follows and discover more about kites.

❖ **Activity 16. Cute Kite**

This activity must let the students realize that the angles have the same 90^{O} measures, thus, they are all right angles. They must make a conjecture that the diagonals of a kite are perpendicular with each other.

The length of the segments given in the table are the same, thus, a diagonal bisects the other diagonal. Ask the question "Do the diagonals have the same length?" The students must be able to say that one diagonal is longer than the other one. Further ask "Which diagonal bisects the other one?" They must be able to tell the conjecture that in a kite, it is the longer diagonal that bisects the shorter one. Guide them in proving the two theorems related to kites as follows:

<u>Theorem 10</u>. The diagonals of a kite are perpendicular to each other.

Theorem 11. The area of a kite is half the product of the lengths of its diagonals.

- **Answer Key to Theorem 9.**

  Statements: 1. Kite WORD with diagonals $\overline{WR}$ and $\overline{OD}$; 4. $\overline{WR} \perp \overline{OD}$
  Reasons: 2. Definition of kite ; 3. Definition of Congruent segments

- **Answer Key to Theorem 10.**

  Statements: 1. Kite ROPE ; 2. $\overline{PR} \perp \overline{OE}$
  Reasons: 5. Substitution ; 6. Distributive Property of Equality ; 7. SAP ; 8. Substitution

- **Activity 17. Play a Kite**

- **Answer Key**

1. Area = 36 cm^2 by applying Theorem 11
2. PA = 30 cm by applying Theorem 11

- **Answer Key to Quiz 3**

A. 1. x = 10 ; 2. y = 13 ; IJ = 16 ; 3. x = 10 ; 4. y = 11 ; HG = 9
5. x = 14 ; IE = 10

B. 1. $\overline{AD}$ and $\overline{CB}$; 2. $\overline{DC}$ and $\overline{AB}$; 3. $\angle$A and $\angle$B ; $\angle$D and $\angle$C
4. m$\angle$B = 70 ; 5. m$\angle$C = 105 ; 6. x = 44 ; 7. y = 54 ; 8. DB = 56 cm ; 9. AC = 26 ; 10. DB = 22

C. 1. $\overline{LM}$ and $\overline{MN}$; $\overline{LK}$ and $\overline{NK}$; 2. MN = 6 ; 3. KM = 10.5 ; 4. Area = 45.5 cm^2 ; 5. KM = 24 ; 6. m$\angle$3 = 27 ; 7. m$\angle$LMN = 62 ; 8. m$\angle$4 = 68 ; 9. m$\angle$MKN = 19.5 ; 10. m$\angle$KLN = 70

This time, instruct students to revisit ***Check Your Guess 4***. If they answered R, they know already the reason. If they answered W, it means they were wrong as per guess made. Discuss with them their reason/s why they were wrong. In this way, the teacher shall establish continuity of or strengthen the knowledge and skills they have learned.

- **<u>What to understand?</u>**

Tell the students that it's high time to further test their learned knowledge and skills. This is to determine their extent of understanding of the key concepts on quadrilaterals. Solving problems involving parallelograms, trapezoids and kite needs the application of the different properties and theorems discussed in the module.

❖ Activity 18. You Complete Me.

✓ Answer Key

Down: 1 – Parallelogram ; 2 – Rhombus ; 3 – Quadrilateral ; 5 – Parallel ; 8 - Kite
Across: 2 – Rectangle ; 4 – Trapezoid ; 6 – Diagonal ; 7 – Square ; 9 – Angle ; 10 – Vertex

❖ Activity 19. It's Showtime!

✓ Answer Key

Quadrilateral	Specific Kind
ABCD	Rhombus
EFGH	Trapezoid
IJKL	Square
MNOP	Kite
QRST	Rectangle

Notes to the teacher:

1. Vertical lines have undefined slope.
2. Horizontal lines have zero slope. (m = 0)
3. Intersecting vertical and horizontal lines are perpendicular.
4. Parallel lines have equal lopes. ($m_1 = m_2$)
5. Perpendicular lines have slopes whose product is -1. $(m_1)(m_2) = -1$

1. Rectangle QRST	Sides				Diagonals	
	TQ	SR	TS	QR	QS	RT
Slope	– 2	– 2	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{7}$
Both pairs of opposite sides are parallel	$\overline{TQ} \parallel \overline{SR}$		$\overline{TS} \parallel \overline{QR}$			
Four pairs of consecutive sides are perpendicular	$\overline{TQ} \perp \overline{QR}$; $\overline{TQ} \perp \overline{TS}$ $\overline{SR} \perp \overline{TS}$; $\overline{SR} \perp \overline{QR}$					
Diagonals are not perpendicular with each other					The product of their slopes is NOT equal to – 1.	

2. Trapezoid EFGH	Sides			
	EF	HG	EH	FG
Slope	1	1	Undefined	– 1
One pair of opposite sides are parallel	$\overline{EF} \parallel \overline{HG}$			
One pair of opposite sides are not parallel			Their slopes are NOT equal.	

3. Kite MNOP	Sides				Diagonals	
	MN	OP	NO	MP	NP	MO
Slope	1	3	– 1	– 3	Undefined	0
Both pairs of opposite sides are not parallel	Their slopes are NOT equal.		Their slopes are NOT equal.			
Diagonals are perpendicular to each other					$\overline{NP} \perp \overline{MO}$	
4. Rhombus ABCD	Sides				Diagonals	
	AB	DC	AD	BC	BD	AC
Slope	$\frac{1}{4}$	$\frac{1}{4}$	4	4	1	– 1
Both pairs of opposite sides are parallel	$\overline{AB} \parallel \overline{DC}$		$\overline{AD} \parallel \overline{BC}$			
Four pairs of consecutive sides are not perpendicular	NO two consecutive sides have slopes whose product is – 1.					
Diagonals are perpendicular to each other					$\overline{BD} \perp \overline{AC}$	
5. Square IJKL	Sides				Diagonals	
	IJ	KL	JK	IL	IK	JL
Slope	Undefined	undefined	0	0	1	– 1
Both pairs of opposite sides are parallel	$\overline{IJ} \parallel \overline{KL}$		$\overline{JK} \parallel \overline{IL}$			
Four pairs of consecutive sides are perpendicular	$\overline{JI} \perp \overline{IL}$; $\overline{IL} \perp \overline{LK}$ $\overline{LK} \perp \overline{JK}$; $\overline{JK} \perp \overline{JI}$					
Diagonals are perpendicular to each other					$\overline{BD} \perp \overline{AC}$	

❖ **Activity 20. Show More What You've Got!**

✓ **Answer Key**

1. a. m∠W = 25 ; b. HS = 18 ; c. dimensions are 11 cm x 17 cm, area is 187cm^2
   d. area of the largest square = 121 cm^2
2. a. The bases measure 10 and 18. ; b. ∠T = 79^O ; c. Each leg is 7.5 cm long ; d. Perimeter = 35 in
3. a. LE = 7 cm ; b. Area = 30 in^2 ; c. IE = 8 ft and LK = 11 ft

➢ **<u>What to transfer?</u>**

To show evidence of learning on quadrilaterals, let them do Activity 21 and submit their outputs as one of their third quarter projects. Ask the expertise of other teachers (preferably TLE teachers) which shall act as their project advisers. Present the prepared rubric for the performance task. After they've submitted their outputs, ask the questions presented as part of the discussions on quadrilaterals. Let them read and further discuss the summary of the key concepts on quadrilaterals and the glossary of terms.

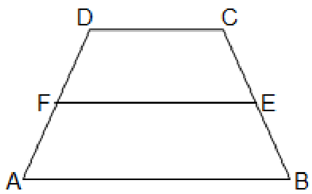
SUMMATIVE TEST

PART I. *Directions*. Choose the letter of the correct answer.

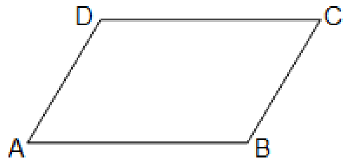
1. Consecutive angles of a parallelogram are
   a. complementary c. adjacent
   b. supplementary d. congruent

2. A quadrilateral with exactly one pair of parallel sides
   a. square c. trapezoid
   b. rectangle d. rhombus

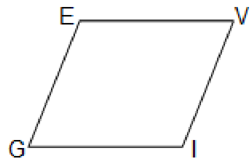
3. In the figure at the right, DC= 20 cm and AB = 36 cm. What is FE?
   a. 16 cm
   b. 56 cm
   c. 28cm
   d. 46 cm



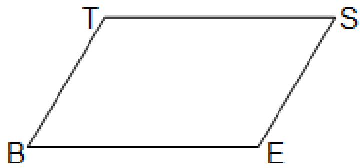
4. The figure below is a parallelogram. If AD = 2x – 10 and BC = x + 30, then BC = ___.
   a. 50
   b. 60
   c. 70
   d. 80



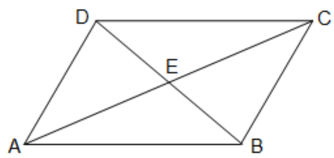
5. The figure below is a rhombus. If m ∠I = (4x)° and ∠E = (2x + 60)°, what is m ∠I?
   a. 100°
   b. 110°
   c. 120°
   d. 130°



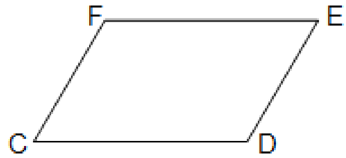
6. Quadrilateral BEST is a parallelogram. If m∠B = (x + 40)° and m ∠E = (2x + 20)°, what is the m ∠B?
   a. 50°
   b. 60°
   c. 70°
   d. 80°



7. The figure below is a parallelogram. The diagonals AC and BD intersect at E. if AE = 2x and EC = 12, what is x?
   a. 5
   b. 6
   c. 7
   d. 8



8. Quadrilateral CDEF is a parallelogram. If m $\angle$C = y° and m $\angle$E = (2y – 40)°, then m $\angle$D is



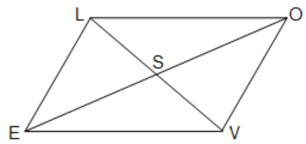
a. 80°
b. 110°
c. 140°
d. 170°

9. How many congruent triangles are formed when a diagonal of parallelogram is drawn?
a. 1 b. 2 c. 3 d. 4

10. Base angles of an isosceles trapezoid are
a. complementary
b. supplementary
c. congruent
d. adjacent

11. All of the following are properties of a parallelogram EXCEPT:
a. diagonals bisect each other.
b. opposite sides are congruent.
c. opposite angles are congruent
d. opposite sides are not parallel

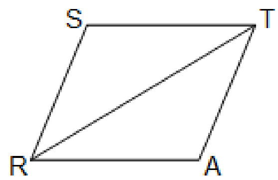
12. If LOVE is a parallelogram and SE = 6, what is SO?



a. 3
b. 6
c. 12
d. 15

13. Which of the following statements ensures that a quadrilateral is a parallelogram?
a. Diagonals bisect each other
b. The two diagonals are congruent
c. Two consecutive sides are congruent.
d. Two consecutive angles are congruent.

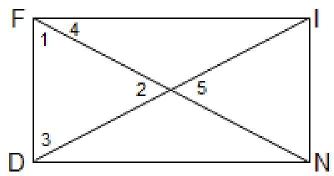
14. STAR is a rhombus with a diagonal $\overline{RT}$. If m$\angle$STR = 3x – 5 and m$\angle$ART = x + 21, what is m$\angle$RAT?



a.13°
b. 34°
c. 68°
d. 112°

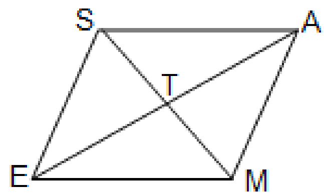
15. The diagonals of a rectangle have lengths 5x – 11 and 2x + 25. Find the lengths of the diagonals.
a. 12 b. 24 c. 49 d. 60

16. Refer to rectangle *FIND*. If m$\angle$4 = 38, what is m$\angle$3?



a. 42
b. 52
c. 62
d. 72

17. Refer to rhombus *SAME*. If AT = 7 cm and TM = 5 cm. Find its area.
    a. 50 cm^2
    b. 60 cm^2
    c. 70 cm^2
    d. 80 cm^2



18. In an isosceles trapezoid, the altitude drawn from an endpoint of the shorter base to the longer base divides the longer base in segments of 5 cm and 10 cm long. Find the lengths of the bases of the trapezoid.
    a. 5 cm and 15 cm c. 15 cm and 25 cm
    b. 10 cm and 20 cm d. 20 cm and 30 cm

19. Find the length of a diagonal of a kite whose area is 176 sq.cm and other diagonal is 16 cm long.
    a. 22 cm b. 24 cm c. 26 cm d. 28 cm

20. One side of a kite is 5 cm less than 7 times the length of another. If the perimeter is 86 cm, find the length of each side of the kite.
    a. 4 cm, 4 cm, 39 cm, 39 cm c. 6 cm, 6 cm, 37 cm, 37 cm
    b. 5 cm, 5 cm, 38 cm, 38 cm d. 7 cm, 7 cm, 36 cm, 36 cm

Part II. *Directions*: Read and understand the situation below then answer or perform what are asked.

Peter, being an SSG President in your school, noticed that there is enough vacant space in the school. He proposed to your school principal to put up rectangular study table for the students to use during vacant time like recess, snacks or even during lunchtime, and as one of his priority projects during his term. To help acquire these rectangular study tables, he solicited construction materials from the alumni such as woods, plywoods, nails, paint materials and many others. After all the materials have been received, the school principal requested your class adviser, Ms. Samonte to divide the class into groups of 5. Each group of students was assigned to do the design of a particular rectangular study table. The designs that the students will prepare shall be used by the carpenter in constructing rectangular study tables.

1. Suppose you are one of the students of Ms. Samonte, how will you prepare the design of the rectangular study table?
2. Make a design of the rectangular study table assigned to your group.
3. Illustrate every part or portion of the rectangular study table including their measures.
4. Using the design of the rectangular study table made, determine all the mathematics concepts or principles involved.

RUBRIC				
Criteria	Poor (1 pt)	Fair (2 pts)	Good (3 pts)	Excellent (4 pts)
Content	Students are able to give 1 or no property of rectangle.	Students are able to give 2 or more properties of rectangle.	Students are able to give at least 3 properties of rectangle.	Students are able to list all properties of rectangle.
Application	No evidence of any strategy is shown or explained.	One or more incorrect approaches attempted or explained.	A complete, appropriate strategy is shown or explained and the solution is shown but not labeled correctly.	A complete, appropriate strategy is shown or explained and the solution is shown and labeled correctly.
Clarity of Presentation	Student work is not legible.	Student work is sloppy and hard to read and very little work is shown.	Student work is mostly legible and clean but with marks or scribbles on work.	Student work is clean, neat, legible, and free of any unnecessary marks.

✓ **Answer Key to Summative Test**

1. b	2. c	3. c	4. c	5. c	6. d	7. b	8. c	9. b	10. c
11. d	12. b	13. a	14. d	15. c	16. b	17. c	18. a	19. a	20. c

Teaching Guide

Module: Similarity

A. Learning Outcomes

All activities and inputs in this module that you have to facilitate are aligned with the content and performance standards of the K to 12 Mathematics Curriculum for Grade 9. Ensuring that students undertake all the activities at the specified time with your maximum technical assistance lies under your care. The table below shows how the standards are unpacked.

Content Standard:

The learner demonstrates understanding of key concepts of similarity.

Performance Standard

The learner is able to investigate, analyze, and solve problems involving similarity through appropriate and accurate representation.

<table>
<tr><td rowspan="2">SUBJECT:
Math 9
QUARTER:
Third Quarter
Topic:
Similarity
WRITER:
MERDEN C. LARGO-BRYANT</td><th colspan="2">LEARNING COMPETENCIES</th></tr>
<tr><td colspan="2">1.describe a proportion
2.illustrate similarity of polygons
3.prove the conditions for
o similarity of triangles
a. AA Similarity Theorem
b. SSS Similarity Theorem
c. SAS Similarity Theorem
d. Triangle Angle Bisector Theorem
e. Triangle Proportionality Theorem
o similarity of right triangles
a.Right Triangle Similarity Theorem
b. Pythagorean Theorem
c. 45-45-90 Right Triangle Theorem
d. 30-60-90 Right Triangle Theorem
4. apply the theorems to show that triangles are similar
5. apply the fundamental theorems of proportionality to solve problems involving proportions
6. solve problems that involve similarity</td></tr>
<tr><td rowspan="2"></td><th>ESSENTIAL UNDERSTANDING</th><th>ESSENTIAL QUESTION</th></tr>
<tr><td>Students will understand that concepts of similarity of objects are useful in solving measurement-related real-life problems.</td><td>How do concepts of similarity of objects help us solve measurement-related real-life problems?</td></tr>
</table>

B. Planning for Assessment

- **Product/Performance**

To assess learning, students should perform a task to demonstrate their understanding of Similarity. It is expected that students, having been equipped with knowledge and skills on inequalities in triangles, would come up with a product—drawing of the floor plan of a house and making a rough estimate of the cost of building it based on the current prices of construction materials. This task is found in Activity No. 27 of the module.

- **Assessment Map**

To ensure understanding and learning, students should be engaged in different learning experiences with corresponding assessment. The table below shows the assessment at different stages of the learning process. Details of this assessment map will guide you which items in each stage of assessment are under specific domains—Knowledge, Process/Skills, Understanding or Performance. Be sure to expose students to varied assessment in this module in order to develop their critical thinking and problem solving skills.

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	• Pre-Test Items No. 1-3	• Pretest Items No. 4-8	• Pretest Items No. 9-14	• Pretest Items No. 15-20
Formative				
Activity No. 2				4 questions
Activity No. 3		Items A, B, C, D Question 1		
Activity No. 4		5 items		
Activity No. 5		4 items		
Activity No. 6	Question No. 11	Question No. 3, 4, 7, 8, 12-16, 21-26	Question No. 1, 2, 5, 6, 9, 10, 17, 18, 19, 20, 27-28	.
Activity No. 7		Question 1	Question 2, 4 Quiz on Proving	Question 3, 5
Activity No. 8		Quiz A	• Proof of AA Similarity Theorem • Quiz B	
Activity No. 9			• Proof of SSS Similarity Theorem • Quiz A & B	
Activity No. 10		Quiz A	• Proof of SAS Similarity Theorem • Quiz B	
Activity No. 11		Quiz A, B	• Proof of SAS Similarity Theorem	

Activity No. 12			Proof of Triangle Proportionality Theorem	
Activity No. 13	Quiz B, D	Quiz A, C, E	Identifying Ratios Quiz F, G	
Activity No. 14		Question 11	Questions 1-10, 12	Question 13-15
Activity No. 15			Questions 1-4	Investigation A&B; 1&2
Activity No. 16	Figure Analysis		Proof of Right Triangle Similarity Theorem	
Activity No. 17		Quiz A: Right Triangles A-D	• Proof of Pythagorean Theorem • Quiz A Question 1-3	Quiz B
Activity No. 18		Activity 18 Quiz	Creating and testing hypothesis; making observations and stating conclusions	
Activity No. 19		Quiz A	Proof of 45-45-90 Right Triangle Theorem	Quiz B
Activity No. 20		Quiz A	• Proof of 30-60-90 Right Triangle Theorem • Quiz B	
Activity No. 21		Question 1	Question 2	Problem
Activity No. 22		Determining coordinates	• Proof • Questions 1-4	
Activity No. 23				Quiz A, B, C
Activity No. 24		• Question 1-3 • Quiz A, B	• Question 4 • Quiz C • Quiz D	• Reading the House Plan in the Activity • Questions 5-9
Activity No. 25		Computations A-G	Questions 1-4	Question 5-11
Activity No. 26			Questions 1-5	Questions 6-8
Activity No. 27				Sketching a Floor Plan and making a rough cost estimate of building it
Activity No. 28			Items 1-10	
Summative	• Post-Test Items No. 1-3	• Post-Test Items No. 4-8	• Post-Test • No. 9-14	• Post-Test Items No. 15-20
Self-Assessment	Items No. 1-5, 7-10	Item No. 6		

Assessment Matrix (Summative Test)

What will I assess?	Post-Test Items by Levels of Assessment			
	Knowledge	Process	Understanding	Product
	3 items	5 items	6 items	6 items
	15%	25%	30%	30%
	Scoring: One point Each			
Competency No. 1: Describe a proportion		4		
Competency No. 2: Illustrate similarity of polygons		5	10	
Competency No. 3: Prove the conditions for similarity of triangles and right triangles	2		13	
	3			
Competency No. 4: Apply the theorems to show that triangles are similar	1		11	
			14	
Competency No. 5: Apply the fundamental theorems of proportionality to solve problems involving proportions			9	16
				18
				19
Competency No. 6: Solve problems that involve similarity		6	12	15
		7		17
		8		20
Competency Nos. 1, 2, 3, and 4	Activity: Sketching a Floor Plan of a House and Cost Estimation Scoring: By Rubrics			

C. Planning for Teaching-Learning

The unit lesson on Geometry for Grade 9 is to be delivered in the Third Quarter of the school year. Similarity is the second chapter of Geometry for Grade 9. You are expected to facilitate this lesson within 25 sessions, non-inclusive of extra time student spend for tasks that you may most likely assign to students to do in their independent/cooperative learning time, free time or after school.

There are several opportunities for applications of learning in the *What to Understand* section of this module. You may decide to focus only on those activities that will support the final transfer task depending on the amount of time available for you.

You are also advised to refer to other references for more opportunities for formative assessments.

- **Introduction**

Before the reading of the introduction, ask the students what they observe from the picture of sailboats found in the introduction of the learning module. They should be able to mention the similarity of the shapes of the sails and the boats.

Arouse the attention and interest of the students by emphasizing the importance of similarity concepts in real life. The introduction, through the essential question, serves as a steering mechanism of the lesson. All sections and activities in the lesson are geared towards the goal of answering it. As the learning facilitator, your role is to emphasize the Essential Question in the introduction and to remind the students about it in every section of the module.

- **Lesson and Coverage**

Remind the students that the concepts and skills they will learn from the learning module.

- **Module Map**

Through the Module Map, you will be able to show to the students that

1. knowledge in proportion is needed in defining and illustrating similarity of polygons;
2. the definition of similarity of polygons is important in illustrating, proving, and verifying the theorems on triangle and right triangle similarity
3. concepts learned on triangle and right triangle similarity help solve problems involving proportion and similarity

- **Pre-Assessment:**

This section features the test that diagnoses what students already know about the topic before the actual teaching of the lesson. This feedback information is valuable to you because it directs you on how to proceed as a facilitator of learning. As a result, you are able to provide the appropriate technical assistance students need as the lesson unfolds.

PRE-ASSESSMENT:

ANSWER KEY							
1.	A	6.	D	11.	A	16.	D
2.	B	7.	B	12.	B	17.	A
3.	D	8.	C	13.	D	18.	C
4.	B	9.	C	14.	C	19.	D
5.	A	10.	B	15.	B	20.	D

WHAT TO KNOW

Be reminded that activities in the what-to-know section are designed to reveal the students' background knowledge on similarity.

Activity No. 1

My Decisions Now and Then Later

Activity No. 1 is an anticipation-reaction guide that helps students assess their prior knowledge on similarity. As the succeeding activities are tackled, they may change their responses. Hence, the checking of their responses occurs near the end of the learning module.

1	2	3	4	5	6	7	8	9	10
A	D	A	D	A	A	D	A	A	A

Activity No. 2

The Strategy: Similarity!

Find out what students know about grid drawing, indirect measurement; determining distances using maps; and understanding a house plan. From their answers, you would know what students already know that can help you how to proceed. You also have to use this activity in increasing the students' interest in the topic because of its practical usefulness, especially in solving measurement-related problems.

After the sharing of knowledge on Activity 2, inform the students that the lesson will enable them to do the final project that requires them to draw the floor plan of a house and make a rough estimate of the cost of building it based on the current prices of construction materials. Your output and its justification will be rated according to these rubrics: accuracy, creativity, resourcefulness, and mathematical justification.

WHAT TO PROCESS

In this section, the students will use the concepts and skills they have learned in the previous grades in ratio and proportion and deductive proof. They will also be amazed with the connections between algebra and geometry as they illustrate or prove the conditions of principles involving similarity of figures, especially triangle similarity. They will also realize that their success in writing proofs involving similarity depends upon their skill in making accurate and appropriate representation of mathematical conditions. In short, this section offers an exciting adventure in developing their logical thinking and reasoning— 21st century skills that will prepare them to face challenges in future endeavors in higher education, entrepreneurship or employment. As the teacher, it is your role to guide the students as they perform the activities. It is within your decision to have the whole class, groups of students or individual students conduct the activities. It is generally suggested that activities be done in groups.

Activity No. 3

Let's be fair—proportion please!

Give students time to study the examples in this activity before letting them cite more proportions. Some answers are given in the table.

Direct students to verify the accuracy of determined proportions by checking the equality of the ratios or rates. Let them study the example given. Remind them that the objective is to show that the ratios or rates are equivalent. Hence, solutions need not be in the simplest form. There are several possible answers. One possible answer for each item is shown on the table below:

Fig.	Proportional Quantities	Checking the equality of ratios or rates in the cited proportions
A	$1\,ft : 12\,in. = 2ft : 24\,in.$	$\frac{1}{12} ? \frac{2}{24} \to \frac{1}{12} = \frac{1}{12}$
B	Shorter Segment **:** Thicker Segment	$\frac{4}{8} ? \frac{2}{4} \to \frac{1}{2} = \frac{1}{2}$
C	$4\,min : 80\,m = 5\,min : 100\,m$	$\frac{4}{80} ? \frac{5}{100} \to \frac{1}{20} = \frac{1}{20}$
D	$3 : 240\,pesos = 5 : 400\,pesos$	$\frac{3}{240} ? \frac{5}{400} \to \frac{1}{80} = \frac{1}{80}$

Questions:

Definition of Proportion: ***Proportion is the <u>equality</u> of two ratios.***

Activity No. 4

Certainly, the ratios are equal!

Discuss briefly but clearly the fundamental rule of proportion and its properties.

Give technical assistance to groups of students as they rewrite the given proportions and as they find out whether rewritten proportions are still equal.

Property of Proportion	Original Proportion	Using cross-multiplication property to find out if ratios equal?
	$\frac{y}{3} = \frac{a}{4}$	$4y = 3a$
Alternation Property	$\frac{y}{a} = \frac{3}{4}$	$4y = 3a$
Inverse Property	$\frac{3}{y} = \frac{4}{a}$	$3a = 4y \rightarrow 4y = 3a$
Addition Property	$\frac{y+3}{3} = \frac{a+4}{4}$	$4(y+3) = 3(a+4)$ $4y + 12 = 3a + 12$ $4y = 3a$

Subtraction Property	$\frac{y-3}{3}=\frac{a-4}{4}$	$4(y-3)=3(a-4)$ $4y-12 = 3a-12$ $4y=3a$
Sum Property of the Original Proportion	$\frac{y}{3}+\frac{a}{4}=\frac{y+a}{7}$	$For\ \frac{y}{3}=\frac{y+a}{7}$: $7y\ ?\ 3(y+a)$ $7y\ ?\ 3y+3a$ $4y=3a$ $For\ \frac{a}{4}=\frac{y+a}{7}$: $7a\ ?\ 4(y+a)$ $7a\ ?\ 4y+4a$ $3a=4y$ $4y=3a$

Activity No. 5

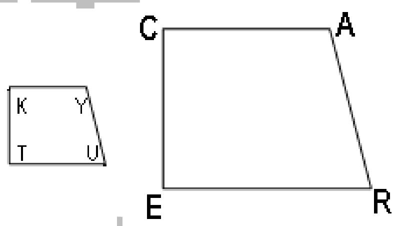
Solving Problems Involving Proportion

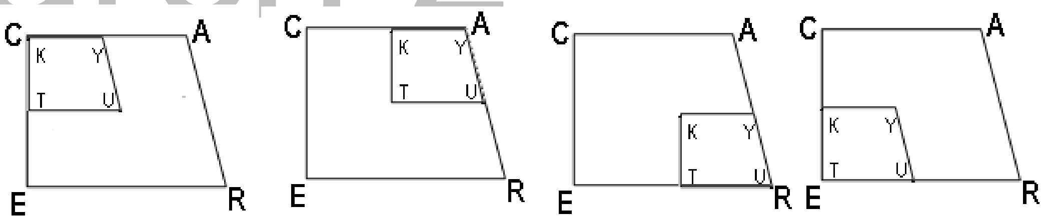
Spearhead the discussion of the examples on how to determine ratios in a proportion and guide the students as they solve related exercises.

Activity No. 6

How are polygons similar?

Each side of Trapezoid KYUT is ***k*** times the corresponding side of Trapezoid CARE. These trapezoids are similar. In symbols, KYUT ~ CARE. One corresponding pair of vertices is paired in each of the figures that follow. Study their shapes, their sizes, and their corresponding angles and sides carefully.





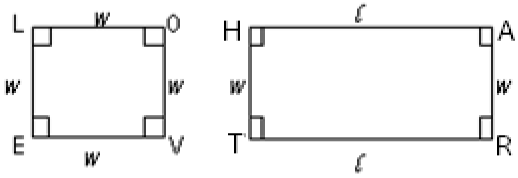
Questions:

1. What do you observe about the shapes of polygons CARE and KYUT?
   <u>Polygons CARE and KYUT have the same shape.</u>
2. What do you observe about their sizes?
   <u>Polygons CARE and KYUT have the different sizes—KYUT is smaller and CARE is larger.</u>

Aside from having the same shape, what makes them similar? Let us answer this question after studying their corresponding sides and angles. Let us first study non-similar

Parallelograms LOVE and HART and Parallelograms YRIC and DENZ before carefully studying the characteristics of Polygons CARE and KYUT.

Let us consider Parallelograms LOVE and HART.



Observe the corresponding angles and corresponding sides of Parallelograms LOVE and HART by taking careful note of their measurements. Write your observations on the given table. Observe the corresponding angles and corresponding sides of Parallelograms LOVE and HART by taking careful note of their measurements. Write your observations on the given table. Two observations are done for you.

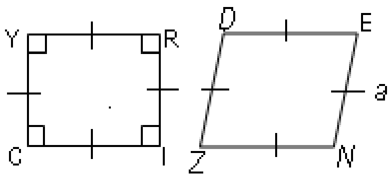
Corresponding Angles	Ratio of Corresponding Sides	Simplified Ratio/s of the sides
$m\angle L = m\angle H = 90$	$\frac{LO}{HA} = \frac{w}{l}$	$\frac{w}{l}$ & 1
$m\angle O = m\angle A = 90$	$\frac{OV}{AR} = \frac{w}{w} = 1$	
$m\angle V = m\angle R = 90$	$\frac{VE}{RT} = \frac{w}{l}$	
$m\angle E = m\angle T = 90$	$\frac{EL}{TH} = \frac{w}{w} = 1$	

3. Are the corresponding angles of Parallelograms LOVE and HART congruent? YES
4. Do they have a common ratio of sides? NO
5. Do Parallelograms LOVE and HART have uniform proportionality of sides? NO

Note: Parallelograms LOVE and HART are not similar.

6. What do you think makes them not similar? Answer this question later.

This time, we consider polygons YRIC and DENZ.



Observe the corresponding angles and corresponding sides of Parallelograms YRIC and DENZ, taking careful note of their measurements. Write your observations using the given table. The first observation is done for you.

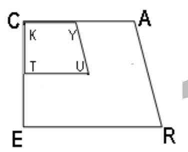
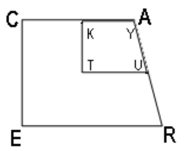
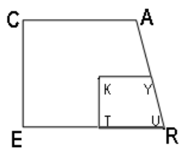
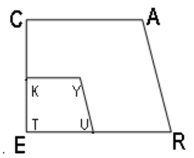
Corresponding Angles	Ratio of Corresponding Sides	Simplified Ratio
$m\angle Y \neq m\angle D$	$\frac{YR}{DE} = \frac{a}{a}$	1

$m\angle R \neq m\angle E$	$\frac{RI}{EN} = \frac{a}{a}$	1
$m\angle I \neq m\angle N$	$\frac{IC}{NZ} = \frac{a}{a}$	1
$m\angle C \neq m\angle Z$	$\frac{CY}{ZD} = \frac{a}{a}$	1

7. Are the corresponding angles congruent? NO
8. Do Parallelograms YRIC and DENZ have uniform proportionality of sides? YES

   Note: YRIC and DENZ are not similar.
9. What do you think makes them not similar? Answer this question later.

10. Now consider again the similar polygons KYUT and CARE ($KYUT \sim CARE$). Notice that by pairing their corresponding vertices, corresponding angles coincide perfectly. It can be observed also that corresponding angles are congruent. In the following table, write your observations about the corresponding overlapping sides as each pair of corresponding vertices is made to coincide with each other.

Ratios of the corresponding sides that overlap			How do you express the proportionality of the overlapping sides using their ratios?	Corresponding Angles
	$KT:CE$	$KY:CA$	$KT:CE = KY:CA = k:1$	$\angle K \cong \angle C$
	$\frac{KT}{CE}$	$\frac{KY}{CA}$	$\frac{KT}{CE} = \frac{KY}{CA} = k$	
	$KY:CA$	$YU:AR$	$KY:CA = YU:AR = k:1$	$\angle Y \cong \angle A$
	$\frac{KY}{CA}$	$\frac{YU}{AR}$	$\frac{KY}{CA} = \frac{YU}{AR} = k$	
	$YU:AR$	$UT:RE$	$YU:AR = UT:RE = k:1$	$\angle U \cong \angle R$
	$\frac{YU}{AR}$	$\frac{UT}{RE}$	$\frac{YU}{AR} = \frac{UT}{RE} = k$	
	$UT:RE$	$KT:CE$	$UT:RE = KT:CE = k:1$	$\angle T \cong \angle E$
	$\frac{UT}{RE}$	$\frac{KT}{CE}$	$\frac{UT}{RE} = \frac{KT}{CE} = k$	

11. Observe that adjacent sides overlap when a vertex of KYUT is paired with a vertex of CARE. It means that for CARE and KYUT that are paired at a vertex, corresponding angles are congruent. Moreover, the ratios of corresponding sides are equal. Hence, the corresponding sides are proportional.

Big question: Do KYUT and CARE have uniform proportionality of sides like YRIC and DENZ? Let us study carefully the proportionality of the corresponding adjacent sides that overlap.

When the following vertices are paired:			
$K \& C$	$Y \& A$	$U \& R$	$T \& E$
$\frac{KT}{CE} = \frac{KY}{CA}$	$\frac{KY}{CA} = \frac{YU}{AR}$	$\frac{YU}{AR} = \frac{UT}{RE}$	$\frac{UT}{RE} = \frac{KT}{CE}$

12. Notice that $\frac{KY}{CA}$ is found in the pairing of vertices K & C and Y & A. It means that

$$\frac{KT}{CE} = \frac{KY}{CA} = \frac{YU}{AR}.$$

13. Observe that $\frac{YU}{AR}$ is found in the pairing of vertices Y & A and U & R. It means that

$$\frac{KY}{CA} = \frac{YU}{AR} = \frac{UT}{RE}.$$

14. Still we can see that $\frac{UT}{RE}$ is found in the pairing of vertices U & R and T & E. It means that

$$\frac{YU}{AR} = \frac{UT}{RE} = \frac{KT}{CE}.$$

15. Notice also that $\frac{KT}{CE}$ is found in the pairing of vertices T & E and K&C. It means that

$$\frac{UT}{RE} = \frac{KT}{CE} = \frac{KY}{CA}.$$

16. Therefore, we can write the proportionality of sides into

$$\frac{KT}{CE} = \frac{KY}{CA} = \frac{YU}{AR} = \frac{UT}{RE} = \frac{KT}{CE}$$

17. If $\frac{KT}{CE} = \frac{KY}{CA} = k$, can we say that the ratios of the other corresponding adjacent sides are also equal to k? Explain your answer.

Yes because all the ratios of the sides of KYUT and CARE are equal.

Since the ratios of all the corresponding sides of Similar Trapezoids CARE and KYUT are equal, it means that they have ***uniform proportionality of sides***. That is, all the corresponding sides are proportional to each other.

The number k that describes the ratio of two corresponding sides of similar polygons such as Trapezoids CARE and KYUT is referred as ***scale factor.*** This scale factor is true to all the rest of the corresponding sides of similar polygons because of the uniformity of the proportionality of their sides.

18. Express the uniform proportionality of sides of Similar Trapezoids CARE and KYUT in one mathematical sentence using the scale factor ***k***?

$$\frac{KT}{CE} = \frac{KY}{CA} = \frac{YU}{AR} = \frac{UT}{RE} = \frac{KT}{CE} = k$$

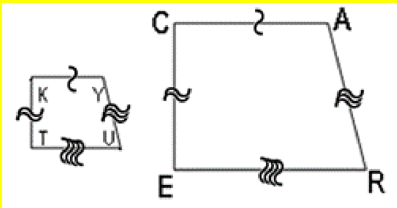
19. The conditions observed in similar trapezoids CARE and KYUT help us point out the characteristics of similar polygons.

How are polygons similar?

Two polygons are similar if:

2. their corresponding angles are congruent.
3. their corresponding sides are proportional.

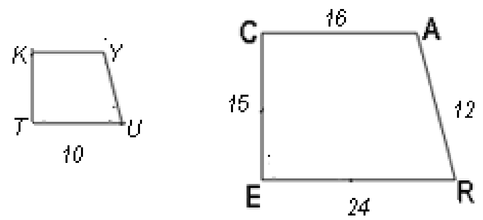
Curved marks can be used to indicate proportionality of corresponding sides of figures such as shown in parallelograms KYUT and CARE below:



20. Now that you know what makes polygons similar, answer the following questions:

Why are Parallelograms LOVE and HART not similar?	Corresponding sides of Parallelograms LOVE and HART are not proportional.
Why are Parallelograms YRIC and DENZ not similar?	Corresponding angles of Parallelograms YRIC and DENZ are not congruent.

KYUT ~ CARE. Given the lengths of their sides in the figure, and their proportional sides on the table, answer the following questions:



Proportional Sides	
$\frac{KT}{CE} = \frac{KY}{CA}$	$\frac{KT}{15} = \frac{KY}{16}$
$\frac{KT}{CE} = \frac{TU}{ER}$	$\frac{KT}{15} = \frac{10}{24}$
$\frac{TU}{ER} = \frac{UY}{RA}$	$\frac{10}{24} = \frac{UY}{12}$
$\frac{UY}{RA} = \frac{KY}{CA}$	$\frac{UY}{12} = \frac{KY}{16}$

21. The scale factor of similar figures can be determined by getting the ratio of corresponding sides with given lengths. Which of the ratios of corresponding sides give the scale factor k? $\frac{TU}{ER}$

22. What is the ratio of these corresponding sides? $\frac{10}{24}$

23. What is the simplified form of scale factor k? $\frac{5}{12}$

24. Solve for KT by equating the ratio of corresponding sides containing KT with the scale factor k?

$$\frac{KT}{15} = \frac{5}{12} \rightarrow 12(KT) = 5(15) \rightarrow \boldsymbol{KT} = \frac{5(15)}{12} = \frac{5(5)(3)}{4(3)} = \frac{\mathbf{25}}{\mathbf{4}}$$

25. Solve for KY by equating the ratio of corresponding sides containing KY with the scale factor k?

$$\frac{KY}{16} = \frac{5}{12} \rightarrow 12(KY) = 5(16) \rightarrow \boldsymbol{KY} = \frac{5(16)}{12} = \frac{5(4)(4)}{4(3)} = \frac{\mathbf{20}}{\mathbf{3}}$$

26. Solve for UY by equating the ratio of corresponding sides containing UY with the scale factor k?

$$\frac{UY}{12} = \frac{5}{12} \rightarrow 12(UY) = 5(12) \rightarrow \boldsymbol{UY} = \frac{5(12)}{12} \rightarrow \boldsymbol{UY} = 5$$

27. Polygons CARE and KYUT, although having the same shape, differ in sizes. Hence, they are not congruent, only similar. Let us remember: What are the two characteristics of similar polygons?

(1) <u>Corresponding angles of similar polygons are congruent</u>

(2) <u>Corresponding sides of similar polygons are proportional</u>

28. What can you say about the two statements that follows:

I. All congruent figures are similar. II. All similar figures are congruent.

A. Both are true. C. Only II is true.

B. Only I is true. D. Neither one is true.

Activity No. 7

Self-Similarity

Questions:

1. There are 11 self-similar hexagons in the figure.
2.

Step 1	Construct a regular hexagon with a side congruent to the side of the outer hexagon of the given figure.
Step 2	Mark the midpoints of the sides of the regular hexagon.
Step 3	Connect all the adjacent midpoints with a segment to form a new regular hexagon.
Step 4	Mark the midpoints of the sides of the second regular hexagon.
Step 5	Connect all the adjacent midpoints with a segment to form another new regular hexagon.
Step 6	Mark the midpoints of the sides of the third hexagon.

3. Product is the same as the original figure.
4. All other regular polygons
5. ***First Sierpinski Triangle***

   Step 1: Mark all the midpoints of each side of the original triangle which is the largest.

   Step 2: Connect all the midpoints to form new smaller triangles.

   Step 3. Repeat steps 1 and 2

   Second Sierpinski Triangle

   Step 1: Mark three equal magnitudes on each side of the original triangle.

   Step 2: Connect all corresponding points of pairs of adjacent sides.

   Step 3. Repeat steps 1 and 2

   - Are the triangles of each of the Sierpinski Triangles similar? Explain.

     Yes because a common scale factor is used to form the next self-similar triangle from the previous one.
   - What is the scale factor used to reduce each triangle of the Sierpinski triangle to the next one in size? Explain.

     First Sierpinski Triangle: ½ because the next triangle is formed by determining the midpoint of the considered triangle

     Second Sierpinski Triangle: $\frac{1}{3}$ because the next triangle is formed by dividing each side of the considered triangle into three equal parts
   - Possible insights from a research on Seirpinski Triangles:

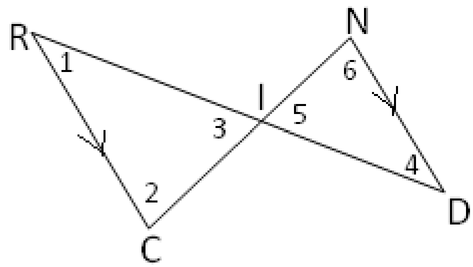
     1. Sierpinski Triangle is a fractal.
     2. Sierpinski Triangles can be generated randomly.

Triangle Similarity

AAA Similarity Postulate
If the three angles of one triangle are congruent to three angles of another triangle, then the two triangles are similar.

Quiz on AA Similarity Postulate

Given the figure, prove that $\Delta RIC \sim \Delta DIN$



Hints:		Statements	Reasons
1	Based on their markings, describe $\overline{RC}$ and $\overline{DN}$?	$\overline{RC} \parallel \overline{DN}$	Given
2	Based on statement 1, describe alternate interior angles if $\overline{CN}$ and $\overline{RD}$ are transversals	$\angle 2 \cong \angle 6$	Alternate interior angles are congruent
		$\angle 1 \cong \angle 4$	
3	Describe the vertical angles	$\angle 3 \cong \angle 5$	Vertical angles are congruent
4	Conclude using statements 1, 2, & 3	$\Delta RIC \sim \Delta DIN$	AAA Similarity Postulate

Activity No.8

AA Similarity Theorem and its Proof

AA Similarity Theorem
Two triangles are similar if two angles of one triangle are congruent to two angles of another triangle.

Given: $\angle U \cong \angle H;\ \angle V \cong \angle Y$

Prove: $\Delta LUV \sim \Delta WHY$

Proof:

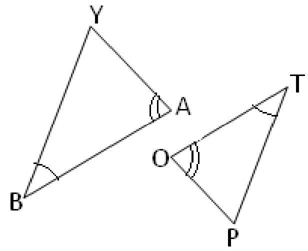
Hints		Statements	Reasons
1	Write all the given	$\angle U \cong \angle H;\ \angle V \cong \angle Y$	Given

2	Describe the measure of the congruent angles in Statement 1	$m\angle U \cong m\angle H;\ m\angle V \cong m\angle Y$	Definition of congruent angles
3	Add $m\angle V$ to both sides of $m\angle U = m\angle H$ in statement 2	$m\angle U + m\angle V \cong m\angle H + m\angle V$	Addition property of equality
4	Substitute $m\angle V$ on the right side of statement 3 using statement 2	$m\angle U + m\angle V \cong m\angle H + m\angle Y$	Substitution Property of Equality
5	Add the measures of all the angles of a triangles LUV and WHY	$m\angle U + m\angle V + m\angle L = 180;$ $m\angle H + m\angle Y + m\angle W = 180$	The sum of the measures of the three angles in a triangle is 180.
6	Equate the measures of the angles of triangles LUV and WHY from statement 5	$m\angle U + m\angle V + m\angle L = m\angle H + m\angle Y + m\angle W$	Transitive Property of Equality
7	Substitute $m\angle H$ on the right side of statement 6 using statement 2	$m\angle U + m\angle V + m\angle L = m\angle U + m\angle V + m\angle W$	Substitution Property of Equality
8	Simplify Statement 7	$m\angle L = m\angle W$	Subtraction Property of Equality
9	Are triangles LUV and WHY similar? Reason should be based from statements 2 and 8	$\Delta LUV \sim \Delta WHY$	AAA Similarity Postulate

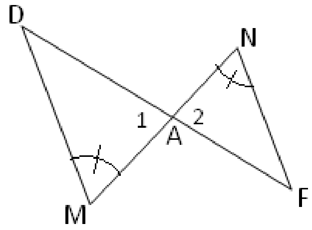
Quiz on AA Similarity Theorem

A.

If :	$\angle E \cong \angle R;\ \angle H \cong \angle P$
Then:	$\Delta HEY \sim \Delta PRO$



B. Prove that $\Delta DAM \sim \Delta FAN$.



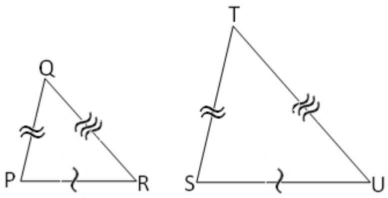
Hints:		Statements	Reasons
1	Congruent angles with markings	$\angle M \cong \angle N$	Given
2	Congruent angles because they are vertical	$\angle 1 \cong \angle 2$	Vertical angles are congruent
3	Conclusion based on statement 1 and 2	$\Delta DAM \sim \Delta FAN$	AA Similarity Theorem

Activity No.9

SSS Similarity Theorem and its Proof

SSS Similarity Theorem
Two triangles are similar if the corresponding sides of two triangles are in proportion.

Proof:

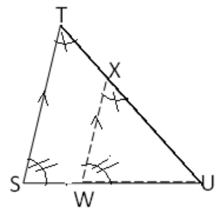


Given: $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$

Prove: $\Delta PQR \sim \Delta STU$

Proof:

- Construct X on $\overline{TU}$ such that $\overline{XU} \cong \overline{QR}$.
- From X, construct $\overline{XW}$ parallel to $\overline{TS}$ intersecting $\overline{SU}$ at W.



Hints:		Statements :	Reasons:
1	Which sides are parallel by construction?	$\overline{XW} \parallel \overline{ST}$	By construction
2	Describe angles WXU & STU and XWU & TSU based on statement 1	$\angle WXU \cong \angle STU$; $\angle XWU \cong \angle TSU$	Corresponding angles are congruent
3	Are WXU and STU similar?	$\Delta WXU \sim \Delta STU$	AA Similarity Theorem
4	Write the equal ratios of similar triangles in statement 3	$\frac{WX}{ST} = \frac{XU}{TU} = \frac{WU}{SU}$	Definition of Similar polygons
5	Write the given	$\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$	Given
6	Write the congruent sides that resulted from construction	$\overline{XU} \cong \overline{QR}$	By construction

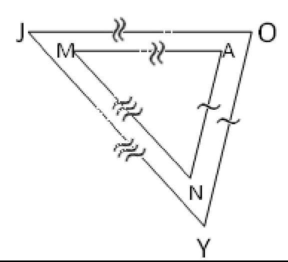
7	Use statement 6 in statement 5	$\frac{PQ}{ST}=\frac{XU}{TU}=\frac{PR}{SU}$	Substitution Property of Equality
8	If $\frac{PQ}{ST}=\frac{XU}{TU}$ (statement 7) and $\frac{WX}{ST}=\frac{XU}{TU}$ (statement 4), then	$\frac{PQ}{ST}=\frac{WX}{ST};\ \frac{PR}{SU}=\frac{WU}{SU}$	Transitive Property of Equality
9	If $\frac{XU}{TU}=\frac{PR}{SU}$ (statement 7) and $\frac{XU}{TU}=\frac{WU}{SU}$ (statement 4), then	$PQ=WX;\ PR=WU$	Multiplication Property of Equality
10	Are triangles PQR and WXU congruent? Base your answer from statements 9 and 6	$\Delta PQR\cong\Delta WXU$	SSS Triangle Congruence Postulate
11	Use statement 10 to describe angles WUX and SUT	$\angle WUX\cong\angle SUT$	Definition of congruent triangles
12	Substitute the denominators of statement 4 using the equivalents in statements 9 and 6, then simplify	$\frac{WX}{WX}=\frac{XU}{XU}=\frac{WU}{WU}=1$	Substitution Property of Equality
13	Using statements 2, 11, and 12, what can you say about triangles PQR and WXU?	$\Delta PQR\sim\Delta WXU$	Definition of Similar polygons
14	Write a conclusion using statements 13 and 3	$\Delta PQR\sim\Delta STU$	Transitivity

Notice that we have also proven that ***congruent triangles are similar***, where the uniform proportionality of sides is equal to one (1).

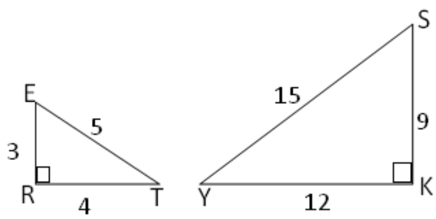
Quiz on SSS Similarity Theorem

A. Use the SSS Similarity Theorem in writing an if-then statement to describe an illustration or completing a figure based on an if-then statement.

If :	$\frac{OJ}{AL}=\frac{JY}{LF}=\frac{OY}{AF}$
Then	$\Delta JOY\sim\Delta LAF$



B. Given the figure, prove that $\Delta ERT\sim\Delta SKY$.



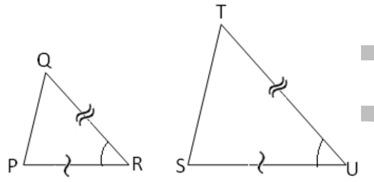
Hints:		Statements	Reasons
1	Do all their corresponding sides have uniform proportionality? Verify by substituting the lengths of sides. Simplify afterwards.	$\frac{ER}{SK} \overset{?}{=} \frac{RT}{KY} \overset{?}{=} \frac{ET}{SY}$ $\rightarrow \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{\mathbf{1}}{\mathbf{3}}$	By computation
2	Conclusion based on the simplified ratios	$\Delta ERT \sim \Delta SKY$	SSS Similarity Postulate

Activity No. 10

SAS Similarity Theorem and its Proof

SAS Similarity Theorem
Two triangles are similar if an angle of one triangle is congruent to an angle of another triangle and the corresponding sides including those angles are in proportion.

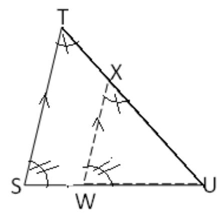
Proof:



Given:
$\frac{QR}{TU} = \frac{PR}{SU}$; $\angle R \cong \angle U$

Prove:
$\Delta PQR \sim \Delta STU$

Proof:
- Construct X on $\overline{TU}$ such that $\overline{XU} = \overline{QR}$.
- From X, construct $\overline{XW}$ parallel to $\overline{TS}$ intersecting $\overline{SU}$ at W



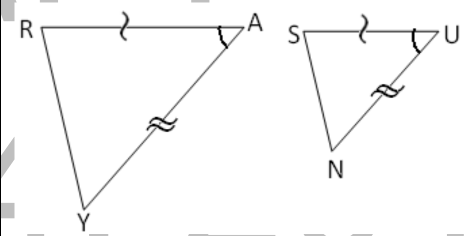
No.	Hints	Statements	Reasons
1	Which sides are parallel by construction?	$\overline{XW} \parallel \overline{ST}$	By construction
2	Describe angles WXU & STU and XWU & TSU based on statement 1	$\angle WXU \cong \angle STU$; $\angle XWU \cong \angle TSU$	Corresponding angles are congruent
3	Are WXU and STU similar?	$\Delta WXU \sim \Delta STU$	AA Similarity Theorem
4	Write the equal ratios of similar triangles in statement 3	$\frac{WX}{ST} = \frac{XU}{TU} = \frac{WU}{SU}$	Definition of Similar polygons
5	Write the congruent sides that resulted from construction	$\overline{XU} = \overline{QR}$	By construction
6	Write the given related to corresponding sides	$\frac{QR}{TU} = \frac{PR}{SU}$	Given
7	Use statement 5 in statement 6	$\frac{\boldsymbol{XU}}{TU} = \frac{PR}{SU}$	Substitution Property of Equality

8	If $\frac{XU}{TU} = \frac{PR}{SU}$ (statement 7) and $\frac{XU}{TU} = \frac{WU}{SU}$ (statement 4), then	$\frac{PR}{SU} = \frac{WU}{SU}$	Transitive Property of Equality
	If $\frac{XU}{TU} = \frac{PR}{SU}$ (statement 7) and $\frac{QR}{TU} = \frac{PR}{SU}$ (statement 6), then	$\frac{QR}{TU} = \frac{XU}{TU}$	
9	Multiply the proportions in statement 8 by their common denominators and simplify	$\overline{PR} = \overline{WU}$ $\overline{QR} = \overline{XU}$	Multiplication Property of Equality
10	Write the given related to corresponding angles	$\angle R \cong \angle U$	Given
11	What can you say about triangles PQR and WXU based on statements 9 &10	$\Delta PQR \cong \Delta WXU$	SAS Triangle Congruence Postulate
12	Write a statement when the reason is the one shown	$\Delta PQR \sim \Delta WXU$	Congruent triangles are similar
13	Write a conclusion using Statements 12 & 3	$\Delta PQR \sim \Delta STU$	Substitution Property

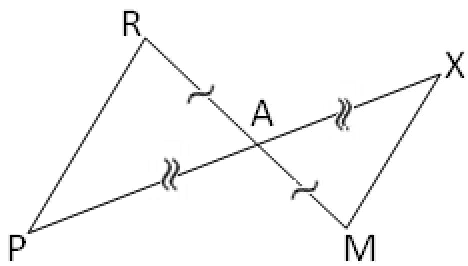
Quiz No.

A.

If :	$\angle A \cong \angle O$; $\frac{AL}{OJ} = \frac{AF}{OY}$
Then	$\Delta LAF \sim \Delta JOY$



B. Given the figure, prove that $\Delta RAP \sim \Delta MAX$.



Hints:		Statements	Reasons
1	Write in a proportion the ratios of two corresponding proportional sides	$\frac{AR}{AM} = \frac{AP}{AX}$	Given

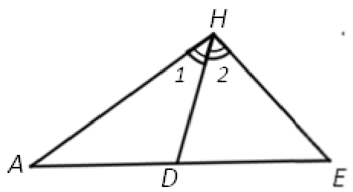
2	Describe included angles of the proportional sides	$\angle RAP \cong \angle MAX$	Vertical angles are congruent.
3.	Conclusion based on the simplified ratios	$\Delta RAP \sim \Delta MAX$	SAS Similarity Theorem

Activity No.11

Triangle Angle Bisector Theorem (TABT) and its Proof

Triangle Angle-Bisector Theorem
If a segment bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

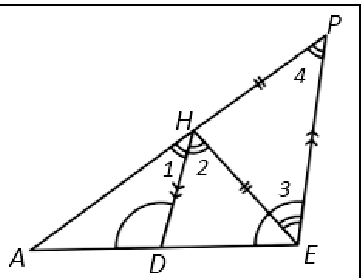
Proof:



Given:
- ΔAHE
- $\overline{HD}$ bisects AHE

Prove:
$$\frac{DA}{DE} = \frac{AH}{EH}$$

Proof:
Extend $\overline{AH}$ to P such that $\overline{EP} \parallel \overline{HD}$.



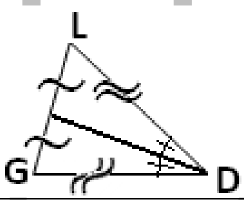
No.	Hints	Statements	Reasons
1	List down the given	$\overline{HD}$ bisects ΔAHE	Given
2	What happens to the bisected $\angle AHE$?	$\angle 1 \cong \angle 2$	Definition of angle bisector
3	What do you say about $\overline{HD}$ and $\overline{EP}$?	$\overline{HD} \parallel \overline{EP}$	By Construction
4	What can you conclude about $\angle ADH$ & $\angle DEP$ and $\angle 1$ & $\angle 4$?	$\angle ADH \cong \angle DEP$ $\angle 1 \cong \angle 4$	Corresponding angles are congruent
5	What can you conclude about $\angle 2$ & $\angle 3$?	$\angle 2 \cong \angle 3$	Alternate interior angles are congruent
6	What can you say about $\angle 3$ & $\angle 4$ based on statements 2, 4, & 5?	$\angle 3 \cong \angle 4$	Transitive Property
7	What can you say about ΔHEP based on statement 6?	ΔHEP is isosceles.	Base angles of Isosceles triangles are congruent.
8	What can you say about the sides opposite $\angle 4$ & $\angle 3$?	$\overline{EH} \cong \overline{HP}$	Definition of isosceles triangles
9	What can you say about ΔAHD & ΔAPE using statement 4?	$\Delta AHD \sim \Delta APE$	AA Similarity Theorem
10	Using statement 3, write the proportional lengths of ΔAPE using sides AP and AE	$\frac{AH}{AP} = \frac{AD}{AE}$	Definition of Similar Polygons

11	Use Segment Addition Postulate for AP and AE.	$\frac{AH}{AH+AP} = \frac{AD}{AD+DE}$	Segment Addition Postulate
12	Use Inversion Property of Proportion to statement 11	$\frac{AH+HP}{AH} = \frac{AD+DE}{AD}$	Inversion Property of Proportion
13	Decompose the fractions	$\frac{AH}{AH}+\frac{HP}{AH} = \frac{AD}{AD}+\frac{DE}{AD}$ $1+\frac{HP}{AH} = 1+\frac{DE}{AD}$	Principles in the operations of Fractions
14	Simplify Statement 13	$\frac{HP}{AH} = \frac{DE}{AD}$	Subtraction Property of Equality
15	Use statement 8 in statement 14	$\frac{EH}{AH} = \frac{DE}{AD}$	Substitution
16	Use symmetric Property in statement 15	$\frac{DE}{AD} = \frac{EH}{AH}$	Symmetric Property of Equality
17	Use Inversion Property in statement 16	$\frac{AD}{DE} = \frac{AH}{EH}$	Inversion Property of Proportion

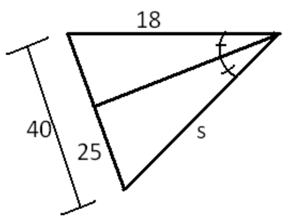
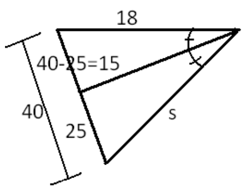
Quiz on Triangle Angle Bisector Theorem (TABT)

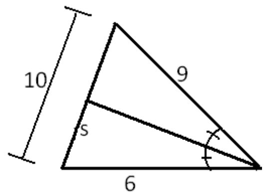
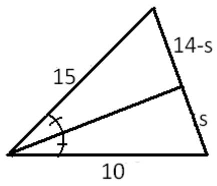
A. Use the TABT in writing an if-then statement to describe the illustration or complete the figure based on the given if-then statement.

If :	$\overline{HP}$ bisects $\angle H$
Then:	$\frac{PA}{PE} = \frac{AH}{EH}$



B. Solve for the unknown side s by applying the Triangle Angle-Bisector Theorem. The first one is done for you. Note that the figures are not drawn to scale.

Figures	Solutions
1. 	 $\frac{25}{15} = \frac{s}{18} \rightarrow 15s = 25(18)$ $s = \frac{25(18)}{15}$ $s = \frac{[(5)(5)][(2)(3)(3)]}{(3)(5)}$ $s = (5)(2)(3) = 30$

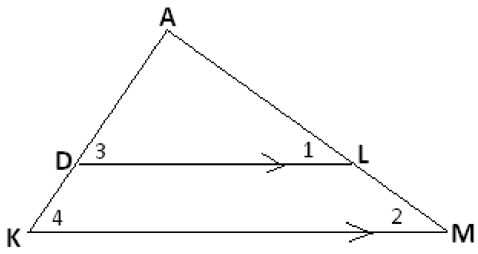
2. 	$\begin{gathered}\frac{s}{10-s}=\frac{6}{9}\\ 9s=60-6s\\ 9s+6s=60\\ 15s=60\\ s=4\end{gathered}$
3. 	$\begin{gathered}\frac{14-s}{s}=\frac{15}{10}\\ 140-10s=15s\\ 140=15s+10s\\ 140=25s\\ \frac{140}{25}=s\\ s=\frac{28}{5}=5\ \frac{3}{5}\end{gathered}$

Activity No.12

Triangle Proportionality Theorem (TPT) and its Proof

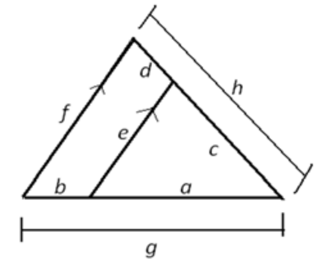
Triangle Proportionality Theorem
If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

Proof:

	**Given:** $\overline{DL} \parallel \overline{KM}$ **Prove:** $\frac{AD}{AK}=\frac{AL}{AM}$
Proof:	
Statements :	**Reasons:**
1. $\overline{DL} \parallel \overline{KM}$	1. Given
2. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	2. Corresponding angles are congruent.
3. $\Delta DAL \sim \Delta KAM$	3. AA Similarity Theorem
4. $\frac{AD}{AK}=\frac{AL}{AM}$	4. Definition of Similar Polygons

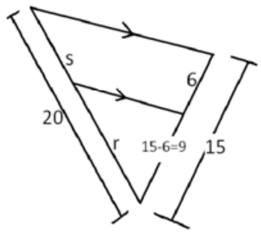
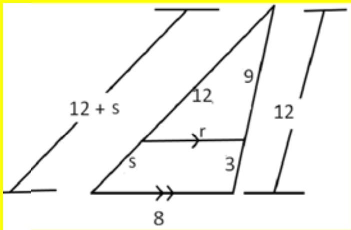
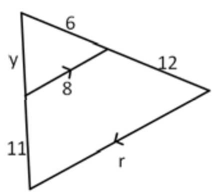
Activity No.13

Determining Proportions Derived from TPT

	$\frac{a}{g} = \frac{c}{h} = \frac{e}{f}$ $\frac{a}{b} = \frac{c}{d} = \frac{e}{f-e}$ $\frac{b}{g} = \frac{d}{h} = \frac{f-e}{f}$

Remind the students that by using the properties of proportion, proportions other than the one shown can be formed. Do this by giving examples.

Quiz on Triangle Proportionality Theorem

Figures	Solutions
1. 	Solving for s: $s = 20 - r = 20 - 12 = \mathbf{8}$ Solving for r: $\frac{r}{20} = \frac{9}{15}$ $15r = 9(20)$ $r = \frac{9(20)}{15}$ $r = \frac{[(3)(3)][(4)(5)]}{(3)(5)} = \mathbf{12}$
2.  **Solving for s:** $\frac{12}{12+s} = \frac{9}{12} \rightarrow \frac{12}{12+s} = \frac{(3)(3)}{(3)(4)} \rightarrow \frac{12}{12+s} = \frac{3}{4}$	By alternation property: $\frac{12}{3} = \frac{12+s}{4} \rightarrow \frac{(3)(4)}{3} = \frac{12+s}{4} \rightarrow 4 = \frac{12+s}{4}$ By cross multiplication: $16 = 12 + s \rightarrow 16 - 12 = s \rightarrow s = 4$ **Solving for r:** $\frac{r}{8} = \frac{9}{12} \rightarrow 12r = (9)(8) \rightarrow r = \frac{(3)(3)(2)(2)(2)}{(2)(2)(3)} = 6$
3. 	• $\frac{y}{11} = \frac{6}{12} = \frac{8}{r-8}$ • $\frac{y}{11} = \frac{6}{12} \rightarrow 12y = 6(11) \rightarrow y = \frac{6(11)}{12} \rightarrow y = \frac{6(11)}{6(2)} \rightarrow y = \frac{11}{2}$ • $\frac{6}{12} = \frac{8}{r-8} \rightarrow 6(r-8) = 8(12) \rightarrow r - 8 = \frac{8(12)}{6}$ $r - 8 = \frac{8(6)(2)}{6} \rightarrow r - 8 = 16 \rightarrow r = 16 + 8 = 24$

B. Triangle Proportionality Theorem states that if a segment divides two adjacent sides of a triangle proportionally, then it is parallel to the third side of the triangle.

C.

1.	$\frac{48}{48+16} ? \frac{56}{56+21}$ $\frac{48}{64} ? \frac{56}{77}$ $\frac{2^4(3)}{2^6} ? \frac{2^3(7)}{(11)(7)}$ $\frac{3}{2^2} ? \frac{8}{11}$ $\frac{3}{4} \neq \frac{8}{11}$ Therefore, $\overline{MZ} \nparallel \overline{AE}$.	2.	$\frac{35}{15+35} ? \frac{28}{28+12}$ $\frac{35}{50} ? \frac{28}{40}$ $\frac{5(7)}{2(5)(5)} ? \frac{2(2)(7)}{2(2)(10)}$ $\frac{7}{10} = \frac{7}{10}$ Therefore, $\overline{AT} \parallel \overline{FH}$
3.	$\frac{24}{18+24} ? \frac{40}{30+40}$ $\frac{24}{42} ? \frac{40}{70}$ $\frac{6(4)}{6(7)} ? \frac{10(4)}{10(7)}$ $\frac{4}{7} = \frac{4}{7}$ Therefore, $\overline{MR} \parallel \overline{AT}$	4.	$\frac{10}{10+32} ? \frac{11}{11+33}$ $\frac{10}{42} ? \frac{11}{44}$ $\frac{2(5)}{2(21)} ? \frac{11}{4(11)}$ $\frac{5}{21} \neq \frac{1}{4}$ Therefore, $\overline{DR} \nparallel \overline{EN}$

C.	1. ES	2. EH	3. EO	4. TY	5. NS

D.	1. Right	2. Right	3. Right	4. Wrong

F.

Solving for t:

$$\frac{48}{t} = \frac{48+40+32}{156} \rightarrow \frac{48}{t} = \frac{120}{156} \rightarrow \frac{48}{t} = \frac{10(12)}{13(12)} \rightarrow \frac{48}{t} = \frac{10}{13} \rightarrow \text{t} = \frac{48(13)}{10} = \frac{624}{10} = 62.4$$

Solving for s:

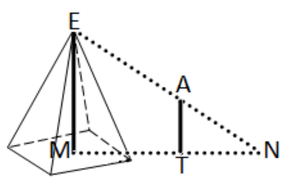
$$\frac{40}{s} = \frac{120}{156} \rightarrow \frac{40}{s} = \frac{10}{13} \rightarrow \text{s} = \frac{40(13)}{10} = 4(13) = 52$$

Solving for r:

$$\frac{32}{r} = \frac{120}{156} \rightarrow \frac{32}{r} = \frac{10}{13} \rightarrow \text{r} = \frac{32(13)}{10} = \frac{416}{10} = 41.6$$

Activity No. 14

Determining Heights without Using a Measuring Tool

<table>
<tr><td rowspan="5">
</td><td>1.height</td><td colspan="2">6. Because the pyramid is very tall, it is difficult to access.</td></tr>
<tr><td>2.pyramid</td><td>7. $\overline{ME} \parallel \overline{AT}$</td><td rowspan="2">10.
True, False, False, True</td></tr>
<tr><td>3. AT</td><td>8. Δ EMN~ΔATN</td></tr>
<tr><td>4.shadow</td><td colspan="2">9. Triangle Proportionality Theorem</td></tr>
<tr><td colspan="3">5. ME (NO), AT (YES), MN (YES), TN (YES)</td></tr>
</table>

11. If MN=80 ft, NT=8 ft, and AT=6ft, what is the height of the pyramid in this activity?

$$\frac{AT}{ME} = \frac{NT}{MN} \rightarrow \frac{6}{ME} = \frac{8}{80} \rightarrow \mathrm{ME} = \frac{80(6)}{8} = \frac{8(10)(6)}{8} = 60\mathrm{ft}$$

12. Yes, because measurements of sides and angles are still the same. The similar triangles are just separated.

13. $\frac{\mathrm{H_s}}{\mathrm{H_{fp}}} = \frac{\mathrm{S_s}}{\mathrm{S_{fp}}} \rightarrow \frac{5}{\mathrm{H_{fp}}} = \frac{3}{12} \rightarrow \mathrm{H_{fp}} = \frac{5(12)}{3} = \frac{5(3)(4)}{3} = 20\ \mathrm{ft}$

14.
$$\frac{4}{12} = \frac{3}{\mathrm{H}}$$
$$4\mathrm{H} = 3(12)$$
$$H = \frac{3(12)}{4}$$
$$H = \frac{3(3)(4)}{4} = 9\,m$$

15.
$$\frac{21}{21+36} = \frac{18}{\mathrm{D}}$$
$$\frac{21}{57} = \frac{18}{\mathrm{D}}$$
$$\frac{3(7)}{8(7)} = \frac{18}{\mathrm{D}}$$

$$\frac{3}{8} = \frac{18}{\mathrm{D}}$$
$$3\mathrm{D} = 8(18)$$
$$D = \frac{8(3)(6)}{3}$$
$$D = 48\,m$$

Activity No. 15

Ratios of Perimeters, Areas and Volumes of Similar Solids

Cube		Larger Cube	Smaller Cube	Ratio (Larger Cube: Smaller Cube)
Side	s	5	3	$5:3$
Perimeter P of the Base	$P = 4s$	20	12	$20:12 = 5:3$
Base Area	$BA = s^2$	25	9	$25:9 = 5^2:3^2$
Lateral Area	$LA = 4s^2$	100	36	$100:36 = 25:9 = 5^2:3^2$
Total Surface Area	$TA = 6s^2$	150	54	$25:9 \rightarrow 5^2 = 3^2$
Total Surface Area	$V = s^3$	125	27	$125:27 \rightarrow 5^3 = 3^3$

Questions:

1. The ratio of the sides of the cubes and the ratio of their perimeters are equal.
2. The ratio of the base areas of cubes is the ratio of the squares of their sides. This is also true to the ratio of lateral surface areas and total surface areas.
3. The ratio of the volumes of cubes is the ratio of the cube of their sides.

4. If the scale factor of two similar cubes is $\boldsymbol{m:b}$, then

(1) the ratio of their perimeters is	$m:b$
(2) the ratio of their base areas, lateral areas or total surface areas is	$m^2:b^2$
(3) the ratio of their volumes is	$m^3:b^3$

Let us find out if the principle is true with spheres and similar rectangular prisms.

A. Sphere

Sphere		Larger Sphere	Smaller Sphere	Ratio (Larger Sphere: Smaller Sphere)	
radius	r	3	6	3:6	1:2
Total Surface Area	$A = 4\pi r^2$	36π	144π	$36\pi : 144\pi$	$1:4 = 1^2 : 2^2$
Volume	$A = \frac{4}{3}\pi r^3$	36π	288π	$36\pi : 288\pi$	$1:8 = 1^3 : 2^3$

B. Rectangular Prism

Rectangular Prism		Smaller Prism	Larger Prism	Ratio (Larger Prism: Smaller Prism)
Length	l	2	6	$2:6 = 1:3$
Width	w	3	9	$3:9 = 1:3$
Height	h	5	15	$5:15 = 1:3$
Perimeter of the Base	$P = 2(l + w)$	10	30	$10:30 = 1:3$
Base Area	$BA = lw$	6	54	$6:54 = 1:9 = 1^2 : 3^2$
Lateral Area	$LA = 2h\,(l + w)$	50	450	$50:450 = 1:9 \rightarrow 1^2 = 3^2$
Total Surface Area	$TA = 2\,(lw + lh + wh)$	62	558	$62:558 = 1:9 \rightarrow 1^2 = 3^2$
Volume	$V = lwh$	30	810	$30:810 = 1:27 \rightarrow 1^3 = 3^3$

Question:

1. Are the ratios for perimeters, areas, and volumes of similar cubes true also to similar spheres and similar rectangular prisms? <u>Yes they are.</u>

2. Do you think the principle is also true in all other similar solids? Explain.
<u>Yes, the principle works in all other similar solids.</u>

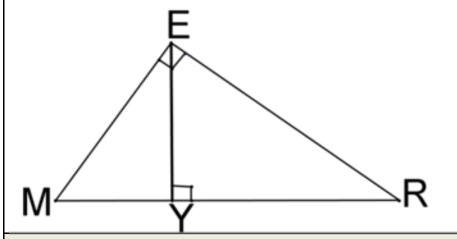
Investigation:

1. Are all spheres and all cubes similar? <u>YES because cubes have congruent sides and spheres have similar shapes.</u>
2. What solids are always similar aside from spheres and cubes? <u>Right circular cones, right square-based pyramids</u>

Activity No. 16

Right Triangle Similarity Theorem and its Proof

Right Triangle Similarity Theorem (RTST)
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original rectangle and to each other.



Given:

1. ΔMER is a right triangle with $\angle MER$ and a right angle and $\overline{MR}$ as the hypotenuse.
2. $\overline{EY}$ is an altitude to the hypotenuse of ΔMER.

Prove: $\boldsymbol{\Delta MER \cong \Delta EYR \cong \Delta MYE}$

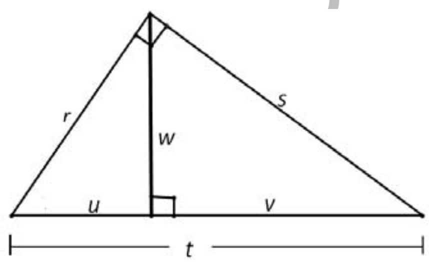
Proof:

Statements :	**Reasons:**
1.1 ΔMER is a right triangle with $\angle MER$ as right angle and $\overline{MR}$ as the hypotenuse. 1.2 $\overline{EY}$ is an altitude to the hypotenuse of ΔMER .	1. Given
2. $\overline{EY} \perp \overline{MR}$	2. Definition of altitude
3. $\angle MYE$ and $\angle EYR$ are right angles.	3. Definition of perpendicular lines
4. $\angle MYE \cong \angle EYR \cong \angle MER$	4. Definition of right angles
5. $\angle YME \cong \angle EMR$; $\angle YRE \cong \angle ERM$	5. Reflexive Property
6. $\Delta MYE \sim \Delta MER$; $\Delta MER \sim \Delta EYR$	6. AA Similarity Theorem
7. $\boldsymbol{\Delta MER \sim \Delta EYR \sim \Delta MYE}$	7. Transitive Property

Special Properties of Right Triangles

When the altitude is drawn to the hypotenuse of a right triangle,

1. the length of the altitude is the geometric mean between the segments of the hypotenuse; and
2. each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



	Using the definition of Similar Polygons in Right Triangles:	
Altitude $\boldsymbol{w}$ is the geometric mean between the $\boldsymbol{u}$ and $\boldsymbol{v}$	B and C	$\frac{v}{w}=\frac{w}{u} \to w^2 =\to \boldsymbol{w}=\sqrt{\boldsymbol{uv}}$
Leg $\boldsymbol{r}$ is the geometric mean between $\boldsymbol{t}$ and $\boldsymbol{u}$	A and C	$\frac{t}{r}=\frac{r}{u} \to r^2=ut \to \boldsymbol{r}=\sqrt{\boldsymbol{ut}}$
Leg $\boldsymbol{s}$ is the geometric mean between $\boldsymbol{t}$ and $\boldsymbol{v}$	A and B	$\frac{v}{s}=\frac{s}{t} \to s^2=vt \to \boldsymbol{s}=\sqrt{\boldsymbol{vt}}$

Quiz No. 4

Figure	Description	Proportion
1.	Altitude of ΔYES, a, is the geometric mean between the m and n.	$\frac{m}{a} = \frac{a}{n} \rightarrow \boldsymbol{a = \sqrt{mn}}$
	Shorter leg s is the geometric mean between m and h.	$\frac{m}{s} = \frac{s}{h} \rightarrow \boldsymbol{s = \sqrt{mh}}$
	Longer leg b is the geometric mean between n and h.	$\frac{n}{b} = \frac{b}{h} \rightarrow \boldsymbol{b = \sqrt{nh}}$

2.

1. The corresponding sides of the similar triangles

	Original Triangle	New Larger Triangle	New Smaller Triangle
Hypotenuse	ES	EY	SY
Longer leg	EY	EZ	YZ
Shorter leg	SY	YZ	SZ

2. Solve for the geometric means $a, b,$ and s.

Geometric Means	Proportion	Answer
altitude a	$a = \sqrt{2\,(8)} = \sqrt{16}$	$a = 4$
Shorter leg s	$s = \sqrt{2\,(10)} = \sqrt{2(2)(5)}$	$s = 2\sqrt{5}$
Longer leg b	$b = \sqrt{8\,(10)} = \sqrt{2^3(2)(5)} = \sqrt{2^4(5)}$	$b = 2^2\sqrt{5}$ $b = 4\sqrt{5}$

3.

1. The corresponding sides of the similar triangles

	Original Right Triangle	Larger New Rectangle	Smaller New Rectangle
Hypotenuse	EI	ER	IR
Longer leg	ER	EC	CR
Shorter leg	IR	CR	CI

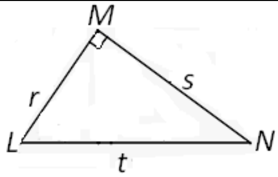
2. Solve for $y, u,$ and b.

y	u	b	m
$\frac{y}{6} = \frac{6}{18}$ $y = \frac{36}{18} = 2$	$u = 18 - y$ $u = 18 - 2$ $u = 16$	$b = \sqrt{u(18)}$ $b = \sqrt{16(9)2}$ $b = 4(3)\sqrt{2}$ $b = 12\sqrt{2}$	$m = \sqrt{u(y)}$ $m = \sqrt{16(2)}$ $m = 4\sqrt{2}$

Activity No. 17

Pythagorean Theorem and its Proof

Pythagorean Theorem
The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

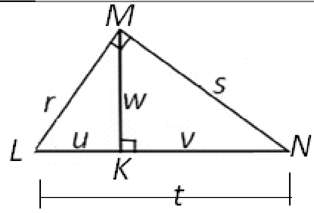


Given:

- $LM = r$ and $MN = s$ as the legs ;
- $LN = t$ as the hypotenuse
- $\angle LMN$ is a right angle.

Prove: $r^2 + s^2 = t^2$

Proof:
Construct altitude $MK = w$ to the hypotenuse $LN = t$, dividing it to $LK = u$ and $KN = v$



	Hints	Statements :	Reasons:
1	Describe triangles LMN, MKN, and LKM when an altitude MK is drawn to its hypotenuse	$\Delta LMN \sim \Delta MKN \sim \Delta LKM$	Right Triangle Similarity Theorem
2	Write the proportions involving the geometric means r and s	$\frac{u}{r} = \frac{r}{t}$ $\frac{v}{s} = \frac{s}{t}$	Special Properties of Right Triangles
3	Cross-multiply the terms of the proportions in statement 2	$r^2 = ut$ $s^2 = vt$	Cross-Multiplication Property of Proportions
4	Add s^2 to both sides of $r^2 = ut$ in statement 3	$r^2 + s^2 = ut + s^2$	Addition Property of Equality
5	Substitute s^2 on the right side of statement 4 using its equivalent from statement 3	$r^2 + s^2 = ut + vt$	Substitution Property of Equality
6	Factor the right side of statement 5	$r^2 + s^2 = t(u + v)$	Common Monomial Factoring
7	Substitute $u + v$ in statement 6 by its equivalent length in the figure	$r^2 + s^2 = t(t)$	Segment Addition Postulate
8	Simplify the right side of statement 7	$r^2 + s^2 = t^2$	Product Law of Exponents

Quiz on the Pythagorean Theorem

A.

A	B	C	D	E
$f^2+g^2=h^2$ $3^2+g^2=5^2$ $9+g^2=25$ $g^2=25-9$ $g^2=16$ $g=4$	$f^2+g^2=h^2$ $5^2+12^2=h^2$ $25+144=h^2$ $g^2=169$ $g=13$	$f^2+g^2=h^2$ $f^2+24^2=25^2$ $f^2+576=625$ $f^2=625-576$ $f^2=49$ $f=7$	$f^2+g^2=h^2$ $8^2+15^2=h^2$ $64+225=h^2$ $289=h^2$ $h=17$	$f^2+g^2=h^2$ $9^2+g^2=41^2$ $81+g^2=1681$ $g^2=1681-81$ $g^2=1600$ $g^2=16(100)$ $g=4(10)=40$

Questions

3. Pythagorean triples are three whole numbers that satisfy the Pythagorean Theorem.
4. Multiples of the Pythagorean Triples are still Pythagorean Triples. Example: 2 (3, 4, 5) =(6, 8, 10). Notice that $6^2+8^2=10^2 \rightarrow 36+64=100 \rightarrow 100=100.$
5. $t^2=r^2+s^2$ is equivalent to $\mathrm{r}^2=(\mathrm{t}+\mathrm{s})(\mathrm{t}-\mathrm{s})$

B.

1. $d^2=16^2+14^2$ $d=\sqrt{16^2+14^2}$ $d=\sqrt{256+196}$ $d=\sqrt{452}=2\sqrt{113}$ $\boldsymbol{d \approx 21.26\ inches}$ **2.** $20^2=H^2+8^2$ $H^2=20^2-8^2$ $H^2=400-16$ $H=\sqrt{384}$ $d=\sqrt{2^6(6)}$ $d=2^3\sqrt{6}=8\sqrt{6}$ $\boldsymbol{d \approx 19.6\ ft}$	**3.** **Solving for AI:** $\mathrm{AI}=\sqrt{3^2+7^2}=\sqrt{58}$ ≈ 7.62 cm **Solving for AE:** $\mathrm{AE}=\sqrt{7^2+5^2}=\sqrt{74}$ ≈ 8.60 cm **Solving for AF:** $\mathrm{AF}=\sqrt{3^2+5^2}=\sqrt{34}$ ≈ 5.83 cm **Solving for AY:** $\mathrm{AY}=\sqrt{5^2+7.62^2}$ $\mathrm{AY}=\sqrt{25+58}=\sqrt{83}$ $=9.11$ cm	**4. Solving for GR:** $\mathrm{GR}=\sqrt{7^2+5^2}$ $=\sqrt{74}\approx 8.60$ cm **Solving for RO:** $\mathrm{RO}=\sqrt{5^2+4^2}$ $=\sqrt{41}\approx 6.40$ cm **5. Solving for KP:** $\frac{1.5}{4.5}=\frac{4.5}{\mathrm{KP}}$ $1.5\ \mathrm{KP}=4.5(4.5)$ $KP=\frac{4.5(4.5)}{1.5}$ $KP=\frac{20.25}{1.5}$ $=13.5ft$	Therefore the distance across the river is 13.5 ft. **5. Solving for x:** $\frac{1.92}{\mathrm{x}}=\frac{0.4}{87.6}$ $0.4\ \mathrm{x}=(1.92)(87.6)$ $\mathrm{x}=\frac{168.192}{0.4}$ $=420.48\ m$ Therefore the height of the skyscraper is 420.48 m.

Activity No. 18

Is the triangle right, acute, or obtuse?

Ask students the kind of triangle formed if the square of the longest side is not equal to the sum of the squares of the shorter sides. Encourage them to make predictions and let them state their predictions as hypotheses.

Kind	Name	s_1	s_2	l	${s_1}^2$	${s_2}^2$	Sum	l^2
Right	JOY	6	8	10	36	64	100	100
Obtuse	SUN	5	7	11	25	49	74	121
Acute	CAR	4	5	6	16	25	41	36

Observations:

1. A triangle is right if the square of the longest side is equal to the sum of the squares of the shorter sides.
2. A triangle is obtuse if the square of the longest side is greater than the sum of the squares of the shorter sides.
3. A triangle is acute if the square of the longest side is less than the sum of the squares of the shorter sides.

Questions:

1. Because hypotheses are just predictions, they can either be accepted or rejected after verifying them.
2. Possible Answer: I find hypothesis-making and testing interesting.

Conclusion:

Given the lengths of the sides of a triangle, to determine whether it is right, acute or obtuse; there is a need to compare the square of the longest side with the sum of the squares of the two shorter sides.

Quiz on Determining the Kind of Triangle according to Angles

Triangle	s_1	s_2	l	${s_1}^2$	${s_2}^2$	Sum	l^2	Kind
1	7	8	10	49	64	113	100	Acute
2	9	12	15	81	144	225	225	Right
3	3	6	7	9	36	45	49	Obtuse

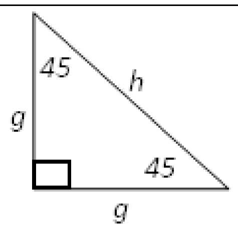
Activity No. 19

45-45-90 Right Triangle Theorem and its Proof

45-45-90 Right Triangle Theorem

In a 45-45-90 right triangle:

- each leg l is $\frac{\sqrt{2}}{2}$ times the hypotenuse h; and
- the hypotenuse h is $\sqrt{2}$ times each leg l



Given:

Right Triangle with

- leg = $\boldsymbol{l}$,
- hypotenuse = $\boldsymbol{h}$,

Prove:

- $h = \sqrt{2}\,l$
- $l = \frac{\sqrt{2}}{2}h$

Clues:	Statements :	Reasons:

1	List down all the given	Right triangle with leg = $\boldsymbol{l}$, hypotenuse = $\boldsymbol{h}$	Given
2	Write an equation about the measures of the legs and the hypotenuse and simplify.	$l^2 + l^2 = h^2 \rightarrow \mathbf{2}\boldsymbol{l}^{\mathbf{2}} = \boldsymbol{h}^{\mathbf{2}}$	Pythagorean Theorem
3	Solving for h in statement 2	$\boldsymbol{h} = \sqrt{2l^2} = l\sqrt{2} = \sqrt{\mathbf{2}}\boldsymbol{l}$	$\sqrt[e]{b^e} = b$
4	Solving for l in statement 3	$h = \sqrt{2}l \rightarrow l = \dfrac{h}{\sqrt{2}}$	Division Property of Equality
		$\boldsymbol{l} = \dfrac{h}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right) = \dfrac{\sqrt{\mathbf{2}}\,\boldsymbol{h}}{\mathbf{2}}$	Rationalization of Radicals

Quiz on 45-45-90 Right Triangle Theorem

A.		B
$\boldsymbol{l} = \dfrac{\mathbf{5}\sqrt{\mathbf{2}}}{\mathbf{2}}$	$\boldsymbol{h} = \mathbf{12}\sqrt{\mathbf{2}}$	1. $\mathrm{d} = 16\sqrt{2} \approx 22.63$ inches 2. $T = 4\left(\frac{8\sqrt{2}}{2}\right) = 16\sqrt{2} \approx 22.63$ in.

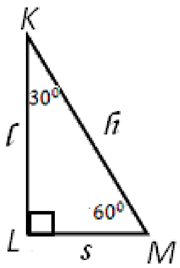
Activity No. 20

30-60-90 Right Triangle Theorem and its Proof

30-60-90 Right Triangle Theorem

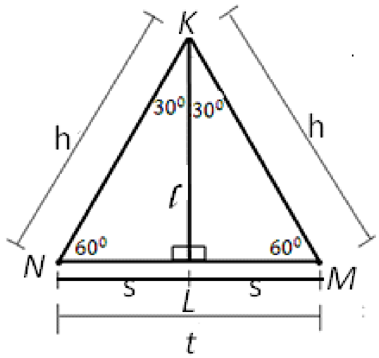
In a 30-60-90 right triangle:

- the shorter leg s is $\frac{1}{2}$ the hypotenuse h or $\frac{\sqrt{2}}{2}$ times the longer leg l;
- the longer leg l is $\sqrt{2}$ times the shorter leg s; and
- the hypotenuse h is twice the shorter leg s

	Given:	Prove:
	Right ΔKLM with • hypotenuse KM = $\boldsymbol{h}$, • shorter leg LM = $\boldsymbol{s}$, • longer leg KL = $\boldsymbol{l}$ • $m\angle LKM = 30$ • $m\angle LMK = 60$	• $\boldsymbol{h} = \mathbf{2}\boldsymbol{s}$ * • $\boldsymbol{s} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{h}$ ** • $\boldsymbol{l} = \sqrt{3}\,\boldsymbol{s}$ *** • $\boldsymbol{s} = \frac{\sqrt{3}}{3}l$ ****

Proof:

Construct a right triangle equivalent to the given triangle with the longer leg $\boldsymbol{l}$ as the line of symmetry such that: $m\angle LKN = 30$ and $m\angle KNL = 60$; $KN = h$, and $LN = s$.



Hints:		Statements :	Reasons:
1	List down all the given	Right ΔKLM with $m\angle LMK = 60$; $m\angle LKM = 30$; $KM = t$; $LM = s$; $KL = l$	Given
2	List down all constructed angles and segments and their measures	$\Delta KLM \cong \Delta KLN$; $m\angle LKN = 30$; $m\angle KNL = 60$; $KN = h$, and $LN = s$.	by Construction
3	Use Angle Addition Postulate to $\angle LKM$ and $m\angle LKN$	$m\angle MKN = m\angle LKM + m\angle LKN$	Angle Addition Postulate
4	What is $m\angle MKN$?	$m\angle MKN = 30 + 30 = 60$	Substitution Property of Equality
5	What do you observe about ΔMKN considering its angles?	ΔMKN is equiangular triangle.	Definition of Equiangular Triangle
6	What conclusion can you make based from statement 5.	ΔMKN is equilateral.	Equiangular Triangle is also equilateral.
7	With statement 6, what can you say about the sides of ΔMKN ?	$KM = KN = MN = h$	Definition of Equilateral Triangle
8	Use Segment Addition Postulate for LN and ML	$LN + ML = MN$	Segment Addition Postulate
9	Replace LN, ML, MN with their measurements	$s + s = h \rightarrow \mathbf{2s = h}$	Substitution Property of Equality
10	What is the value of h?	$\boldsymbol{h = 2s}$ *	Symmetric Property of Equality
11	Solve for r using statement 9	$\boldsymbol{s = \frac{h}{2}}$ **	Division Property of Equality
12	What equation can you write about s, l, and h?	$s^2 + l^2 = h^2$	Pythagorean Theorem

13	Use statement 10 in statement 13	$s^2 + l^2 = (2s)^2$	Substitution Property of Equality
14	Simplify the right side of statement 13	$s^2 + l^2 = 4s^2$	Power of a Product Law of Exponent
15	Solve for l^2	$l^2 = 4s^2 - s^2 \rightarrow \quad l^2 = 3s^2$	Subtraction Property of Equality
16	Solve for l	$l = \sqrt{3s^2} \rightarrow \boldsymbol{l = \sqrt{3}s}$ ***	$\sqrt[e]{b^e} = b$ law of radicals
17	Solving for s in statement 16	$s = \frac{l}{\sqrt{3}} = \frac{l}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \boldsymbol{\frac{\sqrt{3}l}{3}}$ ****	Division Property of Equality and Rationalization of Radicals

Quiz on 30-60-90 Right Triangle Theorem

A.

A.		B.		
1. $If\ s = 6$	$l = 6\sqrt{3}$ $h = 2s = 2(6) = 12$	1.	$L = \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2} \approx 0.87\ ft$	
2. $If\ h = 10$	$s = 5$ $l = 5\sqrt{3}$	2.	$\text{ER} = \frac{\sqrt{3}}{3}(8)$ $= \frac{8\sqrt{3}}{3} \approx 4.62\ \text{cm.}$	$\text{CE} = 2\text{ER} = 2\left(\frac{8\sqrt{3}}{3}\right)$ $= \frac{16\sqrt{3}}{3} \approx 9.24\ \text{cm}$
3. $If\ l = 7\sqrt{3}$	$s = \frac{\sqrt{3}}{3}(7\sqrt{3}) = 7$ $h = 2(7) = 14$		$\text{UR} = \frac{1}{2}(\text{CR})$ $= \frac{1}{2}(8)$ $= 4\ \text{cm.}$	$\text{CU} = \sqrt{3}(\text{UR}) = \sqrt{3}(4)$ $= 4\sqrt{3} \approx 6.93\ \text{cm}$

You have successfully helped in illustrating, proving, and verifying the theorems on similarity. All the knowledge and skills you've learned in this section will be useful in dealing with the next section's problems and situations that require applications of these principles.

WHAT TO REFLECT AND UNDERSTAND:

Explain the purpose of the activities in this section. Note that many of the questions are open-ended questions. Be open to varying responses and process them to help train the students how to think critically.

Activity No. 21

Watch Your Rates

Questions:

1. $\text{Scale Factor} = \frac{L_a}{L_o} = \frac{6}{10} = \frac{3}{5}$
2. *The differences in the dimensions are the same. However, the rates of conversion from the original size to the reduced size and the reduced size back to the original size differ because the initial dimensions used in the computation are different.*

Solution to the Problem:

- **Solving for the dimensions of the original document**
  - $14.3\ = 130\%\,(L_o) \rightarrow L = \frac{14.3}{1.3} = 11\ inches$
  - $10.4\ = 130\%\,(W_o) \rightarrow W = \frac{10.4}{1.3} = 8\ inches$
- **Solving for the desired dimensions**
  - $L_d\ = 120\%\,(L_o) \rightarrow L_d = 1.2\,(11) = 13.2\ inches$
  - $W_d\ = 130\%\,(W_o) \rightarrow W_d = 1.2\,(8) = 9.6\ inches$

- **To rectify, reduce the enlarged document using 92.31% copier settings.**
  - $\mathrm{R} = \frac{\mathrm{D_t}}{\mathrm{D_i}} = \frac{13.2}{14.3} \approx 92.31\%$

Activity No. 22

Dilation: Reducing or Enlarging Triangles

	Reasons:		Reasons		Reasons:
1	Given	4	Substitution	6	Subtraction
2	Corresponding	5	Transitive	7	AA
3	Angles				

The coordinates of each point of the similar triangles are provided in the table below.

Triangles	Coordinates of		Triangles	Coordinates of		Triangles	Coordinates of	
ΔTAB	T	(1, 2)	ΔMER	M	(-4,-4)	ΔTRI	T	(0,1)
	A	(1, 1)		E	(0, 4)		R	(2, 2)
	B	(3, 1)		R	(6, 2)		I	(4, 1)
ΔLER	L	(2, 4)	ΔDIN	D	(-2, -2)	ΔPLE	P	(0, 3)
	E	(2, 2)		I	(0, 2)		L	(6, 6)
	R	(6, 2)		N	(3, 1)		E	(12,3)

Questions:

1. The abscissas of the larger triangles are <u>multiples</u> of the abscissas of the smaller triangles.
2. The ordinates of the larger triangles are <u>multiples</u> of the ordinates of the smaller *triangles.*
3. Scale Factors of the given triangles:

Scale Factor	Similar Triangles		
	ΔTAB to ΔLER	ΔMER to ΔDIN	ΔTRI to ΔPLE
Enlargement	2	2	3
Reduction	½	½	1/3

4. The scale factor is used to determine the coordinates of the points of the larger or smaller version of the original figures.

Scale drawing

Explain to the students what scale drawing is.

Activity No. 23

Avenues for Estimation

Explain the following: (1) maps are diagrammatic representations of the surface of the earth; (2) scale on the map is used to compare the actual distances of locations with their representations on the map; (3) estimation is quite important in finding distances using maps because streets or boulevards or avenues being represented on maps are not straight lines; (4) some parts of these streets may be straight but there are always bends and turns; (5) distance formula.

Show to students how to find distances between locations using the scale on the map. Guide them also as they perform the activity of estimating actual distances. Since answers are estimations, answers may vary. For as long as differences are not too big, answers should be accepted.

Activity No.24

Reading a House Plan

Assist the students as they perform this activity and answer the activity questions.

Parts of the House	Scale Drawing Dimensions		Actual House Dimensions		Floor Area
	Length	Width	Length	Width	
Porch	4s	1s	4m	1m	4 sq.m.
Master's Bedroom with Bathroom	4s	4s	4m	4m	16 sq.m
Bathroom Alone	2s	1.5s	2m	1.5m	3 sq.m.
Living Room	5s	4s	5m	4m	20 sq.m.
Kitchen	4s	3s	4m	3m	12 sq.m.
Children's Bedroom	4s	3s	4m	3m	12 sq.m.
Laundry Area and Storage	4s	2s	4m	2m	6 sq.m.
Whole House	9s	8s	9m	8m	72 sq.m.

Questions:

1. Living Room
2. Kitchen and Children's Bedroom
3. Without considering the area of the bathroom, the master's bedroom is still larger than the other bedroom by 8.33%.

4. Possible Answer: The area of the living room adjacent to the kitchen may also be used as dining area.
5. Note: Ask their opinions on the placing of doors and the direction of the opening. Let them explain their answers.

6-8. Note: This can be an individual or group activity. Allow students to choose.

9. Note: This assignment will reveal the students' socio-economic situation. Your role is to inspire them.

Quiz on Scale Drawing

A. The scale of a drawing is 3 in : 15 ft. Find the actual measurements for:

1. 4 in	**2.** 6 in	**3.** 9 in	**4.** 11 in
20 ft	**30 ft**	**45 ft**	**55 ft**

B. The scale is 1 cm : 15 m. Find the length each measurement would be on a scale drawing.

5. 150 m	**6.** 275 m	**7.** 350 m	**8.** 400 m
10 cm	18.33 cm	23.33 cm	26.67 cm

C. Tell whether the scale reduces, enlarges, or preserves the size of an actual object.

9. 1 m = 10 cm	10. 1 in = 1 ft	11. 100 cm = 1 m
reduces	enlarges	Preserves

D. Problem Solving.

12. Scale= $15\ in{:}\,100\ km = 3\ in{:}\,20\ km$

13.

- $3\ in{:}\,1\ m\ =\ 9\ in{:}\,Length\ L$

  $3\ in{:}\,1\ m\ =\ 9\ in{:}\,Length\ L$

  $3\ L = 9$

  $L = 3\ meters$

- $3\ in{:}\,1\ m\ =\ 6\ in{:}\,Width\ W$

  $3\ W = 6$

  $L = 2\ meters$

Therefore, the actual size of the kitchen is 3 meters by two meters.

14.

$$Actual\ width\ W{:}\,model\ width\ w = Actual\ width\ W{:}\,model\ width\ w$$

$$W{:}\,8\ in = 36\ ft{:}\,1\ ft$$

$$W{:}\,8\ in = 36\ ft{:}\,12\ inches$$

$$W = \frac{(8\text{ in})(36\text{ ft})}{12\text{ inches}} = \frac{3(12)(8\text{ ft})}{12} = 24\text{ ft}$$

15.

- $20\ cm: 76\ m\ = 7\ cm$: Length L

  $20\ L = 7(76)$

  $L = \frac{7(76)}{20} = \frac{532}{20} = 26.6\ meters$

- $20\ cm: 76\ m\ = 4\ cm$: Width W

  $20\ W = 4(76)$

  $W = \frac{4(76)}{20} = \frac{4(76)}{4(5)} = \frac{76}{5} = 15.2\ meters$

- $20\ cm: 76\ m\ = 28\ cm$: Height H

  $20\ H = 28(76)$

  $H = \frac{28(76)}{20} = \frac{7(4)(76)}{4(5)} = \frac{7(76)}{5} = \frac{532}{5} = 106.4\ meters$

Your transfer task requires students to sketch a floor plan of a couple's house. They will also make a rough cost estimate of building the house. In order that they would be able to do the rough cost estimate, explain to them the importance of knowledge and skills in proportion, measurement and some construction standards. Instead of scales, these standards refer to rates because units in these standards differ.

Activity No. 25

Costimation Exercise!

A. Computing for the number of Concrete Hollow Blocks (CHB):

➢ **Solving for the number of CHB needed:**

$$\frac{1\text{ CHB}}{16(8)} = \frac{\text{total no. of CHB}}{2(39.36)(196.8 + 59.04)} \rightarrow \text{total no. of CHB} = \frac{20{,}139.73\text{ sq. in.}}{128\text{ sq. in.}} \approx \mathbf{158\ pieces}$$

However, **160 pieces** of CHB can be purchased in case some pieces get broken.

B. Computing for the no. of bags of cement needed for laying 160 CHB

$$\text{Total no. of bags of cement} = \frac{160\text{ CHB}}{55\text{ CHB}} = 2.91$$

C. Computing for the no. of bags of cement and volume of sand for CHB plaster finish.

➢ **Solving for the number of number of bags of cement needed for CHB plaster finish:**

$$\frac{0.25}{1\text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{2[2(1)(5 + 1.5)]} \rightarrow \text{Total no. of bags of cement} = (0.25\ )(26) = 6.50$$

- **Solving for the volume of sand in cu. m. needed for CHB plaster finish:**

$$\frac{0.0213 \text{ cu. m.}}{1 \text{ sq. m.}} = \frac{\text{Volume of sand in cu. m.}}{26 \text{ sq. m.}} \rightarrow \text{Volume of sand in cu. m} = 0.5538$$

D. Computing for volume of concrete (the no. of bags of 94 lbs cement and volume of sand and gravel) for fish tank flooring using Class A.

- **Solving for the number of number of bags of cement needed for Class A flooring:**

$$\frac{7.84}{1 \text{ cu. m.}} = \frac{\text{Total no. of bags of cement}}{\text{Floor Length x Floor Width x Depth of Concrete}}$$

$$\frac{7.84}{1 \text{ cu.m.}} = \frac{\text{Total no.of bags of cement}}{(5)(1.5)(0.10)} = \text{Total no. of bags of cement} = 5.88$$

- **Solving for the volume of sand in cu. m. needed for Class A flooring:**

$$\frac{0.44 \text{ cu. m.}}{1 \text{ cu. m.}} = \frac{\text{Volume of sand in cu. m.}}{\text{Volume of Concrete in cu. m.}} \rightarrow \text{Volume of sand in cu. m.} = (0.44)(0.75) = 0.33$$

- **Solving for the volume of gravel in cu. m. needed for Class A flooring:**

$$\frac{0.88 \text{ cu. m.}}{1 \text{ cu. m.}} = \frac{\text{Volume of gravel in cu. m.}}{\text{Volume of Concrete in cu. m.}} \rightarrow \text{Volume of gravel in cu. m.} = (0.88)(0.75) = 0.66$$

E. Computing for the no. of bags of cement and volume of sand for mortar of the walls using *4" Fill All Holes and Joints*.

- **Solving for the number of number of bags of cement needed for mortar:**

$$\frac{0.36 \text{ bag of cement}}{1 \text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{13 \text{ sq. m.}} \rightarrow \text{Total no. of bags of cement} = (0.36)(13) = 4.68$$

- **Solving for the volume of sand in cu. m. needed for mortar:**

$$\frac{0.019 \text{ cu. m.}}{1 \text{ cu. m.}} = \frac{\text{Volume of sand in cu. m.}}{13 \text{ sq. m.}} \rightarrow \text{Volume of sand in cu. m.} = (0.019)(13) = 0.247$$

F. Computing for the no. of bags of cement and volume of sand for *plain cement* floor finish using Class A 94-lbs cement

- **Solving for the number of number of bags of cement needed for *plain cement* floor finish using Class A 94-lbs cement**

$$\frac{0.33 \text{ bag of cement}}{1 \text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{7.5 \text{ sq. m.}} \rightarrow \text{Total no. of bags of cement} = 2.475$$

- **Solving for the volume of sand in cu. m. needed for *plain cement* floor finish using Class A 94-lbs cement**

$$\frac{0.00018\text{ cu. m.}}{1\text{ sq. m.}} = \frac{\text{Volume of sand in cu. m.}}{7.5\text{ sq. m.}} \rightarrow \text{Volume of sand in cu. m.} = 0.00135\text{ cu. m.}$$

G. Computing for the no. of needed steel bars

- **Solving for the Total Length of Horizontal Bars (for every 2 layers):**

$$\frac{2.7\text{ m}}{1\text{ sq. m.}} = \frac{\text{Total length of horizontal bar}}{13\text{ sq. m.}} \rightarrow \text{Total length of horizontal bar} = 35.1\text{ m}$$

- **Solving for the Total Length of Vertical bars (at 0.4 spacing):**

$$\frac{3\text{ m}}{1\text{ sq.m.}} = \frac{\text{Total length of vertical bars}}{13\text{ sq.m.}} \rightarrow \text{Total length of vertical bars} = (3)(13) = 39\text{ m}$$

- **Solving for the Total Length of Floor bars (at 0.4 spacing):**

$$\frac{3\text{ m}}{1\text{ sq. m.}} = \frac{\text{Total length of floor bars}}{7.5\text{ sq. m.}} \rightarrow \text{Total length of floor bars} = (3)(7.5) = 22.5\text{ m}$$

- **Solving for the number of steel bars needed:**

$$\text{No. of steel bars needed} = \frac{\text{HB} + \text{VB} + \text{FB}}{\text{Standard Length of bar}} = \frac{35.1 + 39 + 22.5}{6.096} = 15.85 \approx 16\text{ pcs.}$$

Questions:

1. 22.45 bags of cement or 23 bags
2. 8.95 bags of Sahara or 9 bags
3. 1.13 cu.m.
4. 0.66 cu.m.
5. Let the students do canvassing of the prices of the construction materials. If it is not practical in the location, you may provide them yourself with these current prices.

Materials		Quantity	Unit Cost	Total
1	CHB 4” x 8” x 16”	160 pieces		
2	Gravel	0.66 cu.m		
3	Sand	1.13 cu.m.		
4	Portland Cement	23 bags		
5	Steel Bar (10 mm. radius)	16 pcs		
6	Sahara Cement	9 bags		
7	PVC ¾”	5 pcs.		
8	PVC Elbow ¾”	6 pcs		
9	PVC 4”	1 pc.		
10	PVC Solvent Cement	1 small can		
11	Faucet	1 piece		
12	G.I. Wire # 16	1 kg		
13	Hose 5 mm	10 m		
			Grand Total	

Activity No. 26

Blowing Up a Picture into Twice its Size

Questions:

1. Note: Appreciate varied answers.
2. Do you agree that the use of grid makes it possible for everyone to draw?
3. *Yes because the use of grid helps retain the shape and increase the size proportionally*
4. *The scale used to enlarge the original picture in this activity is 2 because the length ℓ of the side of the smallest square in the new grid is twice as long as that of the grid of the original picture.*
5. *The scale to use to enlarge a picture three times its size is 3*
6. *To reduce the size of a picture by 20%, it means that the size of the new picture is only 80% of the size of the original. Therefore, the length ℓ of the side of the smallest square in the new grid is the product of 0.8 and 5 cm. Hence, length ℓ is equal to 4 cms.*
7. *To increase the size of a picture by 30%, it means that the size of the new picture is 130% of the size of the original. Therefore, the length ℓ of the side of the smallest square in the new grid is the product of 1.3 and 10 mm. Hence, length ℓ is equal to 13 mms.*
8. Make this an assignment. Allow students to use any kind of coloring material if they prefer to color their work.

WHAT TO TRANSFER:

Inform the class that the goal of the section is to apply their learning to real life situation. The practical task will enable them to demonstrate their understanding of similarity.

Activity No. 26

Sketchtimating Endeavor

The standard rates used in the cost estimation of the construction materials are the same standards used in Activity No. 25. The knowledge they have learned in Activity No. 24 will serve as a guide on how the students will design the house as they sketch its parts using a specific scale of measurement. Note that subdivisions of the house and its roof are not discussed in the cost estimation activity. It is your task to guide them to refer to engineers, architects or carpenters. You may also encourage them to extend the assignment to include a perimeter fence, furniture and fixtures. Be sure to let them share their ideas on this activity by tackling the questions provided.

SUMMARY:

Let them revisit their responses in Activity No. 1 and the solutions to their answers to the pretest before letting them tackle this wrap-up activity.

Activity No. 27

Perfect Match

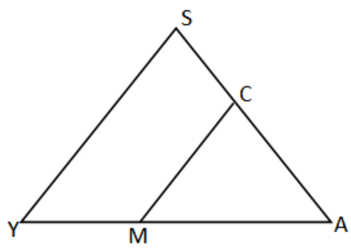
Figure Number	Similarity Concept	Figure Number	Similarity Concept
10	30-60-90 Right Triangle Theorem	7	Right Triangle Similarity Theorem
5	Triangle Angle Bisector Theorem	4	SSS Similarity Postulate
8	Pythagorean Theorem	1	Definition of Similar Polygons
6	Triangle Proportionality Theorem	9	45-45-90 Right Triangle Theorem
3	SAS Similarity Postulate	2	AA Similarity Postulate

Make sure that you have reviewed the students on the answers and solutions of the pre-assessment before letting them answer the post-assessment.

POST-ASSESSMENT:

Let's find out how much you already learn about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items.

1. If $\frac{AC}{AS} = \frac{AM}{AY}$, which of the following data makes $\Delta CAM \sim \Delta SAY$ by SAS Similarity Theorem?



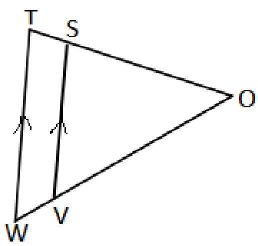
A. $\angle CAM \cong \angle SAY$

B. $\angle ACM \cong \angle ASY$

C. $\angle AMC \cong \angle AYS$

D. $\angle SCM \cong \angle YMC$

2. Which proportion is correct?



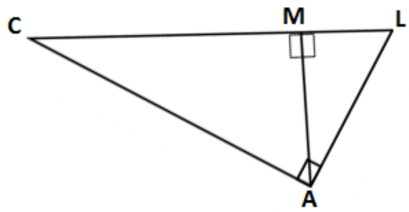
A. $\frac{OS}{OT} = \frac{OV}{VW}$

B. $\frac{SV}{TW} = \frac{OS}{ST}$

C. $\frac{ST}{OT} = \frac{VW}{OW}$

D. $\frac{VW}{OW} = \frac{SV}{TW}$

3. Which of the following statements is true about the figure?



I. $\Delta CAL \sim \Delta CMA \sim \Delta AML$

II. AM is the geometric mean of CM and LM

III. AL is the geometric mean of LM and CL.

IV. AC is the geometric mean of CM and CL.

V. $\overline{AM}$ is an altitude to hypotenuse $\overline{CL}$.

A. I only B. I & V only C. II, III, IV only D. All of the above

4. If $j : p = 4 : 1$, what is the correct order of steps in determining $j^2 + 2p^2 : j^2 - p^2$?

I. $j = 4k; p = k$ III. $\dfrac{(4k)^2 + 2(k)^2}{(4k)^2 - (k)^2}$

II. $j^2 + 2p^2 : j^2 - p^2 = 17 : 15$ IV. $\dfrac{j}{4} = \dfrac{p}{1} = k$

A. I, IV, III, II B. IV, I, III, II C. I, IV, II, III D. I, III, II, I

5. The ratio of the areas of two similar rectangular prisms is $49 : 81$. What is the ratio of their volumes?

A. 343:729 B. 9: 7 C. 7:9 D. 441`: 567

2. The lengths of the sides of a triangle are 4 cm, 5 cm, and 6 cm. What kind of a triangle is it?

A. Regular triangle B. Acute triangle C. Right triangle D. Obtuse triangle

3. What is the perimeter of a 30-60-90 triangle whose hypotenuse is 8 cm long?

A. $4 + 8\sqrt{3}\ cm$ B. $24 + 8\sqrt{3}\ cm$ C. $12 + \sqrt{3}\ cm$ D. $12 + 4\sqrt{3}\ cm$

4. One leg of an isosceles right triangle measures 7 cm. How long is the hypotenuse?

A. $7\sqrt{2}\ cm$ B. $3.5\ cm$ C. $\dfrac{7\sqrt{2}}{2}\ cm$ D. $\dfrac{7\sqrt{3}}{3}\ cm$

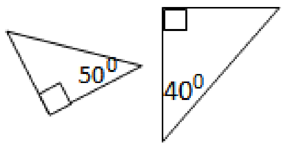
5. What theorem will you use to find the diagonal of a 10 cm by 8 cm rectangle?
   A. Right Triangle Proportionality Theorem
   B. Pythagorean Theorem
   C. Triangle Proportional Theorem
   D. Triangle Angle Bisector Theorem

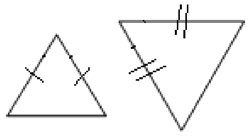
6. Which of the following pairs of solids will not always be similar?

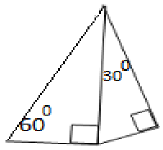
A. Pair of spheres C. Pair of pyramids

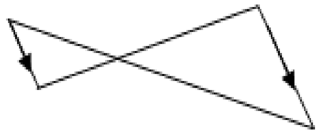
B. Pair of cubes D. Pair of square prisms

7. Which of the following pairs of triangles cannot be proved similar?

A. 

C. 

B. 

D. 

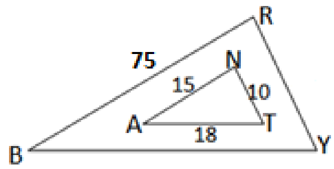
8. The ratio of the sides of the original triangle to its reduced version is $2:1$. The reduced triangle is expected to have

   A. sides that are twice as long as the original
   B. perimeter that is as long as the original
   C. sides that are half as long as the original
   D. angles that are half as large as the original

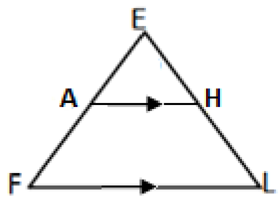
9. $\Delta BRY \sim \Delta ANT$. Which ratio of sides gives the scale factor?

   A. $\frac{AN}{BR}$ C. $\frac{AT}{BR}$

   B. $\frac{AN}{AT}$ D. $\frac{NT}{BR}$



10. What similarity concept justifies that $\Delta AEH \sim \Delta FEL$?



   A. Right Triangle Proportionality Theorem
   B. Triangle Proportionality Theorem,
   C. SSS Similarity Theorem
   D. SAS Similarity Theorem

11. A map is drawn to the scale of 1 cm: 150 m. If the distance between towns A and B is 105 km, how far are they on the map?

   A. 700 cm B. 70 cm C. 7000 cm D. 707 cm

12. The length of the shadow of your 1.6 meter height is 2.8 meters at a certain time in the afternoon. How high is an electrical post in your backyard if the length of its shadow is 20 meters?

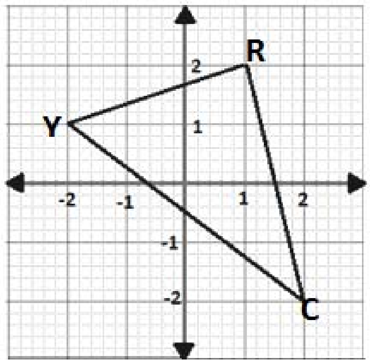
   A. 7.14 m B. 12.5 m C. 22.5 m D. 11.43 m

13. The smallest square of the grid you made on your original picture is 8 cm. If you enlarge the picture on a 18 cm grid, which of the following is NOT true?

   I. The new picture is 225% larger than the original one.
   II. The new picture is two and one-fourth times larger than the original one.
   III. The scale factor between the original and the enlarged picture is 4:9.

A. I, II, and III B. I and II C. II and III D. III only

14. You would like to enlarge ΔYRC by dilation such that the scale factor is 5. Which of the following is NOT the coordinates of a vertex of the enlarged triangle?



A. $(-10, 5)$ B. $(5, 10)$ C. $(10, -10)$ D. $(-10, 10)$

15. A document is 75% only of the size of the original document. If you are tasked to convert this document back to its original size, what copier enlargement settings will you use?

A. 135% B. 133% C. 125% D. 120%

16. You would like to put a 12 ft by 10 ft concrete wall subdivision between your dining room and living room. How many 4-inch thick concrete hollow blocks (CHB) do you need for the subdivision? Note that:

Clue 1: the dimension of the face of CHB is 6 inches by 8 inches.

Clue 2: 1 foot = 12 inches

Clue 3:

$$\frac{\text{1 CHB}}{\text{Area of the face of CHB in sq. in.}} = \frac{\text{total no. of CHB needed}}{\text{Area of the Wall Subdivision in aq. in.}}$$

A. 300 pieces B. 306 pieces C. 316 pieces D. 360 pieces

ANSWER KEY							
1.	**A**	6.	**B**	11.	**C**	16.	**D**
2.	**C**	7.	**D**	12.	**C**	17.	**A**
3.	**D**	8.	**A**	13.	**A**	18.	**D**
4.	**B**	9.	**B**	14.	**B**	19.	**B**
5.	**A**	10.	**C**	15.	**A**	20.	**A**

TEACHING GUIDE

Module 7: Triangle Trigonometry

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of the basic concepts of trigonometry.

Performance Standard:

The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="2">SUBJECT: Math 9
QUARTER: Fourth Quarter
TOPIC: Triangle Trigonometry

LESSONS:
1. The Six Trigonometric Ratios: sine, cosine, tangent,secant, cosecant and cotangent
2. Trigonometric Ratios of Special Angles
3. Angles of Elevation and Angles of Depression
4. Application The Trigonometric Ratios
5. Laws of Sines and Cosines
6. Oblique Triangles</td><td colspan="2">LEARNING COMPETENCIES</td></tr>
<tr><td colspan="2">1. Illustrate the six trigonometric ratios: sine, cosine, tangent, secant, cosecant and cotangent.
2. Find the trigonometric ratios of special angles.
3. Illustrate angles of elevation and angles of depression.
4. Use trigonometric ratios to solve real-life problems involving right triangles.
5. Illustrate laws of sines and cosines.
6. Solve problems involving oblique triangles.</td></tr>
<tr><td rowspan="2">WRITERS:

GILDA GARCIA
ROSELLE A. LAZARO</td><td>ESSENTIAL UNDERSTANDING:

Students will understand that basic concepts of trigonometry are usefull in formulating and solving real-life problems with precision and accuracy.</td><td>ESSENTIAL QUESTION:

How can trigonometric ratios be used in formulating and solving real-life problems?</td></tr>
<tr><td colspan="2">TRANSFER GOAL:
Students will be able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy.</td></tr>
</table>

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to come up with in this module.

a. Definition of the six trigonometric ratios.
b. Exact values of trigonometric ratios involving special angles
c. Diagrams and solutions to real-life problems involving angles of elevation and depression
d. Application of the trigonometric ratios in solving real-life problems
e. The Laws of sines and cosines in solving problems involving oblique triangles.
f. Performance tasks where the basic concepts of trigonometry is applied.

Assessment Map

TYPE	KNOWLEDGE	PROCESS/ SKILLS	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	**Pre-Test** Naming and identifying sides and angles through illustrations	**Pre-Test** Determining the missing sides and angles on the given triangle using the trigonometric ratios	**Pre-Test** Solving problems involving trigonometric ratios.	**Pre-Test** Products and performances involving the basic concepts of trigonometry
Formative	Lesson 1 Quiz: Use drawings, pictures, diagrams to define and illustrate the six trigonometric ratios	Lesson 1 Quiz: Use the trigonometric ratios in determing the missing sides and angles of a right triangle	Lesson 1 Quiz: Describing and discussing when and how the six trigonometric ratios be used.	
	Lesson 2 Quiz: Deriving the exact values of the trigonometric ratios involving special angles	Lesson 2 Quiz: Evaluating expressions that involve the trigonometric ratios of special angles	Lesson 2 Quiz: Analizing and explaining clearly the concepts on the values of the trigonometric ratios of special angles.	

			Illustrating and solving problems that involve trigonometric ratios of special angles	
	Lesson 3 Quiz: Defining and illustrating angles of elevation and angle of depression		Lesson 3 Quiz: Solving real-life problems that involve angles of elevation and depression	Lesson 3 Doing the performance task using the trigonometric ratios involving the angles of elevation and angles of depression
Summative	**Post-Test** Naming and identifying sides and angles through illustrations	**Post-Test** Determining the missing sides and angles on the given triangle using the trigonometric ratios	**Post-Test** Solving problems involving trigonometric ratios.	**Post-Test** Products and performances involving the basic concepts of trigonometry
Self - Assessment				

Assessment Matrix (Summative Test)

<table>
<tr><th>Levels of Assessment</th><th>What will I assess?</th><th>How will I assess?</th><th>How will I score?</th></tr>
<tr><td>Knowledge
15%</td><td rowspan="4">The learner demonstrates understanding of the basic concepts of trigonometry.

Illustrate the six trigonometric ratios: sine, cosine, tangent, secant, cosecant and cotangent.

Determine the trigonometric ratios involving special angles.

Illustrate angles of elevation and angles of depression.

Use trigonometric ratios to solve real-life problems involving right triangles

Illustrate laws of sines and cosines.

Solve problems involving oblique triangles</td><td>Paper and Pencil Test

Items 1, 2 and 3</td><td>1 point for every correct answer</td></tr>
<tr><td>Process/Skills
25%</td><td>Items 4, 5, 6, 7, 8</td><td>1 point for every correct answer</td></tr>
<tr><td>Understanding
30%</td><td>Items 9, 10, 11, 12, 13, 14</td><td>1 point for every correct answer</td></tr>
<tr><td>Product/ Performance
30%</td><td>Items 15, 16, 17, 18, 19, 20</td><td>1 point for every correct answer</td></tr>
</table>

C. Planning for Teaching – Learning

Introduction:

This module covers the key concepts in triangle trigonometry. It involves five lessons, which are The Six Trigonometric Ratios, the trigonometric ratios of special angles, the angle of elevation and depression, The Laws of Sines and Cosine, the Oblique Triangle and their application to real life situation. To apply their knowledge in triangle trigonometry, a variety of activities is provided in this module.

In Lesson 1 of this module, the students will define the six trigonometric ratios through proper illustrations. They will also use these to determine the missing sides and angles of the given right triangle.

The students will learn how to determine the trigonometric ratios involving special angles in Lesson 2 of this module. They will also compute for the numerical values of trigonometric expressions involving special angles.

In Lesson 3 of this module, the students will distinguish between angle of elevation and angle of depression. Through the activities provided for them they will be able to define angles of elevation and depression using their own understanding to help them illustrate and solve simple word problems.

Lesson 4 focuses on application problems involving concepts of trigonometric ratios and angles of elevation and depression. Through the activities that are provided in this lesson, the students will be able to discern when and how to apply these concepts in solving word problems involving right triangles. Thus this lesson will help in the development of the students critical thinking skills.

Lesson 5 introduces students to the concept of oblique triangles. This lesson is divided into two, namely: Lesson 5.1 – The Law of Sines and Its Applications and Lesson 5.2 – The Law of Cosines and Its Applications. In Lesson 5.1, the students will know and understand how the Law of sines came about and how the trigonmetric ratio involving sines can be utilized in finding areas of triangles through some exploratory activities. Lesson 5.2 offers illustrative examples on when and how the Law of Cosines can be utilized in solving oblique triangles. In both lessons, students will solve word problems that involve situations in real-life.

In all lessons, students are given a chance to use their prior knowledge and skills to help them through in this module. Variety of activities also given to process their knowledge and skills acquired, to deepen their understanding and transfer these to real-life situations.

As an introduction to this module, prior to the discussion you have asked the students to bring pictures of mountains, buildings, ships, airplanes, etc., unless you decided to bring these pictures yourself. Let them show/see the pictures and ask the following questions:

1. Have you ever wondered how towers and buildings are constructed?
2. How do you determine the distance traveled as well as the height of an airplane as it takes off?
3. What about determining the height of the mountain?

You may add some more questions if necessary, depending on the pictures they brought. Encourage the students to find out their answers to these questions and to identify the real-life applications of basic concepts of trigonometry through this module.

Objectives:

After the students have gone through the lessons in this module, they are expected to:

1. illustrate the six trigonometric ratios;
2. apply trigonometric ratios to solve the right triangle given:
    a. the length of the hypotenuse and length of one leg
    b. the length of the hypotenuse and one of the acute angles
    c. the length of one leg and one of the acute angles
    d. the length of both sides
3. determine the trigonometric ratios involving special angles;
4. compute the numerical values of trigonometric expressions involving special angles;
5. illustrate angles of elevation and depression;
6. solve problems involving angles of elevation and depression;
7. use the trigonometric ratios in solving real-life problems involving right triangles;
8. illustrate the laws of sines and cosines;
9. solve problems involving oblique triangles

Pre – Assessment:

Allow the students to take the Pre-assessment first before studying the lessons. This will check their prior knowledge, skills and understanding regarding the concepts related to trigonometry. Remind the students about their goal in completing the lessons in this module.

Answer Key

1. D	6. C	11. A	16. C
2. C	7. A	12. D	17. A
3. A	8. B	13. B	18. C
4. B	9. A	14. C	19. C
5. D	10. C	15. B	20. B

LEARNING GOALS AND TARGETS:

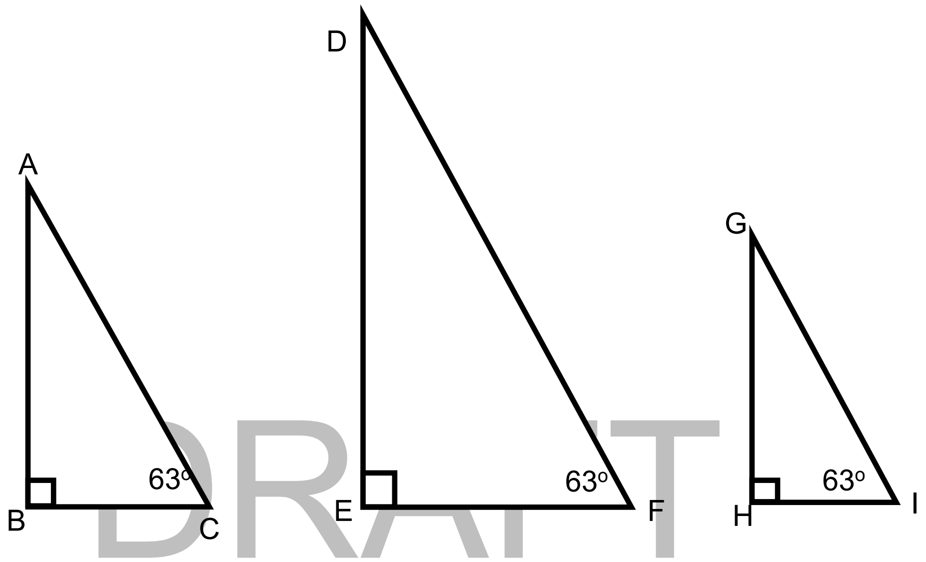
The students are expected to demonstrate understanding of the basic concepts of trigonometry. They are also expected to apply the concepts of trigonometric ratios to formulate and solve real life problems with precision and accuracy.

LESSON 1 : THE SIX TRIGONOMETRIC RATIOS: SINE, COSINE, TANGENT, SECANT, COSECANT, AND COTANGENT

What to KNOW

Let the students recall the different concepts they have learned about triangles. Guide them to do Activity 1 for them to define and illustrate the six trigonometric ratios.

Activity 1. Triangle of Different Sizes (answers of the students may vary)



Measures	in ΔABC	in ΔDEF	in ΔGHI
leg opposite the 63° angle	4	6	33
leg adjacent the 63° angle	2.3	3.5	18
Hypotenuse	4.6	6.9	38
$\frac{\text{leg opposite the } 63° \text{ angle}}{\text{hypotenuse}}$	$\frac{4}{4.6}$ = 0.87	$\frac{6}{6.9}$ = 0.87	$\frac{3.3}{3.8}$ = 0.87
$\frac{\text{leg adjacent to } 63° \text{ angle}}{\text{hypotenuse}}$	$\frac{2.3}{4.6}$ = 0.5	$\frac{3.5}{6.9}$ = 0.5	$\frac{1.8}{3.8}$ = 0.47
$\frac{\text{leg opposite the } 63° \text{ angle}}{\text{leg adjacent to } 63° \text{ angle}}$	$\frac{4}{2.3}$ = 1.74	$\frac{6}{3.5}$ = 1.71	$\frac{3.3}{1.8}$ = 1.83

In this activity, students can recall the concepts of a right triangle. They will also discover the AA similarity theorem for triangles, that is all right triangles with a given acute angle measure are similar. Give further discussion on the different ratios.

That is, $\frac{AB}{AC} = \frac{DE}{DF} = \frac{GH}{GI}$. This ratio is named sine 63° ≈ 0.87. Note that the ratio is constant for the given angle regardless of the size of the triangle. The same is true for the other two ratios. These three ratios are the primary trigonometric ratios of 63°, namely: sine, cosine and tangent, respectively. Give further discussion on their output.

Activity 2 will help the students to validate the definition of the primary trigonometric ratios they have identified in the first activity. Their output of this activity will lead them to the discussion of the six trigonometric ratios.

Activity 2. Measuring and Calculating (Answers may vary subject to measuremet error. You may consider answers that are within 0.1 units of the values in the table.)

	Lengths of sides			Trigonometric ratios		
Angle (°)	Opposite	Adjacent	Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
a) 20	2.5	6.8	7.2	0.35	0.94	0.37
b) 33	3.4	5.9	6.8	0.50	0.87	0.58
c) 42	5.9	6.3	8.6	0.69	0.73	0.94
d) 71	6.2	2.2	6.6	0.94	0.33	2.82
e) 39	4.0	5.1	6.6	0.61	0.77	0.78
f) 58	6.9	4.2	8.0	0.86	0.53	1.64

Let the students read and understand the notes of the six trigonometric ratios and study the given examples. Guide them if needed. You can still give more examples for their further understanding of the lesson. Also discuss with them the importance of the use of scientific calculator in determining the values of the trigonometric ratios and their equivalent angle measure.They may proceed to the next activity when they are ready.

Try this: (Answer key)

1.
   a. 0.09
   b. 0.12
   c. 0.70
   d. 0.42
   e. 0.42

2.
   a. 17° 48'
   b. 48° 31'
   c. 63° 42'
   d. 108° 20'
   e. 35° 14'

3.
   a. $\theta = 32^{\circ}$
   b. $\theta = 60^{\circ}$
   c. $\theta = 61^{\circ}$
   d. $\theta = 15^{\circ}$
   e. $\theta = 36^{\circ}$

4.
   a. $\theta = 64^{\circ}\ 9'$
   b. $\theta = 89^{\circ}\ 15'$
   c. $\theta = 43^{\circ}\ 46'$
   d. $\theta = 59^{\circ}\ 56'$
   e. $\theta = 66^{\circ}\ 30'$

What to Process

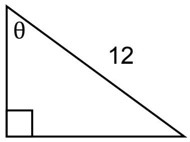
To check students' understanding on the six trigonometric ratios, you can do the Bingo game. After this activity you can ask the processing questions, if ever they still have questions regarding the lesson then give additional input through examples.

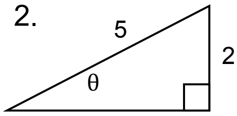
Activity 3. Surfing the Safari for Trigonometry

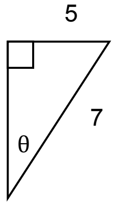
In this activity the students will apply the trigonometric ratios to solve the missing sides and angles of a right triangle. After performing the activity ask them the processing questions that follow.

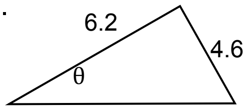
These are the questions inside the Bingo Board.

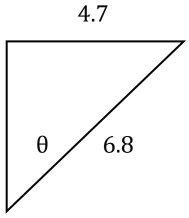
Find the size of the θ to the nearest degree.

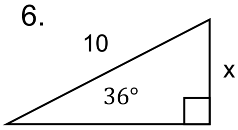
1.


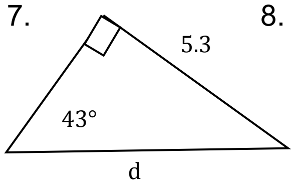
2.


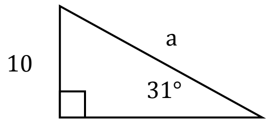
3.


4.


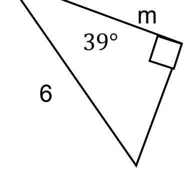
5.


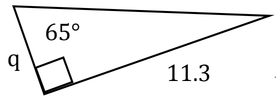
6.


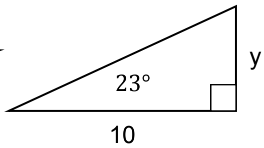
7.


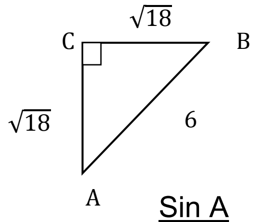
8.


Find the missing side. Give your answer to 1 decimal place.

9.


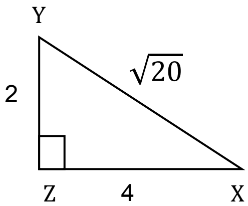
10.


11.


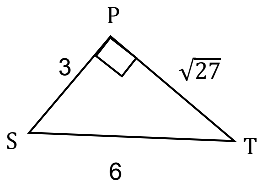
12.


<u>Sin A</u>

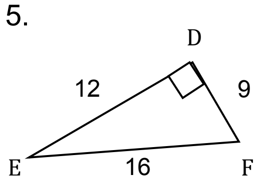
Find the value of the following ratios. (Rationalize the denominator of your answer.)

13.


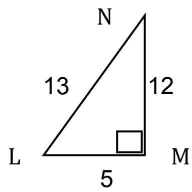
<u>tan Y</u>

14.


<u>Cos S</u>

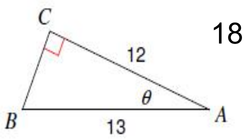
15.


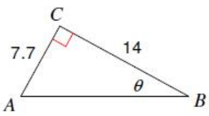
<u>Tan E</u>

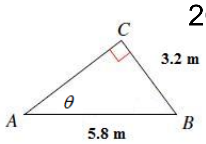
16.


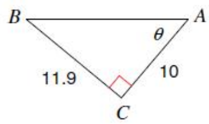
<u>Cos L</u>

Find the indicated angle measures to the nearest degree.

17.


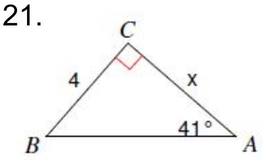
18.


19.


20.


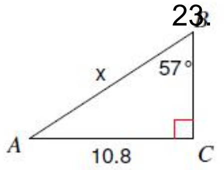
For each right triangle below, find the indicated side length (round to the tenths place).

21.

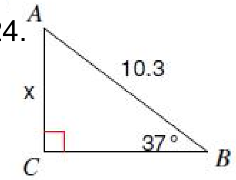


22.

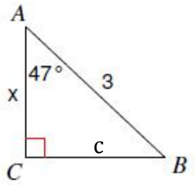
23.



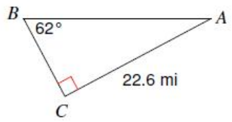
24.

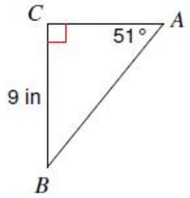


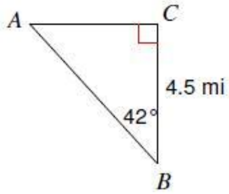
25.



Find all missing side lengths of the triangle. (round answers to the tenths place).







Answer Key:

1. 54°	6. 5.9	11. 4.2	16. $\frac{5}{13}$	21. 4.6
2. 24°	7. 7.8	12. $\frac{\sqrt{2}}{2}$	17. 23°	22. 12.9
3. 46°	8. 19.4	13. 2	18. 29°	23. 6.2
4. 37°	9. 4.7	14. $\frac{1}{2}$	19. 33°	24. 2.0
5. 44°	10. 5.3	15. $\frac{3}{4}$	20. 50°	25. 26.6

Activity 4. Try Me!

One of the important skills in mathematics is to solve accurately. This activity will develop the students' skills in determining the value of the missing sides and angles using the trigonometric ratios.

Ask the students to form groups of six. Assign each group the set of problems provided in the module and answer the processing questions that follow. In this activity, students may develop cooperative learning. To discuss their answers ask them the processing questions. Give further explaination for their queries or clarrification if in case there are some.

Answer key:

1. a = 37 sin 15°
2. $b = \frac{13}{\tan 76°}$
3. c = $\frac{10}{\sin 49°}$
4. b = $\frac{21.2}{\tan 71°}$
5. c = $\frac{13}{\cos 16°}$
6. c = $\frac{11}{\sin 19°}$
7. b = $\sqrt{16^2 - 7^2}$
8. a = $\sqrt{20^2 - 10^2}$
9. $A = tan^{-1}(7/12)$
10. $A = cos^{-1}(8/12)$

1.a = 15.5
∠A = 42° 21’
∠B = 47° 39’
2.b = 14.4
∠A = 25° 57’
∠B = 64° 3’
3.a = 17.3
∠A = 60°
∠B = 30°
4.a = 11.5
∠A = 62° 31’
∠B = 27° 29’
5.b = 5
∠A = 67° 23’
∠B = 22° 37’

1.∠B = 75°
a = 9.6
b = 35.7
2.∠ A = 26°
a = 8.4
b = 17.3
3.∠B = 75°
a = 6.5
b = 24.1
4.∠B = 45°
a = 11.3
b = 11.3
5.∠A = 34° 5’
a = 8.9
b = 13.3

1.∠B = 14°
b = 3.2
c = 13.4
2.∠B = 68°
a = 8.9
c = 23.7
3.∠A = 60°
a = 19.1
c = 22
4.∠A = 72°
b = 5.9
c = 18.9
5.∠B = 13°
a = 181.9
c = 186.7

1.∠A = 36° 57’
∠B = 53° 3’
c = 26.3
2.∠A = 30° 15’
∠B = 59° 45’
c = 13.9
3.∠A = 15° 57’
∠B = 74° 3’
c = 7.3
4.∠A = 60°
∠B = 30°
c = 3.5
5.∠A = 45°
∠B = 45°
c = 353.6

What to Reflect or Understand

Ask the students to check their understanding on how the six trigonometric ratios are used. Let them think, deepen and test this understanding about the lesson by doing the next activity.

Activity 5. Use! List! Explain!

Let the students use a figure or diagram to list the six trigonometric ratios and explain how to determine which ratio to use when solving for an unknown measure of a right triangle. In this activity, students will develop their critical thinking skill.

What to Transfer

Introduce to the students how to make a clinometer and how to use it. This will help them accomplish the next activity. A clinometer is a device used to measure angles of elevation or depression.

Activity 6. The Clinometer

Let the students do this activity. Guide them so that they can properly and accurately use the clinometer in determining the height of an object. Discuss the processing questions that follow.

Activity 7. Trigonome - Tree

This activity is an opportunity for the students to demonstrate their understanding of the six trigonometric ratios. They will be asked to measure the height of a tree in their community using a trigonometric ratio and finding the value of the other ratios as well. Give them the criteria on how are you going to rate their work. A sample rubric is given below.

Assessment Rubric

Criteria	Understanding of concept, can apply accurately (4)	Understanding of concept, can apply but commit errors in calculation (3)	Understanding of concept, but not able to apply (2)	No understanding (1)
Able to draw figure or illustration of the problem.				
Able to apply knowledge of trigonometric ratios.				

SUMMARY/SYNTHESIS/GENERALIZATION

This lesson was about the six trigonometric ratios. The lesson provided the students with a variety of activities to help them illustrate and define the six trigonometric ratios. Students also learned how to use them in finding the missing sides and angles of a right triangle and applied them to real-life situations. Their knowledge in this lesson will help them understand the next topic, which is, the trigonometric ratios involving special angles.

Lesson 2: Trigonometric Ratios of Special Angles

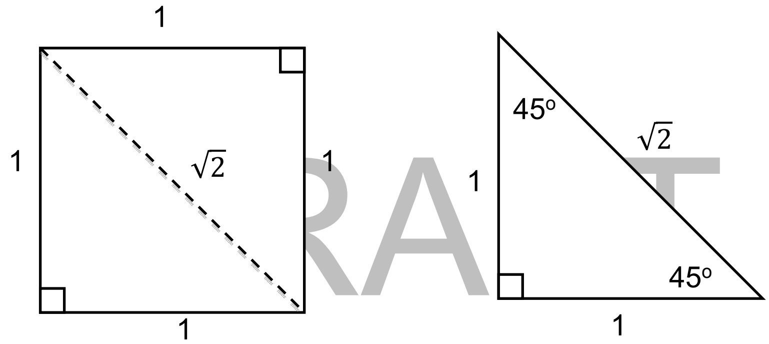
What to Know

In this lesson students will use the concepts they have learned previously to evaluate trigonometric ratios involving special angles. Ask them to perform the succeeding activities to develop mastery of this topic.

Activity 1. Special Triangles of Exact Values

Start the lesson with some manipulative activities. This will enable the students to discover the exact values of the trigonometric ratios of special angles. Ask them to discuss their work by answering the following questions: How do you find the activity? What have you discovered from the activity? Do you think this will be useful as you proceed to the next activity? Why?

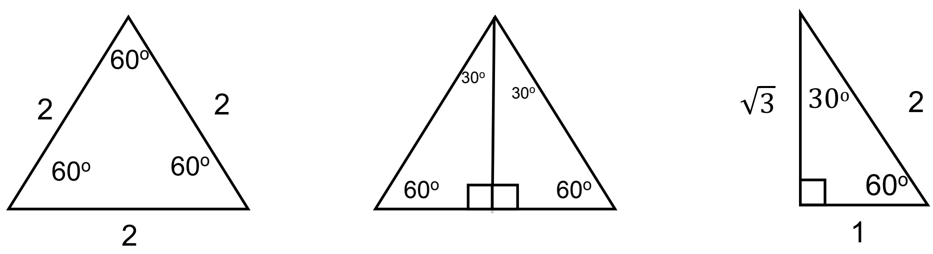
Case 1. Answer Key:



The SOH – CAH – TOA for 45°.

sin 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ cos 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ tan 45° = $\frac{1}{1} = 1$

Case 2.



sin 30° = $\frac{1}{2}$ cos 30° = $\frac{\sqrt{3}}{2}$ tan 30° = $\frac{\sqrt{3}}{3}$

sin 60° = $\frac{\sqrt{3}}{2}$ cos 60° = $\frac{1}{2}$ tan 60° = $\sqrt{3}$

Activity 2. Compare my Size!

In this activity, the students will measure and compare the angles of the given triangles. This will validate the concepts they have learned in the previous activity. After doing the activity, they will write the mathematical concepts they have discovered and this will develop their critical thinking skills.

Guide the students in studying the key concepts about the lesson trigonometric ratio of special angles. Provide more examples to ensure students' understanding of the lesson.

Activity 3. Practice Makes Perfect

Develop the students' skills in determining the missing length of the sides and measure of the angles in the given right triangle. They will use the concepts of trigonometric ratios involving special angles they have learned in the prevoius activity.

Answer key:

1. t = 8
   r = $8\sqrt{2}$
2. i = 2
   g = $\sqrt{2}$
3. o = $60\sqrt{2}$
4. n = 15
   o = 15
5. m = $3\sqrt{2}$
   e = 3
6. t = $\sqrt{10}$
7. r = 20
   i = $20\sqrt{3}$
8. c = 2
   r = 4
9. a = $5\sqrt{3}$
   t = 5
10. i = 24
    o= $12\sqrt{3}$
11. s = 2
    x = 4
12. o = 9
    f = 18
13. s = 7
    p = 14
    e = 7
    c = $7\sqrt{3}$
14. i = 6
    a = $2\sqrt{3}$
    l = $6\sqrt{2}$
    e = 6
15. a = $10\sqrt{3}$
    n = $5\sqrt{3}$
    g = 15
    l = 5

What to Process

In this section, the students will study how to find the trigonometric ratios involving special angles. The succeeding activities will help the students to deepen their understanding of the concepts learned in the previous lesson.

Activity 4. What Makes You Special?

In this activity, ask the students to determine the values of the six trigonometric ratios involving the special angle 30^o, 45^o and 60^0. Then let them answer the questions that follow for discussion.

Answer key:

TRIGONOMETRIC RATIOS OF THE ANGLES						
θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
30^O	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45^O	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60^O	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

What to Reflect or Understand

In this section, the students will learn the process of determining the values of trigonometric ratios of special angles even without the use of the trigonometric table or calculator. Remind them that the two special right triangles are the 30º – 60º – 90º triangle and the isosceles (45º – 45º – 90º) triangle. The angles 30º, 60º and 45º are the most frequently used angle measures in many problems and are referred to special angles.

Provide them with a variety of examples to facilitate their understanding of the lesson. To help them memorize the values, refer them to the URL below.
http://function-of-time.blogspot.com/2009/08/trig-reference-angle-cheat-hand.html

Activity 5. You Complete Me!

To check the students' understanding about this lesson, let them answer this activity. Guide them if needed.

Answer key:

1. sin 60º
2. 0
3. ½
4. tan 30º
5. 1
6. 7/4
7. tan 45º or 1
8. sec 45º
9. cot 30º
10. $\frac{\sqrt{2}}{4}$
11. ½
12. csc 60º
13. $6\sqrt{2}$
14. sin 30º or 1/2
15. 9/4

What to TRANSFER

In this section, the students will apply their understanding of the lesson to real – life situations. They will be given a task to demonstrate their learning.

Activity 6. Application

The students will learn how to use the lesson learned in the different cases. This will develop their critical thinking skills and reasoning ability. Students will perform this activity by group.

Answer key:

Case 1. Error Analysis. Students will analyze the given situation based on what they have learned about trigonometric ratios involving special angles.

Case 2. Formulating Own Problem. Students will write a real-life problem using the given information and applying the concepts they have learned.

Case 3. House Repair. In this case, students will experience how this lesson can be used and applied to daily life situation. They will also realize how important these concepts are to real-life.

Case 4. Geometry in 3 Dimensions. Students in this case will connect the concepts learned to Geometry problems.

Case 5. In this case, students will see the connections of the lesson to some situations in real-life and realize its importance.

SUMMARY/SYNTHESIS/GENERALIZATION

This lesson was about the trigonometric ratios of special angles. The students have illustrated the special acute angles 30^o, 45^o and 60^o and determined the values. They have also studied how to evaluate the trigonometric ratios of these special angles. Students were also given the chance to do some activities that applies their learning in this lesson. Understanding this lesson and other previously learned concepts will help the students to facilitate their learning in the next lesson; The angle of elevation and depression.

Lesson 3: Applications: Angles of Elevation and Angle of Depression

What to KNOW

Start this lesson by asking the following questions: "Suppose you are on top of a mountain, looking down at a certain village, how will you directly measure the height of the mountain? An airplane is flying a certain height above the ground, is it possible to find directly the distance along the ground from the airplane to an airport?" The trigonometric ratios they have learned in the previous lesson will help them answer these questions. Let the students perform the succeeding activities for them to apply these concepts in solving real – life problems.

Activity 1. Look Up! Look Down!

The students learn best when they discover the concepts instead of giving to them. This activity will help them discover, define and illustrate angles of elevation and angles of depression. Let them explore doing the activity even outside the room. As a facilitator in the classroom, try to guide them when they are about to lose track during the activity. Ask the students the processing questions for further discussion. Students' answers in this activity may vary.

You can provide them with more examples illustrating angle of elevation and angle of depression to further their understanding of the concepts.

What to Process

In this section, the students will learn how to illustrate problems that involve angle of elevation and angle of depression. They are now ready to use this concept in solving such problems. They will be provided with an opportunity to explore and solve problems in daily life through the following activities.

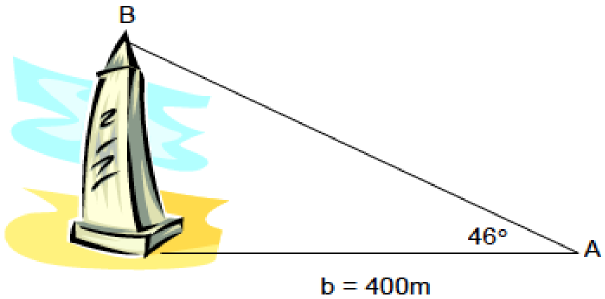
Activity 2. Identify Me!

The students will identify the angle of elevation, angle of depression and the line of sight using the given figures. This will enable the students to reflect on and revise their understanding of angles of elevation and depression. Guide them if needed. Facilitate discussion by using the processing questions that follow.

Discuss the examples provided in the module. Explain further how to solve word problems involving angle of elevation and angle of depression. Guide them in answering the "Try this out" portion. You can give more examples if still needed.

Try this out! Answer key

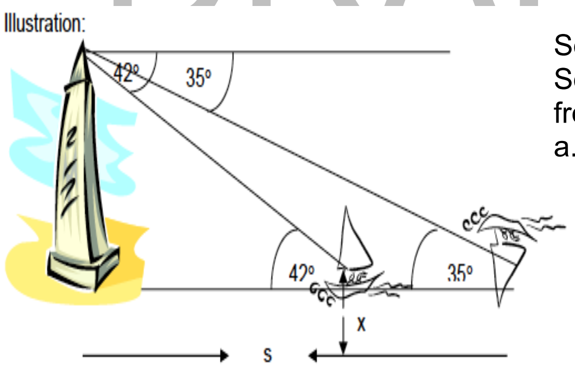
1.



Solution:
Let x be the height of the tower in meters.
Use the mnemonic TOA. In the triangle above, x is the opposite side and b is adjacent sede of $\angle$A.

$\tan 46^o = x/400$
$x = 400 \tan 46^o$
$x = 400\ (1.0355)$
$x = 414.2$ m

2.



Solution:
Solve for the distance of each boat from the light house.

a. For the farther boat

$\tan 35^o = \frac{48.8}{s}$

$s \tan 35^o = 48.8$

$s = \frac{48.8}{\tan 35}$ - equation 1

b. For the nearest boat

$\tan 42^o = \frac{48.8}{s-x}$

$s - x = \frac{48.8}{\tan 42}$

$s = x + \frac{48.8}{\tan 42}$ - equation 2

Equate expressions for s from equation 1 and equation 2

$$\frac{48.8}{\tan 35°} = x + \frac{48.8}{\tan 42°}$$

$$\frac{48.8}{\tan 35°} - \frac{48.8}{\tan 42°} = x$$

$x = 69.69 – 54.20$

$x = 15.49$ m

Activity 3. Draw My Problem!

In this activity, let the students practice their skill in illustrating the information presented in the problem through drawings. This is to prepare them for solving similar word problems in the succeeding activities. Guide them if needed. Ask them the processing questions that follow for discussion. Suggested rubric to rate their output is given below.

Criteria	Understanding of concept, can apply accurately (4)	Understanding of concept, can apply but commit errors in calculation (3)	Understanding of concept, but not able to apply (2)	No understanding (1)
Able to draw figure for the given word problem and explain.				

What to Reflect or Understand

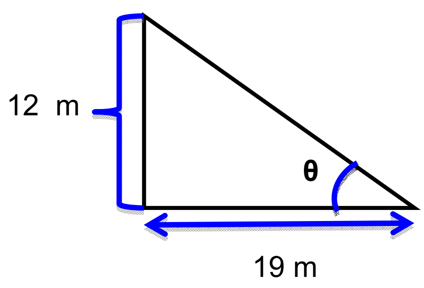
In this section, let the students solve problems involving the angle of elevation and angle of depression. They will also use the concepts that acquire in the previous activity.

Activity 4. Illustrate and Solve Me!

Let the students perform Activity 3. The students will illustrate and solve the set of real-life problems using the concepts of angles of elevation and depression. This will enable them to further validate and deepen their understanding of the lesson. Ask the students the processing questions that follow. You can also assess their work using the suggested rubric below.

Answer key

1.



$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan \theta = \frac{12}{19}$

$\tan \theta = 0.6316$

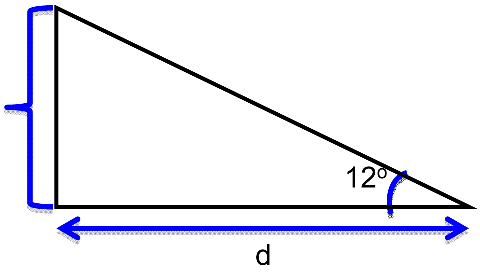
$\theta = \tan^{-1} 0.6316$

$\theta = 32.28^{\circ}$

$\theta = 32^{\circ}\, 17'$

∴ The angle of elevation of the sun is $32^{\circ}\, 17'$

2.



$\tan 12^\circ = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 12^\circ = \frac{92}{d}$

$d \tan 12^\circ = 92$

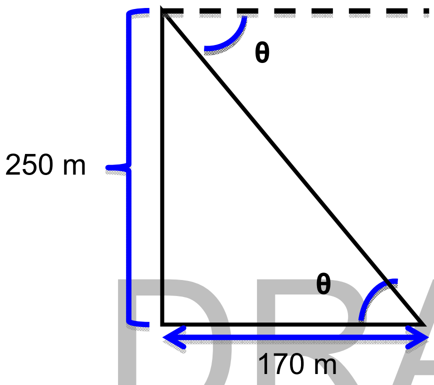
$d = \frac{92}{\tan 12^\circ}$

$d = \frac{92}{0.2126}$

$d = 432.83$ m

∴ The distance of the boat from the base of the hill is 432.83 meters.

3.



$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan \theta = \frac{250}{170}$

$\tan \theta = 1.47$

$\theta = \tan^{-1} 1.47$

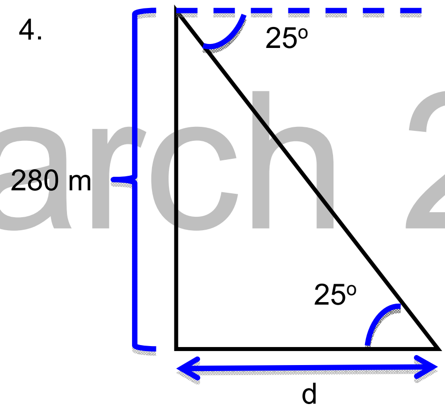
$\theta = 55.78^\circ$

$\theta = 55^\circ 47'$

∴ $55^\circ 47'$

The angle of depression of the rock from the top of the control tower is $55^\circ\ 47'$.

4.



$\tan 25^\circ = \frac{opposite}{adjacent}$

$\tan 25^\circ = \frac{280}{d}$

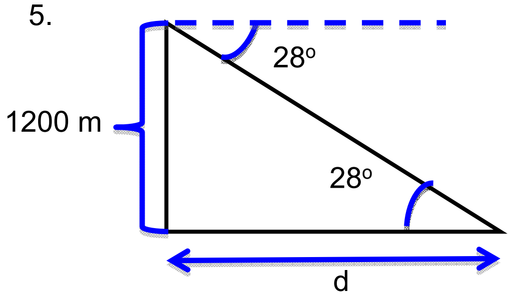
$d = \frac{280}{\tan 25^\circ}$

$d = \frac{280}{0.4663}$

$d = 600.47$ m

∴ The distance of the boat from the cliff is 600.47 meters.

5.



$\tan 28^\circ = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 28^\circ = \frac{1200}{\tan 28^\circ}$

$d = \frac{1200}{0.5317}$

$d = 2256.87$ m

∴ The distance of the rock from the plane is 2256.87 m meters.

Assessment Rubric

Criteria	Understanding of concept, can apply accurately 4	Understanding of concept, can apply but commit errors in calculation 3	Understanding of concept, but not able to apply 2	No understanding 1
Able to draw figure or illustration of the problem.				
Able to apply knowledge of trigonometric				

What to Transfer

In this section, ask the students to apply their understanding of angles of elevation and depression through an activity that reflects meaningful and relevant problems or situations. Their output will be rated according to the rubrics given above.

Activity 5. What can I learn from my SHADOW?

In this activity, students will use trigonometric ratios to determine the angle of elevation to the sun and the height of the unknown object. Student's groups will use the height of a person and the length of his or her shadow to find the angle of elevation to the sun. Then the students will use that angle of elevation and one length of an unknown object's to find the height of that object. Let them accomplish the provided activity sheet in the module. After they have performed the given task you may ask them the processing questions that follow.

SUMMARY/SYNTHESIS/GENERALIZATION

This lesson was about the angles of elevation and angles of depression and how they are illustrated in real life. The students have been given a chance to do activities that helped them identify and define the angle of elevation and angle of depression through illustrations. Also students learned that solving problems requires them to draw a detailed diagram to help them visualize the given information. These concepts that they have just learned will help them get through to the next lesson, the law of sine s and cosines.

Lesson 4

Word Problems Involving Right Triangles

What to Know

Provide the students the opportunity to recall the different trigonometric ratios, angle of elevation, and angle of depression since these concepts are essential in solving word problems involving right triangles.

Activity 1 elicits prior knowledge of the learners on trigonometric ratios. Results of the activity assess the learners' knowledge on the concept of trigonometric ratios. They should have a firm background regarding the concept to be able to solve problems on right triangles successfully.

Answer in Activity 1: *The Perfect Match*

1. e 2. a 3. b 4. d 5. c

Activities 2 and 3 activate the learners' prior knowledge of the concepts of angle of elevation and angle of depression. In Activity 2, the learners are asked to identify whether the given picture illustrates an angle of elevation or an angle of depression. In Activity 3, a real-life situation is provided and you will ask students to discern which should be used to solve the given problem situation. Is it angle of elevation or angle of depression?

Answers in Activity 2: *Who am I?*

1. angle of elevation
2. angle of depression
3. angle of depression
4. angle of elevation
5. angle of elevation

Answers in Activity 3: *Elevation or Depression*

a. angle of depression for both problems within b and c because according to the given situation the observer (boy) is above the object being observed (dog).
b. 4.8 m
c. 4.4 m

What to Process

Let the learners realize the importance of the concepts of angle of elevation and angle of depression in solving real-life problems involving right triangles by letting them do the next two activities.

In Activity 4, illustrations depicting real-life situations involving the concepts of angle of elevation and depression are provided. Ask the students to use these illustrations to find what is asked in each item.

Answers in Activity 4: *Find Me*

1. $\angle A = 26.6°$ 2. 10.2 km 3. 12.1 m 4. 8.2 m 5. 9.8 m

Answers in Activity 5: *Let's Work-it-Out*

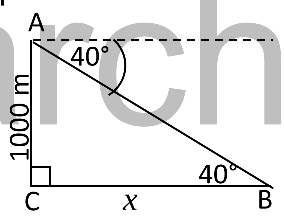
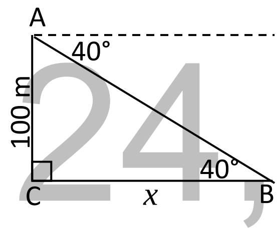
1. 3 m 2. 14.36 m 3. 8.44 m 4. 3.94 m 5. 64.83°

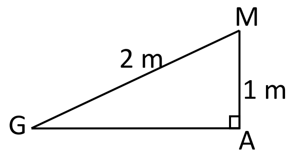
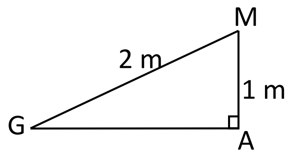
What to Reflect or Understand

Assess the learners' understanding of the concepts of angle of elevation, angle of depression, and trigonometric ratios in general by letting them do Activity 6 and Activity 7.

Activity 6 will provide your students opportunities for critical thinking. In this activity two real-life problems are presented. A table containing suggested solutions to the given problems is presented. Instruct your students to study the suggested solutions and look for errors. Ask them to write their solution on the column with the column heading "My Solution". Their explanation why their solution is correct should be written on the column with the column heading "My Explanation".

Answers in Activity 6: *Where have I Gone Wrong?*

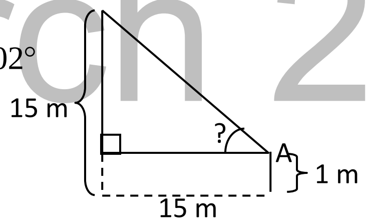
Suggested Solution	My Solution	My Explanation
Problem 1: Let A represent the airplane and B the airport.  $\tan 40° = \frac{opposite}{adjacent}$ $\tan 40° = \frac{x}{1000}$ $x = 1000 \tan 40°$ $= 1000(0.8391)$ $x = 839.1$ ∴ *The horizontal distance between the airplane and the airport is* ***839.1 m.***	 $\tan 40° = \frac{opposite}{adjacent}$ $\tan 40° = \frac{100}{x}$ $x = \frac{100}{\tan 40°} = \frac{100}{0.8391}$ $x = 119.2\text{ m}$ ∴ *The horizontal distance between the airplane and the airport is* ***119.2 m.***	(*Note: Explanation may vary from student to student.*)

<table>
<tr>
<td>Problem 2:

$\sin G = \dfrac{adjacent}{hypotenuse}$
By Pythagorean Theorem,
$GA = \sqrt{GM^2 - MA^2}$
$GA = \sqrt{2^2 - 1^2} = \sqrt{3}$
$\sin G = \dfrac{\sqrt{3}}{2}$
$G = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$
$G = \sin^{-1}(0.8660)$
$\therefore m\angle G = 60°$</td>
<td>
$\sin G = \dfrac{opposite}{hypotenuse}$
$\sin G = \dfrac{1}{2} = 0.5$
$G = \sin^{-1} 0.5$
$\therefore m\angle G = 30°$</td>
<td></td>
</tr>
</table>

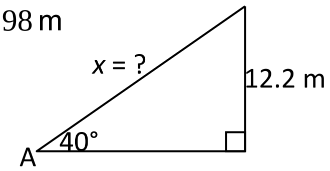
In Activity 7, real-life problems involving right triangles are provided to the students. This time, there are no suggested solutions. They have to solve the problems by themselves using the concepts they have learned.

Answers in Activity 7: *Problem Solving*

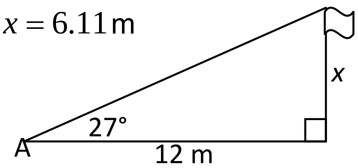
1. $A = 43.02°$



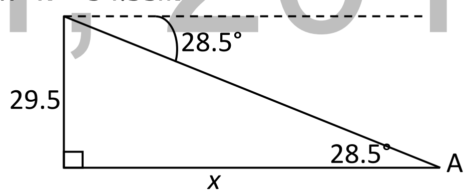
2. $x = 18.98$ m



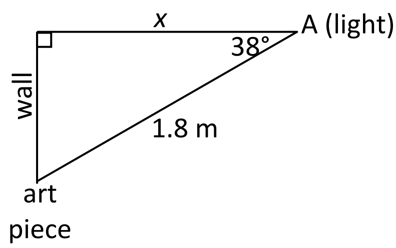
3. $x = 6.11$ m



4. $x = 54.33m$



5. $x = 1.4$ m



What to Transfer

Learning is not only confined in the four walls of the classroom. Let your students experience learning at its best by allowing them to perform outdoor activities where they can apply what they have learned inside the classroom to situations in the outside world.

Answers in Activity 8 *(Go the Distance)* may vary from school to school.

Activity 9 *(Formulating Real-Life problems)* shall be rated by both the students and the teacher. You may use the following rubrics 1 and 2 to rate your students in Activity 9 or you may device your own rubrics for rating.

It is suggested that Rubric 1 be used by the students and Rubric 2 be used by the teacher. Final score of the student for the activity shall be the sum of the average rating of the group (from Rubric 1) and the rating of the teacher (from Rubric 2).

Rubric 1:

Directions: Encircle the suitable number after each sentence. (Every member of the group should rate their group as a whole. The rating of the group shall be based on the average of all total scores as rated by the group members.)

Legend: 4 – Exemplary 2 – Good
3 – Very Good 1 – Requires Improvement

1.	All members take part in the group activity.	4	3	2	1
2.	Members support and encourage others in the group.	4	3	2	1
3.	Group members do not find fault in one another when drawbacks occur during the activity.	4	3	2	1
4.	Group members are open to comments and criticism.	4	3	2	1
5.	Group members perform well as a whole.	4	3	2	1
Add all encircled numbers for Total Score ____________ (out of 20)					

Rubric 2:

CRITERIA	4	3	2	1
Formulation of the Problem	Statements are well organized making the problem easy to comprehend. Uses correct grammar and spelling.	Organization of statements is somewhat jumpy making the problem less easy to comprehend. Most grammar and spelling are correct.	Poor organization of the statements making the problem hard to comprehend. With few errors in grammar and spelling.	Not organized; not comprehensible. Frequent grammar and spelling errors.
Solving the Problem	Provides appropriate process for solving the problem; without errors in computations.	Utilizes moderately correct process with minimal omission or error in procedure.	Employs partial appropriate process but with considerable error in procedure.	Uses entirely incorrect plan.
Presentation of the Output	Student's work shows full grasp and use of the concept taught. Student's output communicates clear thinking with the use of some combination of written, symbolic or visual means.	Student's work shows essential grasp of the concept taught. Student's output in large communicates clear thinking.	Student's work shows partial but limited grasp of the concept taught. Student's output is incomplete, somewhat misdirected, or not clearly presented.	Student's work shows little or no grasp of the concept taught. Student's output is barely, if at all, comprehensible.

Summary:

Lesson 4 is all about solving word problems involving right triangles. The different trigonometric ratios, angle of elevation and angle of depression were recalled in this lesson since these are vital concepts which you must understand to successfully solve right triangles. Activities that will help you remember the difference between angles of elevation and angles of depression were included in this lesson. Opportunities for cooperative learning where you can develop team work and application problems using real-life situations that will help you understand how the concept of trigonometry can be applied in the real world are likewise provided through some of the activities in this lesson. The concept of right triangles discussed in this lesson can serve as springboard for the next lesson which is solving oblique triangles.

Note:
Please change the highlighted page number depending upon where said illustrations are found after finally collating all the LMs.

Lesson 5 Oblique Triangles

Introduction:

To start the lesson on oblique triangles, ask the students to look at the illustrations found on page (49) of the LM. Ask the students if there are triangular patterns in the given illustrations. If they say "yes", ask them if they see right angles in the triangular patterns.

Next, ask them to observe the surroundings and perform the first activity.

Answers in Activity 1:

1. Items contained in the list prepared by the students may vary, from room to room, depending upon how the room was structured.

2. The triangles do not contain a right angle.

3. The triangles may be classified as acute triangles or obtuse triangles. The basis for classification would be the kind of angles that they contain.

4. Your students may state the definition of oblique triangles in several ways. What is important is that they realize that all oblique triangles do not contain a right angle.

5. Oblique triangles may be classified into two. These are: acute triangles and obtuse triangles.

Answers in Oral Exercises:

1. 105°; obtuse triangle
2. 90°; neither
3. 85°; acute triangle
4. 60°; 60°; acute triangle
5. 55°; acute triangle
6. 110; obtuse triangle
7. 65°; neither
8. 25°; obtuse triangle
9. 92°; obtuse triangle
10. 80°; acute triangle

Lesson 5.1 The Law of Sines and Its Applications

What to Know

It is important that students understand how the Law of Sines can be applied in solving problems involving oblique triangles. But knowing when to apply the Law is equally important to ensure success in solving a particular problem.

Activity 1 will serve as a springboard for Activity 2 which is aimed at discovering the Law of Sines.

Answers in Activity 1:

1. Any of the following answers is correct –
   - ✓ All triangles are oblique.
   - ✓ 3 parts of each triangle are given, while the other 3 are missing.
   - ✓ In some triangles, the measures of two angles and one side are given. In others, the measures of two sides and an angle opposite one of these sides are given.

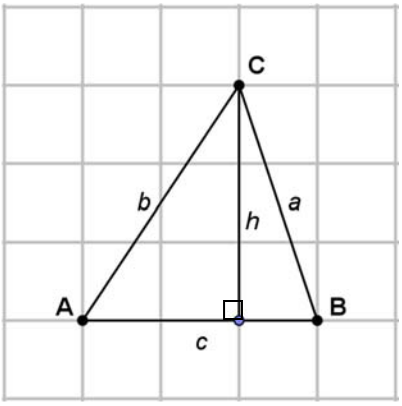
2. Any of the following answers is correct –
   - ✓ Based on the measures of the angles and the general appearance of the triangles, some students may group them as acute triangles or obtuse triangles.
   - ✓ Based on the parts whose measures are given, some students may group them as follows:
     - triangles whose measures of two sides and an angle opposite one of these sides are given
     - triangles whose measures of two angles and a side are given

   (Note: The second basis for grouping is what is desired in this activity. If in case you cannot elicit this answer from your students, ask them additional questions that will lead them to it.)

3. No to both

Allow students to explore possibilities by performing Activity 2. This activity provides students with lots of opportunities to exercise their analytical thinking. The activity will lead to the discovery of the Law of Sines, a law that is very important in solving oblique triangles.

Answers in Activity 2: *Exploring Possibilities 1*

1. 

2. $\sin A = \dfrac{h}{b}$

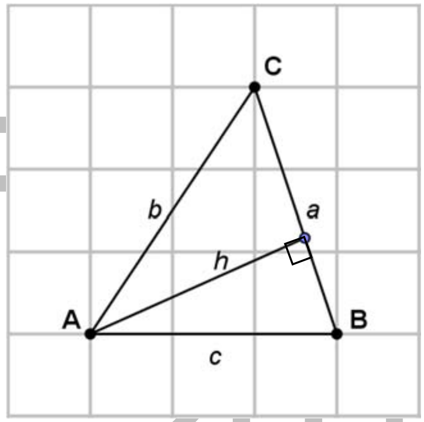
$\sin B = \dfrac{h}{a}$

3. $h = b\sin A$

$h = a\sin B$

Yes, because $b\sin A$ and $a\sin B$ are equal to the same altitude (h) of the triangle.

4. $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

5. 

6. $\sin B = \dfrac{h}{c}$

$\sin C = \dfrac{h}{b}$

7. $h = c\sin B$

$h = b\sin C$

Yes, because $c\sin B$ and $b\sin C$ are equal to the same altitude (h) of the triangle.

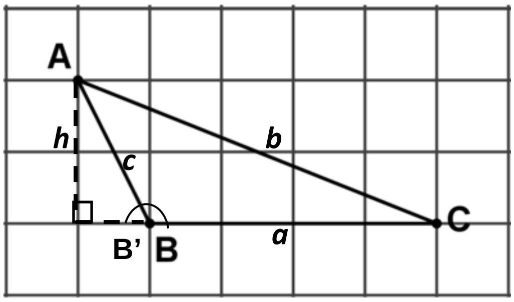
8. $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

9. They are all equal. $\left(\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}\right)$

Reiterate that the equation obtained in No. 9 above corresponds to the Law of Sines. This law states that –

In any triangle, the sine of any angle divided by its opposite side is equal to the sine of any other angle divided by its opposite side.

The derivation presented in the preceding activity was made using an acute triangle, In the case of an obtuse triangle, one of the altitudes may lie outside the triangle as shown in the following figure.



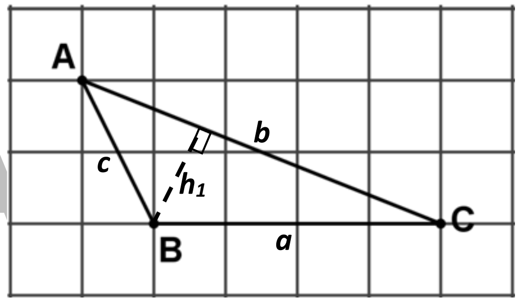
Note: In the figure, B is an interior angle of ΔABC and B’ is its exterior angle at vertex B.

Using the trigonometric ratio $\sin\theta = \dfrac{opposite}{hypotenuse}$, $\sin B' = \dfrac{h}{c}$ and $\sin C = \dfrac{h}{b}$.

Since B and B’ are supplementary angles because they form a linear pair, then $\sin B = \sin B'$. Therefore, $\sin B = \dfrac{h}{c}$.

Solving for *h* in the equations $\sin B = \dfrac{h}{c}$ and $\sin C = \dfrac{h}{b}$, we have

$$h = c\sin B \text{ and } h = b\sin C$$
$$c\sin B = b\sin C$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$



Now, draw an altitude to AC from vertex B as shown in the figure on the right.

From the figure, $\sin A = \dfrac{h_1}{c}$ and $\sin C = \dfrac{h_1}{a}$. Solving for h_1, we have

$$h_1 = c\sin A \text{ and } h_1 = a\sin C$$
$$c\sin A = a\sin C$$
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Therefore,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Discuss the possibilities of obtaining exactly one solution, or two solutions (ambiguous case) through the illustrative examples 1, 2, 3 and 4. Let the students realize through these examples that the law of Sines can be used in solving oblique triangles given any of the following –

- two angles and one side
- two sides and an angle opposite one of the given sides

Also, discuss the possibility of not getting any solution.

Prior to the discussion of the illustrative example 4 on the ambiguous case, let the students answer the preliminary exercises provided. Through these exercises, the students will be able to determine that the sine values of two supplementary angles are equal.

Answers: Preliminary Exercises

A.
1. 0.3420
2. 0.7660
3. 0.8192
4. 0.3090
5. 0.9998
6. 0.3420
7. 0.7660
8. 0.8192
9. 0.3090
10. 0.9998

B. The values obtained in numbers 1 to 5 are exactly the same values obtained in numbers 6 to 10. Furthermore, the given angles in numbers 1 to 5 are the respective supplements of the angles given in numbers 6 to 10. Therefore, one can conclude that if two angles are supplementary, then their sine values are equal.

Answers in Exercises A and B:

A.
1. $\frac{\sin 36°}{x} = \frac{\sin 41°}{10}$
2. $\frac{\sin X}{5} = \frac{\sin 125°}{20}$
3. $\frac{\sin 95°}{x} = \frac{\sin 25°}{5.5}$
4. $\frac{\sin X}{37} = \frac{\sin 30°}{30}$
5. $\frac{\sin 100°}{x} = \frac{\sin 50°}{25}$

B.
1. 9 units
2. 11.8°
3. 13 units
4. This is an ambiguous case with a possibility of two answers: 38.1° and 141.9°. But since the figure shows that the angle is obtuse, then 38.1° should be ruled out. Therefore, the correct answer is 141.9°.
5. 32.1 units

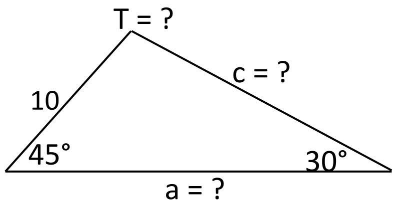
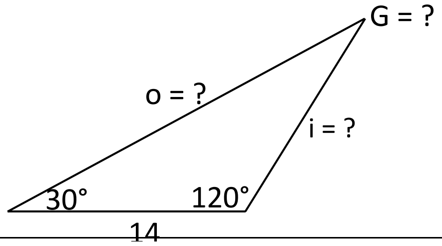
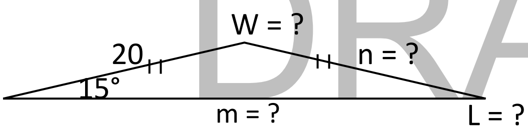
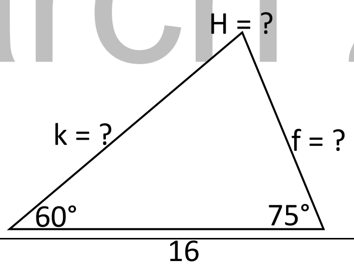
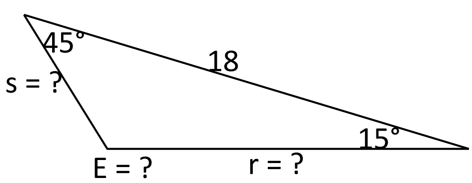
Answers to More Practice Exercises:

1. a) $AB = 22.2$ units
1. b) $AB = 18.0$ or 18 units
2. a) $A = \sin^{-1}(0.5937) = 36.4°$
2. b) $A = \sin^{-1}(0.6225) = 38.7°$

What to Process

Let the students apply the Law of Sines in finding the missing parts of given oblique triangles. Ask the students to complete the puzzle in Activity 3.

Answers in Activity 3: *Pair Me Up*

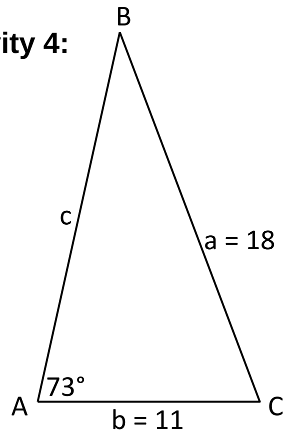
Given Triangle	Missing Parts
	$\angle T = 180° - (45° + 30°) = 180° - 75°$ $\angle T = 105°$ $a = \dfrac{10\sin 105°}{\sin 30°} = 19.3$ $c = \dfrac{10\sin 45°}{\sin 30°} = 14.1$
	$\angle G = 180° - (30° + 120°) = 180° - 150°$ $\angle G = 30°$ $i = 14$ $o = \dfrac{14\sin 120°}{\sin 30°} = 24.2$
	$n = 20$ $\angle L = 15°$ $\angle W = 180° - (15° + 15°) = 180° - 30°$ $\angle W = 150°$ $m = \dfrac{20\sin 150°}{\sin 15°} = 38.6$
	$\angle H = 180° - (60° + 75°) = 180° - 135°$ $\angle H = 45°$ $k = \dfrac{16\sin 75°}{\sin 45°} = 21.9$ $f = \dfrac{16\sin 60°}{\sin 45°} = 19.6$
	$\angle E = 180° - (45° + 15°) = 180° - 60°$ $\angle E = 120°$ $r = \dfrac{18\sin 45°}{\sin 120°} = 14.7$ $s = \dfrac{18\sin 15°}{\sin 120°} = 5.4$

Answer to the Puzzle: <u>CLOCK FOR TELLING WHAT TIME IT WAS</u>

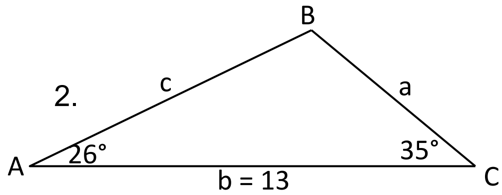
Ask the students to do Activity 4 where they will draw triangles based on given conditions. Let them solve for the missing parts afterwards.

Answers in Activity 4:

1. $\angle B = 35.8°$
   $\angle C = 71.2°$
   $c = 17.8$



2.



$\angle B = 119°$
$a = 6.5$
$c = 8.5$

. Before you let your students perform Activity 5, provide exercises on parallel lines cut by transversals. Also, review students regarding the different angle pairs formed by parallel lines cut by a transversal. Provide exercises that would help students recall the relationship existing between the alternate interior angles/alternate exterior angles formed by parallel lines cut by a transversal.

Ask the students to read carefully the problem given in Activity 5. Let them analyze the diagram presented before answering the questions within the activity.

Answers in Activity 5:

a) The $\angle B = 40°$ because its alternate interior angle is also 40°.

b) $\angle WOS = 90°$	$\overline{NS} \perp \overline{WE}$
$\angle WOB + m\angle BOS = \angle WOS$	by Angle Sum Property
$\angle WOB + 40° = 90°$	by Substitution
$\angle WOB = 50°$	by Subtraction Property of Equality
$\angle AOB = \angle AOW + \angle WOB$	by Angle Sum Property
$\angle AOB = 30° + 50°$	by Substitution
$\therefore \angle AOB = 80°$	by Addition
c) $\angle A + \angle B + \angle AOB = 180°$	The sum of the three angles of any triangle is 180°.
$\angle A + 40° + 80° = 180°$	by Substitution
$\angle A + 120° = 180°$	by Addition
$\therefore \angle A = 60°$	by Subtraction Property of Equality

To find the distance of the ship from town B, use the formula $\frac{\sin A}{OB} = \frac{\sin O}{AB}$.

$$\frac{\sin A}{OB} = \frac{\sin O}{AB}$$

$$OB = \frac{AB \sin A}{\sin O} = \frac{30 \sin 60°}{\sin 80°} = \frac{30(0.8660)}{0.9848} = 26.38$$

$\therefore$ *The distance of the ship from town B is 26.38 km*

What to Reflect or Understand

Provide opportunities for the students to reflect on how the Law of Sines can be applied to solve real-life problems involving oblique triangles.

Let students explore on the possibility of using the concept of Sines in determining areas of oblique triangles by doing Activity 6.

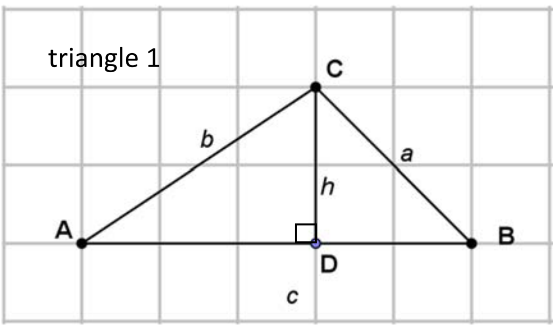
Answers in Activity 6: *Exploring Possibilities 2*

1. In figure 1, *h = c sin A* and *h = a sin C.*
   In figure 2, *h = b sin A.*
   In figure 3, *h = b sin C*

2. *A = ½ bh, thus in –*
   figure 1, A_Δ = *½ bc sin A* and A_Δ = *½ ab sin C*
   figure 2, A_Δ = *½ bc sin*
   figure 3, A_Δ = *½ ab sin C*

3. They are areas of the same triangle. Hence, they will all yield the same value.

4. Testing the formulas:



In triangle 1, AD = 3, DB = 2, AB = *c* = 5, and h = 2.

Using right Δ ADC and the Pythagorean Theorem:

$$b^2 = AD^2 + h^2$$
$$b^2 = 3^2 + 2^2 = 9 + 4 = 13$$
$$b = \sqrt{13} units$$

$$\sin A = \frac{opposite}{hypotenuse} = \frac{h}{b} = \frac{2}{\sqrt{13}}$$
$$A = \sin^{-1}\left(\frac{2}{\sqrt{13}}\right) = 33.7°$$

In right Δ CDB, the legs are congruent, therefore, it is a special right Δ, a 45°-45°-90° Δ. Thus, $\angle B = 45°$ and its hypotenuse (*a*) is $2\sqrt{2}$ units long.

$$\angle C = 180° - (33.7° + 45°) = 101.3°$$

Using the formula $A_\Delta = \frac{1}{2}ab\sin C$, we have

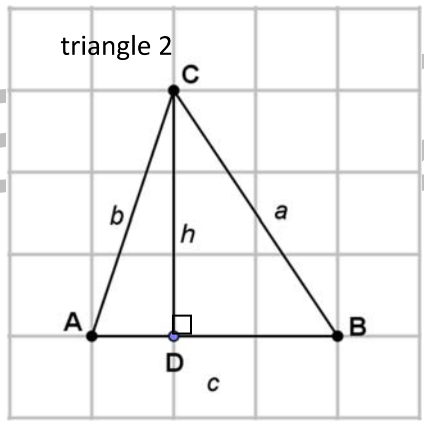
$$A_\Delta = \frac{1}{2}\left(2\sqrt{2}\right)\left(\sqrt{13}\right)\left(\sin 101.3°\right) = \left(\sqrt{26}\right)\left(0.9806\right) = 5 \text{ sq. units}$$

Using the formula $A_\Delta = \frac{1}{2}bc\sin A$, yields

$$A_\Delta = \frac{1}{2}\left(\sqrt{13}\right)(5)\left(\frac{2}{\sqrt{13}}\right) = 5 \text{ sq. units}$$

Using the formula $A_\Delta = \frac{1}{2}ac\sin B$, gives us

$$A_\Delta = \frac{1}{2}\left(2\sqrt{5}\right)(5)\left(\frac{1}{\sqrt{2}}\right) = 5 \text{ sq. units}$$



In triangle 2, AD = 1, DB = 2, AB = *c* = 3, and *h* = 3.

Using right Δ ADC and the Pythagorean Theorem,

$$b^2 = AD^2 + h^2 = 1^2 + 3^2 = 1 + 9 = 10$$
$b = \sqrt{10}$ units
$$\sin A = \frac{opposite}{hypotenuse} = \frac{h}{b} = \frac{3}{\sqrt{10}}$$
$$A = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right) = 71.57°$$

Using right Δ CDB and the Pythagorean Theorem,

$$a^2 = DB^2 + h^2 = 2^2 + 3^2 = 4 + 9 = 13$$
$$a = \sqrt{13}$$

$$\sin B = \frac{opposite}{hypotenuse} = \frac{3}{\sqrt{13}}$$
$$B = \sin^{-1}\left(\frac{3}{\sqrt{13}}\right) = 56.31°$$

$$\angle C = 180° - (71.57° + 56.31°) = 52.12°$$

Using the formula $A_\Delta = \frac{1}{2}ab\sin C$, will yield the following;

$$A_\Delta = \frac{1}{2}\left(\sqrt{13}\right)\left(\sqrt{10}\right)\left(\sin 52.12°\right) = \frac{1}{2}\left(\sqrt{130}\right)\left(0.7893\right) = 4.5 \text{ sq. units}$$

Using the formula $A_\Delta = \frac{1}{2}bc\sin A$, we have

$$A_\Delta = \frac{1}{2}\left(\sqrt{10}\right)\left(3\right)\left(\frac{3}{\sqrt{10}}\right) = \frac{1}{2}\left(9\right) = 4.5 \text{ sq. units}$$

Using the formula $A_\Delta = \frac{1}{2}ac\sin B$, will give us

$$A_\Delta = \frac{1}{2}\left(\sqrt{13}\right)\left(3\right)\left(\frac{3}{\sqrt{13}}\right) = \frac{1}{2}\left(9\right) = 4.5 \text{ sq. units}$$

5. Using triangle 1 and the formula $A_\Delta = \frac{1}{2}bh$, yields

$$A_\Delta = \frac{1}{2}\left(5\right)\left(2\right) = 5 \text{ sq. units}$$

Using triangle 2 and the formula $A_\Delta = \frac{1}{2}bh$, yields

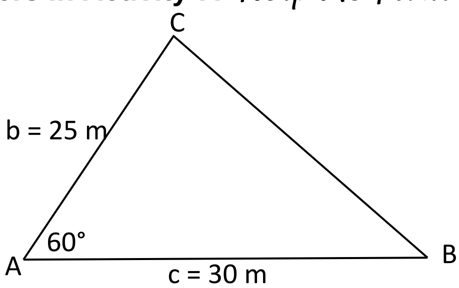
$$A_\Delta = \frac{1}{2}\left(3\right)\left(3\right) = \frac{1}{2}\left(9\right) = 4.5 \text{ sq. units}$$

Yes, the formulas are accurate.

Now, see if the students really understood the concepts that you have taught them. Let the students work in pairs in doing the next activity. This would allow your students to clarify any misconceptions when they discuss the problems with a partner.

Answers in Activity 7: *Help Me Find a Solution*

1.

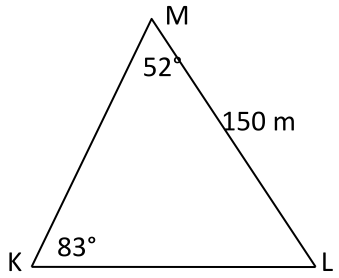


$$A_\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\left(25\right)\left(30\right)\left(\sin 60°\right)$$

$$A_\Delta = \left(25\right)\left(15\right)\left(0.8660\right) = 324.75 \text{ sq. m}$$

Cost = (324.75)(₱2000.00)
Cost = ₱649, 500.00

2.



$$\angle L = 180° - (52° + 83°) = 45°$$

$$\frac{\sin L}{MK} = \frac{\sin K}{150}$$

$$MK = \frac{150\sin L}{\sin K} = \frac{150\sin 45°}{\sin 83°} = 106.9\text{ m}$$

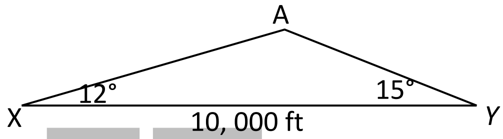
$$\frac{\sin M}{KL} = \frac{\sin K}{150}$$

$$KL = \frac{150\sin M}{\sin K} = \frac{150\sin 52°}{\sin 83°} = 119.1\text{ m}$$

$P = ML + MK + KL = 150m + 106.9m + 119.1\ m = 376\ m$ of fencing materials

$A_{\Delta} = \frac{1}{2}(MK)(ML)\sin\angle M = \frac{1}{2}(106.9)\ (150)(\sin 52°) = 6,317.88\ sq.m$

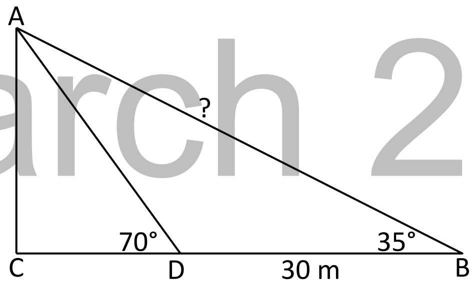
3. $\angle A = 180° - (12° + 15°) = 153°$



$$\frac{\sin A}{XY} = \frac{\sin X}{AY}$$

$AY = \dfrac{(XY)\sin X}{\sin A} = \dfrac{(10000)(\sin 12°)}{\sin 153°} = 4579.65$ ft. → distance of the airplane to the nearer end of the runway

4.



$\angle$ADC and $\angle$ADB form a linear pair, hence, they are supplementary.

$$\angle ADC + \angle ADB = 180°$$
$$70° + \angle ADB = 180°$$
$$\angle ADB = 180° - 70° = 110°$$

$\angle ADB$, $\angle ABD$, and $\angle DAB$ are angles of ΔADB. Thus, the sum of their measures is 180°.

$$\angle ADB + \angle ABD + \angle DAB = 180°$$
$$110° + 35° + \angle DAB = 180°$$
$$\angle DAB = 180° - 145° = 35°$$

Since ΔADB contains two congruent angles, then it also has two congruent sides and these are the sides opposite the congruent angles. Therefore, AD = DB = 30 m.

Using the oblique Δ ADB and the Law of Sines,

$$\frac{\sin \angle ADB}{AB} = \frac{\sin \angle DAB}{DB} \qquad AB = \frac{30 \sin 110°}{\sin 35°} = \frac{30(0.9397)}{0.5736} = 49.15 \text{ m}$$

$$\frac{\sin 110°}{AB} = \frac{\sin 35°}{300}$$

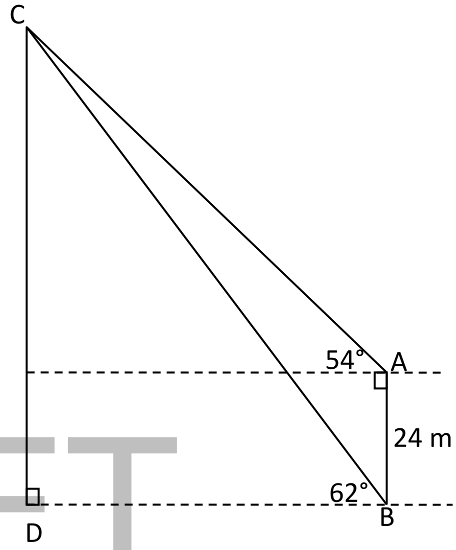
$\therefore$ The distance of the top of the tower from the second observation point is 49.15 m.

5. $\angle CAB = 54° + 90° = 144°$

$\angle CBA = 90° - 62° = 28°$

$\angle ACB = 180° - (144° + 28°)$

$\angle ACB = 180° - 172° = 8°$



Using Δ ABC and the Law of Sines,

$$\frac{\sin \angle ACB}{AB} = \frac{\sin \angle CAB}{BC}$$

$$\frac{\sin 8°}{24} = \frac{\sin 144°}{BC}$$

$$BC = \frac{24 \sin 144°}{\sin 8°} = \frac{24(0.5878)}{0.1392} = 101.34 \text{ m}$$

Using right Δ CDB,

$$\sin \angle CBD = \frac{opposite}{hypotnuse}$$

$$\sin 62° = \frac{CD}{BC}$$

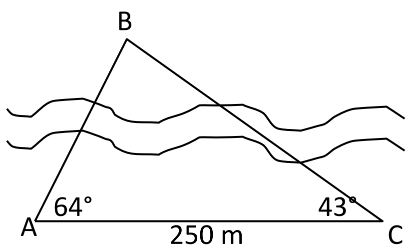
$$CD = BC \sin 62° = (101.34)(0.8829) = 89.48$$

$\therefore$ The distance of the weather balloon from the ground is 89.48 m.

At this point, encourage your students work on the next activity alone. This will further strengthen their understanding of the concepts which have been taught.

Answers in Activity 8: *All Alone but Still Going Strong*

1.



$\angle B = 180° - (64° + 43°) = 73°$

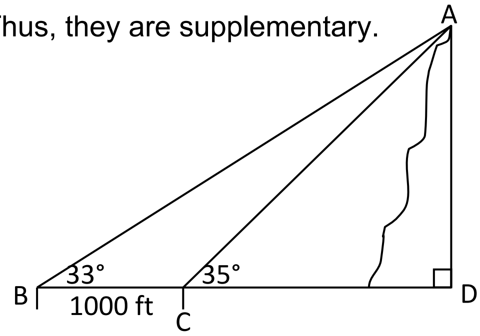
Using the Law of Sines,

$$\frac{\sin B}{AC} = \frac{\sin C}{AB}$$

$$\frac{\sin 73°}{250} = \frac{\sin 43°}{AB}$$

$$AB = \frac{250\sin 43°}{\sin 73°} = 178.29 \text{ m}$$

2. $\angle$ ACD and $\angle$ ACB form a linear pair. Thus, they are supplementary.



$$\angle ACD + \angle ACB = 180°$$
$$35° + \angle ACB = 180°$$
$$\angle ACB = 180° - 35° = 145°$$

In Δ ABC,

$$\angle BAC + \angle ABC + \angle ACB = 180°$$
$$\angle BAC + 33° + 145° = 180°$$
$$\angle BAC = 180° - 178° = 2°$$

Using Δ ABC and the Law of Sines,

$$\frac{\sin\angle ABC}{AC} = \frac{\sin\angle BAC}{BC}$$
$$\frac{\sin 33°}{AC} = \frac{\sin 2°}{1000}$$
$$AC = \frac{1000\sin 33°}{\sin 2°} = \frac{1000(0.5446)}{0.0349} = 15604.58 \text{ ft.}$$

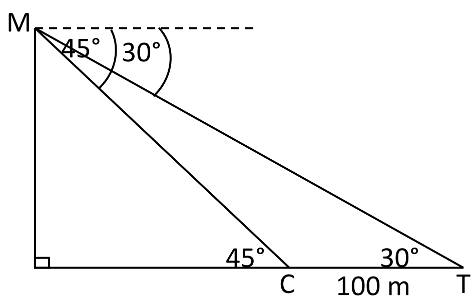
Using right Δ ADC,

$$\sin\angle ACD = \frac{opposite}{hypotenuse}$$
$$\sin 35° = \frac{AD}{15604.58}$$
$$AD = (15604.58)(\sin 35°) = (15604.58)(0.5736) = 8950.79 \text{ ft.}$$

$\therefore$ The mountain is 8950.79 ft. high.

3. In Δ MCT, $m\angle TMC = 45° - 30° = 15°$ and $\angle MCT = 180° - 45° = 135°$.

Using the Law of Sines,



$$\frac{\sin\angle TMC}{CT} = \frac{\sin\angle MCT}{MT}$$
$$\frac{\sin 15°}{100} = \frac{\sin 135°}{MT}$$

$$MT = \frac{100 \sin 135°}{\sin 15°} = \frac{100(0.7071)}{0.2588}$$
$$MT = 273.22m$$

What to Transfer

Give the students the opportunities to demonstrate their understanding of concept of sines. Let them perform Activity 9. You can ask the students to work individually or in groups. Emphasize to them that they need to incorporate the concepts they have learned in the story that they will create.

Following are suggested rubrics which can be used to rate the outputs of the students in Activity 9.
Suggested Rubrics for Activity 9:

RUBRIC FOR WRITTEN OUTPUT				
CRITERIA	4	3	2	1
Creativity	The story has imaginative beginning, middle, and ending. Proper chaining of events is highly visible.	The story has strong beginning, middle and ending. Proper chaining of events is visible.	Story has sufficient beginning, middle, and ending but somewhat vague. Chaining of events is straightforward.	Story is vague. Chaining of events is hard to follow.
Mathematical Concept Used	Shows full understanding of the concept as evidenced in the work presented.	Shows nearly complete understanding of the concept as evidenced in the work presented.	Shows some degree of understanding of the concept as evidenced in the work presented.	Shows very limited understanding of the concept as evidenced in the work presented.
Accuracy of Computations	The computations were presented logically and done accurately. Has presented alternative solution/s leading to the same answer.	The computations were presented logically and done accurately.	The computations were presented logically but there were minimal errors in computations.	The computations were incoherent and erroneous.

RUBRIC FOR ORAL PRESENTATION			
	Very Good 3	**Satisfactory 2**	**Poor 1**
Provides a fascinating introduction.			
Offers a comprehensible explanation of the topic.			
Offers information in proper sequencing.			
Uses complete sentences. Speaks precisely and confidently.			
Handles queries and clarifications from the class very effectively.			
TOTAL __________ (out of 15)			

A performance task in GRASPS form is given in Activity 10. Let the students perform the activity alone. Rate their outputs using a rubric. The following is a suggested rubric for rating students' outputs.

RUBRIC FOR PERFORMANCE TASK				
CRITERIA	4	3	2	1
Mathematical Concept Used	Has shown full understanding of the concept as evidenced in the work presented. Also used other concept leading to the same solution.	Has shown full understanding of the concept as evidenced in the work presented.	Has shown some degree of understanding of the concept as evidenced in the work presented.	Has no knowledge of the concept as evidenced in the work presented.
Accuracy of Computations	The computations were presented logically and done accurately. Has presented alternative solution/s leading to the same answer.	The computations were presented logically and done accurately.	The computations were presented logically but there were minimal errors in computations.	The computations were incoherent and erroneous.
Presentation	Sketch was drawn precisely according to the scale used.	Sketch was drawn precisely according to the scale used.	Sketch was drawn precisely according to the scale used.	Sketch was not drawn to scale and is hard to follow.

	Presentation of the sketch is clear and easy to follow. Has add-on features (such as flowers and trees) making the sketch attractive.	Presentation of the sketch is clear and easy to follow.	Presentation of the sketch is fairly easy to follow.	

Lesson 5.2 The Law of Cosines and Its Applications

What to Know

Introduce the Law of Cosines to the students. Reiterate that this law can also be used in solving oblique triangles. Discuss the illustrative examples found in the LM. Through these examples, students will realize when this law can be applied to solve oblique triangles.

What to Process

Let students apply the Law of Cosines in solving oblique triangles. Let them perform Activity 1 to practice their skills in solving oblique triangles.

Answers in Activity 1: *Practice Makes Perfect*

1. $a = 10.33$ units
   $\angle B = 16.06°$
   $\angle C = 55.94°$

2. $\angle A = 30°$
   $\angle B = 30°$
   $c = 6.93$ units

3. $\angle A = 21.79°$
   $\angle B = 60°$
   $\angle C = 98.21°$

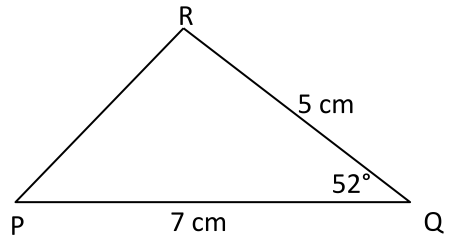
4. $\angle C = 49.25°$
   $\angle A = 65.375°$
   $\angle B = 65.375°$

5. $a = 11.85$ units
   $\angle B = 18.86°$
   $\angle C = 31.14°$

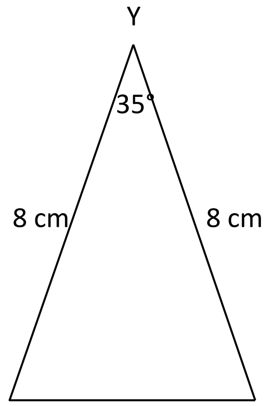
Provide students with opportunities to practice drawing triangles based on specific conditions. This would help prepare them in dealing with application problems. If the students are able to illustrate a problem through diagrams, then this would help connect them with the problem which will make solving easier.

Answers in Activity 2:

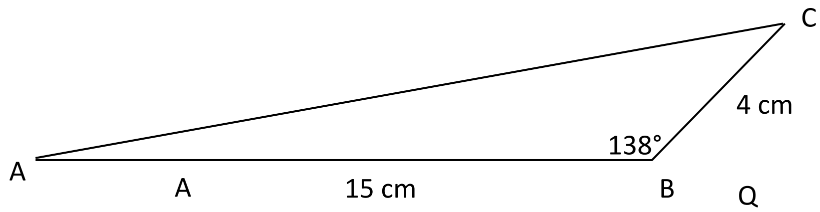
1.



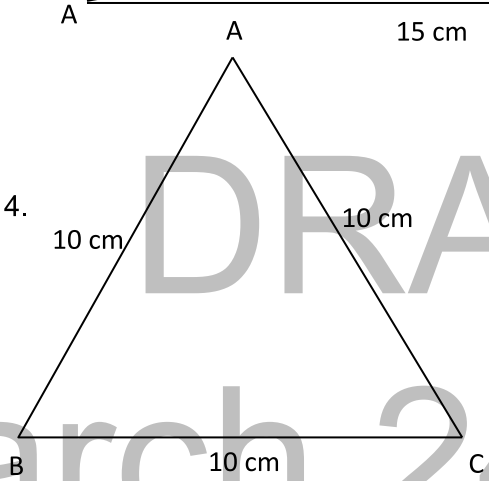
2.



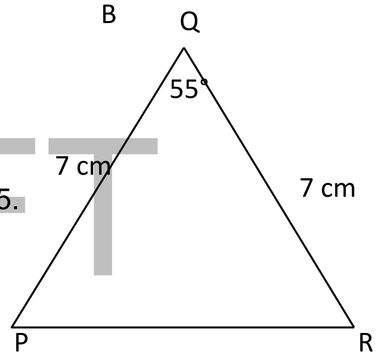
3.



4.

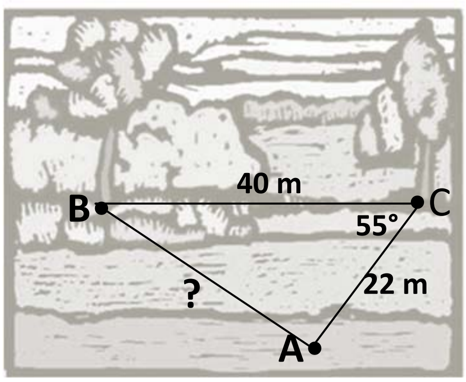


5.

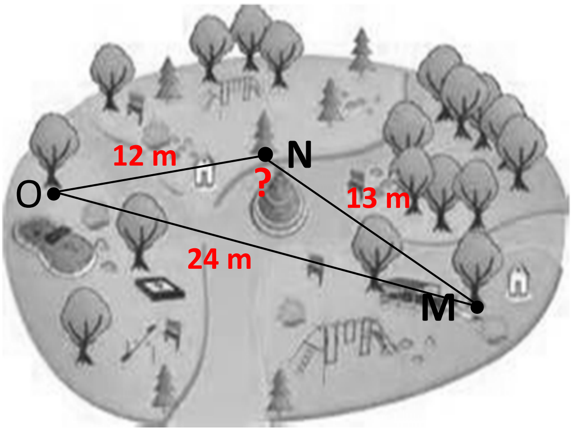


In the next activity, illustrations based on real-life situations are already provided. Let the students answer Activity 3 by applying the Law of Cosines.

Answers in Activity 3:



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 40^2 + 22^2 - 2(40)(22)(\cos 55°)$$
$$c^2 = 1600 + 484 - 1760(0.5736)$$
$$c^2 = 2084 - 1009.536$$
$$c^2 = 1074.464$$
$$c = 32.78\text{ m}$$



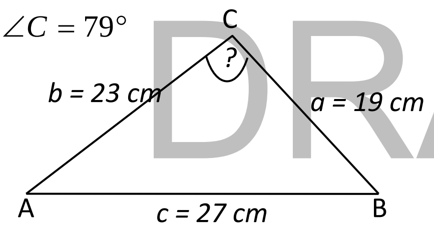
$$OM^2 = ON^2 + MN^2 - 2(ON)(MN)(\cos N)$$
$$24^2 = 12^2 + 13^2 - 2(12)(13)(\cos N)$$
$$576 = 144 + 169 - 312(\cos N)$$
$$312(\cos N) = 313 - 576$$
$$\cos N = \frac{-263}{312}$$
$$N = \cos^{-1}\left(\frac{-263}{312}\right) = 147.45°$$

What to Reflect or Understand

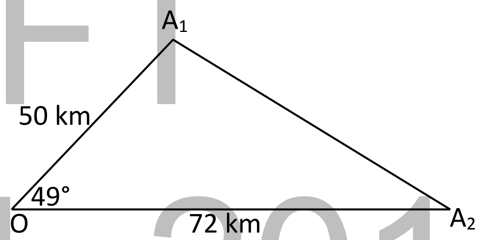
Task the students to reflect on how the Law of Cosines can be applied to solve real-life problems involving oblique triangles.

Answers in Activity 4:

1. $\angle C = 79°$

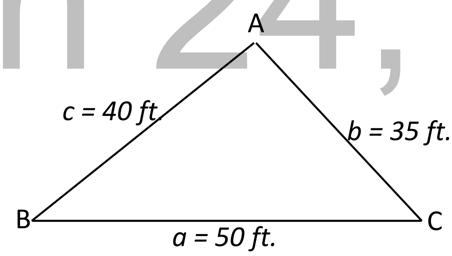


2. $A_1A_2 = 54.4$ km



?

3. $\angle A = 83°$
$\angle B = 44°$
$\angle C = 53°$



What to Transfer

At this stage, ask the students to formulate a problem using a real-life situation. Let the students perform the next activity.

Outputs of the students in Activity 5 may be rated using the suggested rubric that follows.

Suggested Rubric for Activity 5:

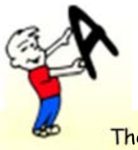
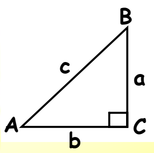
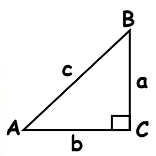
RUBRIC FOR ACTIVITY 5 – What's your problem?				
CRITERIA	**4**	**3**	**2**	**1**
Math Content	Demonstrates clear knowledge and application of math skills.	Demonstrates general knowledge and application of math skills.	Demonstrates limited knowledge and application of math skills.	Demonstrates little or no knowledge or application of math skills.
Mathematical Communication	Accurately communicates solutions to problems and concepts.	Satisfactorily communicates solutions to problems and concepts.	Limited communication of solutions to problems and concepts.	Inaccurately communicates solutions to problems and concepts.
Problem Solving	Provides appropriate process for solving the problem without errors in computations.	Utilizes moderately correct process with minimal omission or error in procedure.	Employs partially correct process but with considerable error in procedure.	Uses entirely incorrect plan/strategy.
Presentation	Solution is presented in an easy follow step-by-step model.	Solution is presented in a logical manner.	Solution is difficult to follow at times.	The reader is unable to follow the steps taken in the solution.

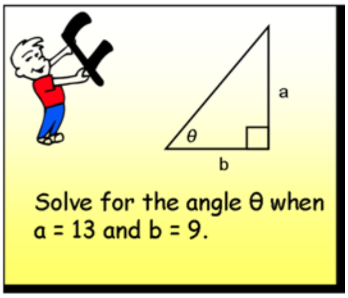
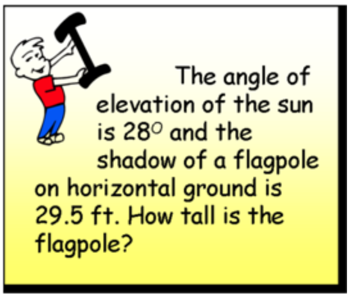
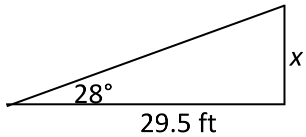
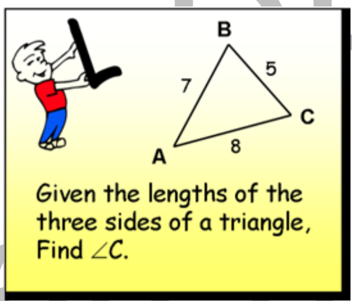
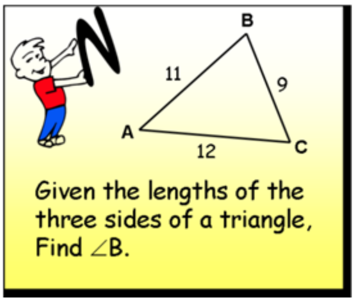
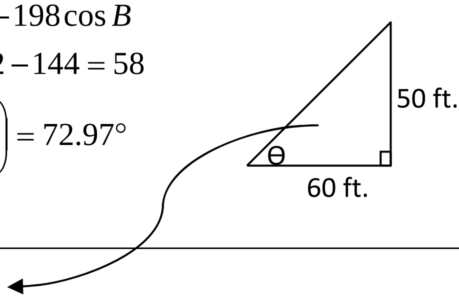
Summary

Lesson 5 deals with solving word problems involving oblique triangles. The lesson is divided into two – Lesson 5.1 and Lesson 5.2. Lesson 5.1 is about the Law of Sines and Its Applications while Lesson 5.2 focuses on the Law of Cosines and Its Applications. These laws are essential in the solution of oblique triangles. Different activities regarding these concepts are given to ensure that students can apply these concepts in solving problem situations in real life. Through the given activities, students will have opportunities to develop their critical thinking skills which play a vital role in solving problems we encounter in the real world.

As a culminating activity ask the students to answer Activity 6 entitled Laws to Validate. This activity will help firm-up the concepts they have learned in this module.

Answers in Activity 6:

GIVEN PROBLEM	ANSWER
 The trigonometric ratio of the acute angle θ of a right triangle denoted by sine is __________.	$\frac{opposite}{hypotenuse}$
GIVEN PROBLEM	**ANSWER**
  Solve the value of ***a*** in the right Δ above with ***c*** = 39 and ∠*A* = 40°.	$\frac{\sin C}{c} = \frac{\sin A}{a}$ $a = \frac{c\sin A}{\sin C} = \frac{39\sin 40°}{\sin 90°} = 25.07$
 The trigonometric ratio of the acute angle θ of a right triangle denoted by cosine is __________.	$\frac{adjacent}{hypotenuse}$
  Solve for b when ***a*** = 6 and ***c*** = 10.	$a^2 + b^2 = c^2$ $6^2 + b^2 = 10^2$ $b^2 = 100 - 36 = 64$ $b = 8$

	$c^2 = a^2 + b^2$ $c^2 = 13^2 + 9^2$ $c^2 = 169 + 81 = 250$ $c = 5\sqrt{10}$ $\frac{\sin C}{c} = \frac{\sin\theta}{a}$ $\sin\theta = \frac{a\sin C}{c} = \frac{13\sin 90°}{5\sqrt{10}}$ $\theta = \sin^{-1}\left(\frac{13}{5\sqrt{10}}\right) = 55.3°$
	$\tan 28° = \frac{x}{29.5}$ $x = 29.5(\tan 28°)$ $x = 29.5(0.5317)$ $x = 15.69\,ft.$ 
	$c^2 = a^2 + b^2 - 2ab\cos C$ $7^2 = 5^2 + 8^2 - 2(5)(8)(\cos C)$ $49 = 25 + 64 - 80\cos C$ $80\cos C = 89 - 49 = 40$ $\cos C = \left(\frac{40}{80}\right) = 0.5$ $c = \cos^{1}(0.5) = 60°$
	$b^2 = a^2 + c^2 - 2ac\cos B$ $12^2 = 9^2 + 11^2 - 2(9)(11)(\cos B)$ $144 = 81 + 121 - 198\cos B$ $198\cos B = 202 - 144 = 58$ $B = \cos^{-1}\left(\frac{58}{198}\right) = 72.97°$ 

Suppose a 50-feet high tree casts a shadow of length 60 feet, what is the angle of elevation from the end of the shadow to the top of the tree with respect to the ground?

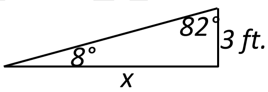
$$c^2 = 60^2 + 50^2$$
$$c^2 = 3600 + 2500$$
$$c^2 = 6100$$
$$c = 10\sqrt{61}$$

$$\frac{\sin\theta}{50} = \frac{\sin 90^\circ}{10\sqrt{61}}$$
$$\sin\theta = \frac{50\sin 90^\circ}{10\sqrt{61}} = \frac{50}{10\sqrt{61}}$$
$$\theta = \sin^{-1}\left(\frac{50}{10\sqrt{61}}\right) = 39.8^\circ$$

The trigonometric ratio of the acute angle θ of a right triangle defined by tangent is __________.

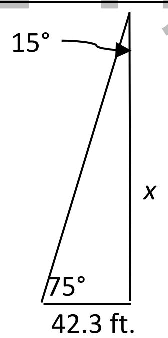
$$\frac{opposite}{adjacent}$$

How far from the door must a ramp begin in order to rise 3 feet with an 8° angle of elevation?

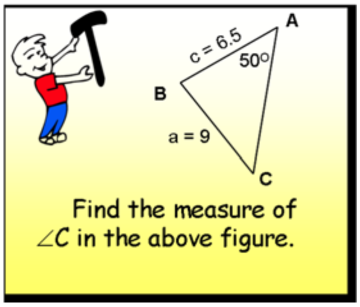


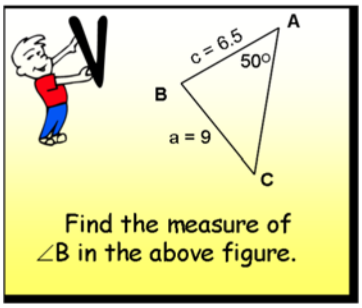
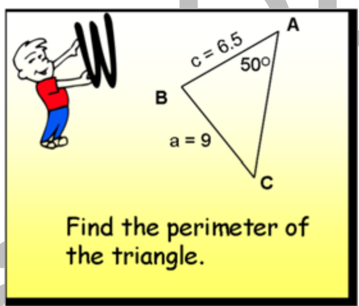
$$\frac{\sin 82^\circ}{x} = \frac{\sin 8^\circ}{3}$$
$$x = \frac{3\sin 82}{\sin 8^\circ}$$
$$x = 21.35\,ft.$$

At a point 42.3 feet from the base of a building, the angle of elevation of the top is 75°. How tall is the building?



$$\frac{\sin 15^\circ}{42.3} = \frac{\sin 75^\circ}{x}$$
$$x = \frac{42.3\sin 75^\circ}{\sin 15}$$
$$x = 157.9\,ft.$$

	$\frac{\sin A}{a} = \frac{\sin C}{c}$ $\sin C = \frac{c \sin A}{a} = \frac{6.5 \sin 50°}{9} = 0.5533$ $C = \sin^{-1}(0.5533) = 33.59°$

	$m\angle A + m\angle B + m\angle C = 180°$ $50° + m\angle B + 33.59° = 180°$ $m\angle B = 180° - 83.59°$ $m\angle B = 96.41°$
	$\frac{\sin 96.41}{b} = \frac{\sin 50°}{9}$ $b = \frac{9 \sin 96.41°}{\sin 50°} = 11.68$ $P = a + b + c$ $P = 9 + 11.68 + 6.5$ $P = 27.18$

DECODER: Triangle trigonometry can be used to solve problems involving angles of **ELEVATION** and **DEPRESSION**. However, many interesting problems involve non-right triangles. The laws needed to solve these types of problems are the **LAWS OF SINES AND COSINES**.

POST TEST:

1. Determine the correct formula for the tan ratio of ∠B.

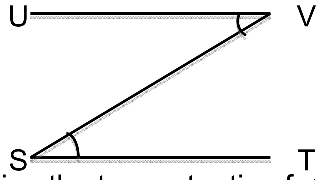
   a. tan B = $\frac{length\ of\ side\ adjacent\ to\ \angle B.}{length\ of\ the\ hypotenuse}$

   c. tan B = $\frac{length\ of\ hypotenuse}{length\ of\ side\ opposite\ to\ \angle B}$

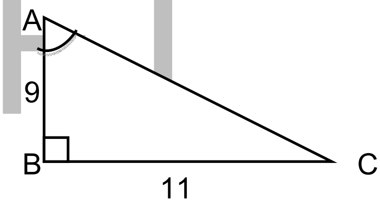
   b. tan B = $\frac{length\ of\ side\ opposite\ to\ \angle B.}{length\ of\ side\ adjacent\ to\ \angle B}$

   d. tan B = $\frac{length\ of\ side\ opposite\ to\ \angle B.}{length\ of\ hypotenuse}$

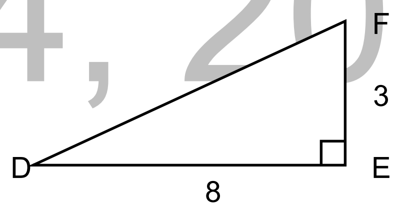
2. Which statement is incorrect?
   a. The hypotenuse is the longest side in a right triangle.
   b. The hypotenuse is always the opposite side of the 90^o in a right triangle.
   c. The Pythagorean theorem applies to all right triangles.
   d. You can solve for the unknown side in any triangle, if you know the lengths of the other two sides, by using the Pythagorean theorem
3. Given the figure on the right, which refers to the angle of elevation?
   a. ∠UVS
   b. ∠VST
   c. Both ∠UVS and ∠VST
   d. none of the above



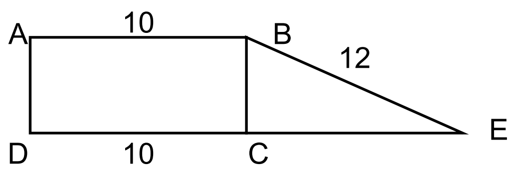
4. In ΔABC, AB = 9 cm and BC = 11 cm. Determine the tangent ratio of ∠A to nearest hundredths.
   a. 0.64
   b. 0.73
   c. 1.36
   d. 1.22



5. In the ΔDEF, determine the measure of ∠D, to the nearest degree.
   a. 22^o
   b. 21^o
   c. 20^o
   d. 19^o



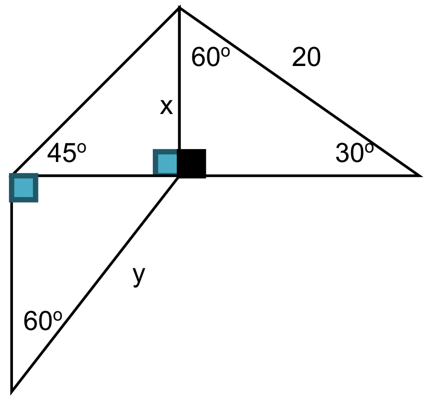
6. In the figure at the right, ABCD is a rectangle whose perimeter is 30. The length of BE is 12. Determine the measure of ∠E to the nearest degree.
   a. 24^o
   b. 25^o
   c. 26^o
   d. 27^o



Use the figure at the right to answer item numbers 7 and 8.

7. Find x.
   a. 10
   b. 14.1
   c. 17.3
   d. 21.5

8. Find y.
   a. 10
   b. 11.5
   c. 15.2
   d.17.3



9. Evaluate: 2 sin 30° + 4cos 60° – 3 cot 45° + sec 45°

   a. $\sqrt{2}$ b. 0 c. $\sqrt{3}$ d. 1

10. rom a point in the ground 7.62 m from the foot of the tree, the angle of elevation from the top of the tree is 32°. Determine the height of the tree to the nearest hundredths.
    a. 3.81 b. 6.46 c. 4.76 d. 12.19

11. From an airplane at an altitude of 1500 m, the angle of depression to a rock on the ground measures 29°. Determine the distance from the plane to the rock.
    a. 3, 094 m b. 3, 304 m c. 3, 422 m d. 3, 549 m

12. A 10-foot ladder is placed so as to form an angle of 25° with the wall of a building. How far is the ladder from the base of the building?
    a. 4.1 ft. b. 4.2 ft. c. 4.4 ft. d. 4.5 ft.

13. PQRS is a rhombus with a side 5 cm long. If $m\angle P = 140°$, find the length of its longer diagonal.
    a. 9.1 cm b. 9.2 cm c. 9.3 cm d. 9.4 cm

14. In right triangle ABC, BC = 8 cm and AC = 17 cm. Find the value of sin C.
    a. $\frac{15}{17}$ b. $\frac{17}{15}$ c. $\frac{17}{8}$ d. $\frac{8}{17}$

15. Mr. de Guzman has a triangular-shaped backyard. Find the area of the backyard if two of its sides measure 8 m and 10 m, and the angle between these sides is 54°.
    a. 36.23 m^2 b. 36.32 m^2 c. 32.36 m^2 d. 32.63 m^2

16. An airplane takes off at a constant angle in a straight line. If it has travelled 1000 feet of ground distance when it is 200 feet in the air, how far has it travelled in the air during that time?
    a. 1019.8 ft. b. 1019.9 ft. c. 1018.8 ft. d. 1018.8 ft.

17. A lighthouse 20 m high with the base at sea level is sighted by a ship. If the angle of elevation from the ship to the top of the lighthouse is 12°, what is the distance from the ship to the lighthouse? Round off your answer to the nearest meter.
    a. 92 m b. 93 m c. 94 m d. 95 m

18. A surveyor finds that the angle between the base and the top of a flagpole on top of a building is 7° when the measurement is made from a point 150 feet from the base of a building. If the building is 100 feet tall, what is the height of the flagpole? Round off your answer to the nearest feet.
    a. 29 ft. b. 28 ft. c. 27 ft. d. 26 ft

19. Two roads intersect at an angle of 100°. Mildred's mailbox is 8 m from the intersection. Jacque's mailbox is on the other road and is 12 m from the intersection. How far is Jacque's mailbox from that of Mildred's?
    a. 14.99 m b. 15.45 m c. 15.35 m d. 15.54 m

20. A triangular field has sides 500 feet, 450 feet, and 720 feet. Find the measure of the angle between the sides measuring 500 feet and 720 feet. Round off your answer to the nearest degree.
    a. 37° b. 38° c. 39° d. 40°

Answer Key (Post – Test):

1. b	6. b	11. a	16. a
2. d	7. a	12. b	17. c
3. b	8. b	13. d	18. a
4. d	9. a	14. a	19. d
5. b	10. c	15. c	20. b