TEACHING GUIDE

Module 1: Special products and Factors

A. Learning Outcomes

1. Grade Level Standard

The learner demonstrates understanding of key concepts and principles of algebra, geometry, probability and statistics as applied, using appropriate technology, in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.

2. Content and Performance Standards

Content Standards:

The learner demonstrates understanding of the key concepts of special products and factors of polynomials.

Performance Standards:

The learner is able to formulate real-life problems involving special products and factors and solves these with utmost accuracy using a variety of strategies.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="3">SUBJECT:
Grade 8 Mathematics
QUARTER:
First Quarter
STRAND:
Algebra
TOPIC:
Special Products and Factors
LESSONS:
1. Special Products
2. Factoring</td><td colspan="2">LEARNING COMPETENCIES</td></tr>
<tr><td colspan="2">a. Identify polynomials which are special products: polynomials with common monomial factors, trinomials that are product of two binomials, trinomials that are product of squares of a binomial and products of sum and difference of two terms
b. Find special products and factors of certain polynomials: product of two binomials, product of a sum and difference of two terms, square of a binomial, cube of a binomial and product of special case of multiplying a binomial with a trinomial
c. Factor completely different types of polynomials (polynomials with common monomial factors, difference of two squares, sum and difference of two cubes, perfect square trinomials, general trinomials) using special formulas, grouping and other techniques.
d. Solve problems involving polynomials and their products and factors.</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:
Students will understand that unknown quantities in geometric problems can be determined by using patterns of special products and factors.</td><td>ESSENTIAL QUESTION:
How can unknown quantities in geometric problems be solved?</td></tr>
<tr><td></td><td colspan="2">TRANSFER GOAL:
Apply the concepts of special products and factors to model various real-life situations and solve related problems.</td></tr>
</table>

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to come up within this module.

a. Punnet square containing the desired genes using the concepts of special products.
b. Pictures and / or sketch of a figure that makes use of special products.
c. Cylindrical containers as trash can model which uses the idea of factoring.
d. Rectangular prism that can be used as packaging box which will demonstrate students' understanding of special products and factoring.

Assessment Map

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Pre-Assessment/ Diagnostic	PRE – TEST →			→
	Background Knowledge *(Interpretation, Explanation)* →			
	Gallery Walk (Interpretation, explanation, Self – knowledge)			
	Knowledge Inventory (Self – knowledge) →			
		IRF Worksheet (Interpretation, Explanation)		
Formative	Written Exercises / Drills *(Interpretation, Explanation)*	Quiz *(Interpretation, Explanation)*	IRF Worksheet *(Explanation, Self – knowledge)*	Pattern finding in real world *(Application, explanation, interpretation)*
		Flash Card Drill *(Interpretation, Self – knowledge)* We have! *(Oral Questioning) (Interpretation)*	Decision Making (Written exercises) *(Interpretation, Explanation, Application, Perspective, Empathy)*	
			Debate *(Interpretation, explanation, Application, Empathy, Self – knowledge, perspective)*	
			Graphic Organizer *(Self – knowledge, Explanation, interpretation)*	
		IRF Worksheet *(Interpretation, Explanation)*		

Summative	Unit Test *(Interpretation, Explanation, Self – knowledge, Application)* →			Packaging activity *(Self – knowledge, Interpretation, Application, Explanation)*
		Misconception checking *(Spotting Errors)* *(Self – knowledge, Explanation, Empathy)* →		
	Summative Test *(Interpretation, Application, Self – knowledge, Empathy)* →			→
Self-Assessment			3 – 2 – 1 Chart *(Explanation, Application, Self – knowledge, Perspective)*	
			Muddiest point *Self – knowledge, Explanation, Perspective*	
			Journal Writing *(Self – knowledge, Explanation, Perspective)*	

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	• Identifying polynomials which are special products: polynomials with common monomial factors, trinomials that are product of two binomials, trinomials that are product of squares of a binomial and products of sum and difference of two terms. • finding special products and factors of certain polynomials: product of two binomials, product of a sum and difference of two terms, square of a binomial, cube of a binomial and product of special case of multiplying a binomial with a trinomial • factor completely different types of polynomials (polynomials with common monomial factors, a difference of two squares, sum and difference of two cubes, perfect square trinomials, general trinomials) using special formulas, grouping and other techniques.	Paper and pen Test (refer to attached post – test) Items 1, 2 & 3	1 point for every correct response
Process/Skills 25%		Paper and pen Test (refer to attached post – test) Items 4, 5, 6, 7 & 8	1 point for every correct response
Understanding 30%	• Students will understand that unknown quantities in geometric problems can be determined by using patterns of special products and factors. • Misconceptions regarding special product and factors.	Paper and pen Test (refer to attached post – test) Items 9, 10, 11, 12, 13 & 14	1 point for every correct response

Product/ Performance 30%	**GRASPS** Solve real – life problems involving polynomials and their products and factors.	Paper and pen Test (refer to attached post – test) Items 15, 16, 17, 18, 19 & 20	1 point for every correct response
	The learner is able to formulate real-life problems involving special products and factors and solves these with utmost accuracy using a variety of strategies.	**TRANSFER TASK IN GRASPS FORM** The RER packaging company is in search for the best packaging for a new dairy product that they will introduce to the market. You are a member of the design department of RER Packaging Company. Your company is tapped to create the best packaging box that will contain two identical cylindrical containers with the box's volume set at 100 in^3. The box has an open top. The cover will just be designed in reference to the box's dimensions. You are to present the design proposal for the box and cylinder to the Chief Executive Officer of the dairy company and head of the RER Packaging department.	Rubric on packaging box. Criteria: 1. Explanation of the proposal 2. Accuracy of computations 3. Utilization of the resources 4. Appropriateness of the model

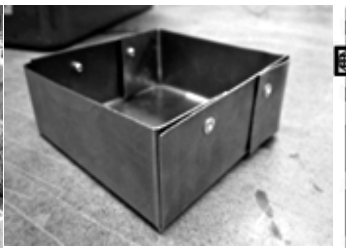
C. Planning for Teaching-Learning

Introduction:

This module covers key concepts in special products and factors of polynomials. It is divided into two lessons namely: (1) Special products, and (2) Factoring. In lesson 1, students will identify polynomials which are special products, find the product of polynomials using patterns, solve real – life problems involving special products and identify patterns in real – life which involves special products. In lesson 2, students will factor polynomials completely using the different rules and techniques in factoring, including patterns and concepts on special products. They will also apply their knowledge in factoring to solve some real – life problems.

In all lessons, students are given the opportunity to use their prior knowledge and skills in multiplying and dividing polynomials. Activities are also given to process their knowledge and skills acquired, deepen and transfer their understanding of the different lessons.

As an introduction to the module, ask the students following questions:


Have you at a certain time asked yourself how a basketball court was painted using less paint? Or how the architect was able to maximize the space of a building and yet was able to place what the owners want? Or how a carpenter was able to create a utility box using minimal materials? Or how some students were able to multiply some polynomial expressions in a least number of time?

Allow the students to give their response and process them after. Emphasize to the students their goal after completing this module and the lessons.

I. PRE – ASSESSMENT

1. Which mathematical statement is correct?
   a. $(2x - y)(3x - y) = 6x^2 - 5x^2y^2 + y^2$
   b. $(4x - 5)(4x - 5) = 16x^2 + 25$
   c. $(3x - 4)(2x + 7) = 6x^2 + 13x - 28$
   d. $(2x + 5)^2 = 4x^2 + 25$

   Answer: C

2. Which of the following DOES NOT belong to the group?

a. $\frac{1}{4}x^4 - 1$

b. $x^2 - 0.0001y^4$

c. $1.6(x - 1)^2 - 49$

d. $(x + 1)^4 - 4x^6$

Answer: C

3. Which of the following factors gives a product of $x^2 + 5x + 4$?

a. $(x + 1)(x + 4)$

b. $(x + 2)(x + 2)$

c. $(x + 5)(x - 1)$

d. $(x + 2)^2$

Answer: A

4. A polynomial expression is evaluated for the x- and y-values shown in the table below. Which expression was evaluated to give the values shown in the third column?

X	Y	**Value of the Expression**
0	0	0
-1	-1	0
1	1	0
1	-1	4

a. $x^2 - y^2$

b. $x^2 + 2xy + y^2$

c. $x^2 - 2xy + y^2$

d. $x^3 - y^3$

Answer: C

5. Find the missing term: $(x + \_\_\_)(3x + \_\_\_) = 3x^2 + 27x + 24$

a. 6, 4

b. 4, 6

c. 8, 3

d. 12, 2

Answer C

6. The length of a box is five inches less than twice the width. The height is 4 inches more than three times the width. The box has a volume of 520 cubic inches. Which of the following equations can be used to find the height of the box?

a. $W(2L - 5)(3H + 4) = 520$
b. $W(2W + 5)(3W - 4) = 520$
c. $W(2W - 5)(3W - 4) = 520$
d. $W(2W - 5)(3W + 4) = 520$

Answer: D

7. One of the factors of $2a^2 + 5a - 12$ is $a + 4$. What is the other factor?

a. $2a - 3$ b. $2a + 3$ c. $2a - 8$ d. $2a + 8$

Answer: A

8. The area of a square is $4x^2 + 12x + 9$ square units. Which expression represents the length of the side?

a. $(3x + 2)$ units
b. $(2x + 3)$ units
c. $(4x + 9)$ units
d. $(4x + 3)$ units

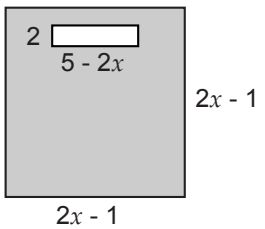
Answer: B

9. The side of a square is x cm long. The length of a rectangle is 5 cm. longer than the side of the square and the width is 5 cm shorter. Which statement is true?

a. the area of the square is greater than the area of the rectangle
b. the area of the square is less than the area of the rectangle
c. the area of the square is equal to the area of the rectangle
d. the relationship cannot be determined from the given information

Answer: A

10. A square piece of land was rewarded by a master to his servant. They agreed that a portion of it represented by the rectangle inside, should be used to construct a grotto. How large is the area of the land that is available for the other uses?



a. $4x^2 - 9$
b. $4x^2 + x + 9$
c. $4x^2 - 8x - 9$
d. $4x^2 + 9$

Answer: C

11. Which value for x will make the largest area of the square with a side of $3x + 2$?

a. $-\frac{3}{4}$

b. 0.4

c. $-\frac{1}{3}$

d. -0.15

Answer: C

12. Which procedure could not be used to solve for the area of the figure below?

a. $A = 2x\,(2x + 6) + \frac{1}{2}\,(2x)(x + 8)$

$A = 4x^2 + 12x + x^2 + 8x$

$A = 5x^2 + 20x$

b. $A = 2x(3x + 14) - 2(\frac{1}{2})(x)(x + 8)$

$A = 6x^2 + 28x - x^2 - 8x$

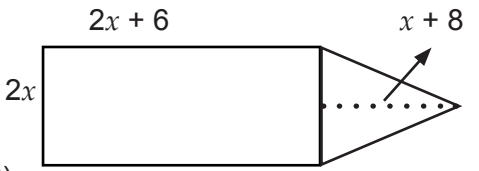
$A = 5x^2 + 20x$

c. $A = [2x(2x + 6) + (x + 8)(2x)] - 2(\frac{1}{2})(x)(x + 8)$

$A = [4x^2 + 12x) + (2x^2 + 16x) - (x^2 + 8x)$

$A = 6x^2 + 28x - x^2 - 8x$

$A = 5x^2 + 20x$



Answer: D

d. $A = 2x(2x + 6)+(\frac{1}{2})(2 + x)(x + 8)$

$A = 4x^2 + 12x + x^2 + 8x$

$A = 5x^2 + 20x$

13. Your classmate was asked to square $(2x - 3)$, he answered $4x^2 - 9$. Is his answer correct?

a. Yes, because squaring a binomial always produces a binomial product.

b. Yes, because product rule is correctly applied.

c. No, because squaring a binomial always produces a trinomial product.

d. No, because the answer must be $4x^2 + 9$.

Answer: C

14. Expression A: $4x^2 - 81$
Expression B: $(2x - 9)(2x + 9)$
If $x = 2$, which statement is true about the given expressions?
a. $A > B$
b. $A < B$
c. $A = B$
d. $A \neq B$
Answer: C

15. Your sister plans to remodel her closet. She hired a carpenter to do the task. What should your sister do so that the carpenter can accomplish the task according to what she wants?
a. Show a replica of a closet.
b. Download a picture from the internet.
c. Leave everything to the carpenter.
d. Provide the lay out drawn to scale.
Answer: D

16. Which of the following standards would best apply in checking the carpenter's work in item number 15?
a. accuracy of measurements and wise utilization of materials
b. accuracy of measurements and workmanship
c. workmanship and artistic design
d. workmanship and wise utilization of materials
Answer: B

17. The city mayor asked you to prepare a floor plan of the proposed day care center in your *barangay*. The center must have a small recreational corner. As head of the city engineering office, what will you consider in preparing the plan?

a. Feasibility and budget.
b. Design and budget
c. Design and Feasibility
d. Budget and lot area

Answer: A

18. Suppose there is a harvest shortage in your farm. What will you do to ensure a bountiful harvest in your farmland?
    a. Hire lot of workers to spread fertilizers in the farmland.
    b. Buy numerous sacks of fertilizers and spread it in his farmland.
    d. Find the area of the farmland and buy proportionate number of fertilizers.
    c. Solve for the number of fertilizers proportionate to the number of workers.
    Answer: C

19. The *Punong Barangay* in your place noticed that garbage is not properly disposed because of the small bins. As the chairman of health committee, you were tasked to prepare a garbage bins which can hold 24 ft^3 of garbage. However, the spot where the garbage bins will be placed is limited, how will you maximize the area?
    a. Find the dimensions of the planned bin according to the capacity given.
    b. Make a trial and error bins until the desired volume is achieved
    c. Solve for the factors of the volume and use it in creating bins.
    d. Find the area of the location of the bins
    Answer: A

20. As head of the marketing department of a certain construction firm, you are tasked to create a new packaging box for the soap products. What criteria will you consider in creating the box?
    a. Appropriateness and the resources used.
    b. Resources used and uniqueness
    c. Appropriateness and uniqueness
    d. Appropriateness and capacity
    Answer: D

LESSON 1 SPECIAL PRODUCTS

What to Know

Let us start our study of this module by reviewing first the concepts on multiplying polynomials, which is one of the skills needed in the study of this module. Discuss the questions below with a partner.

$97 \times 103 =$
$25 \times 25 =$
$99 \times 99 \times 99 =$

Allow the students to answer the following process questions:

1. What do you notice about the given expressions?
2. Did you solve them easily? Did you notice some patterns in solving their answers?
3. What technique/s did you use? What difficulties did you encounter?

You can present the following solution to the students:

97 x 103 = (100 – 3)(100 + 3)
= 1002 – 32
= 10000 – 9
= 9991

25 x 25 = (20 + 5)(20 + 5)
= 202 + 2(20)(5) + 52
= 400 + 200 + 25
= 625

99 x 99 x 99 = (100 – 1)3
= 1003 + 3(100)2(- 1) + 3(100)(-1)2 + (-1)3
= 10000000 – 30000 + 300 – 1
= 970 299

Discuss the given solution to the students and give more numerical problems and allow students to present their solutions, challenge them to look for another pattern to solve problems presented. Do this mentally, e.g. **(42)(38), (57)(63), (42)(42).**

Lesson 1 Special Products

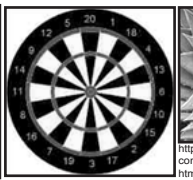
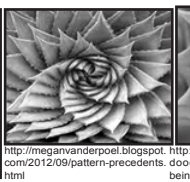
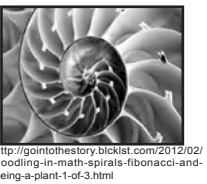
What to Know

Let us start our study of this module by reviewing first the concepts on multiplying polynomials, which is one of the skills needed in the study of this module. Discuss the questions below with a partner.

PATTERNS WHERE ARE YOU?

Have you ever looked around and recognized different patterns? Have you asked yourself what the world's environment would look like if there were no patterns? Why do you think our Creator includes patterns around us?

Look at the pictures below and identify the different patterns on each picture. Discuss these with a partner and see whether you observe the same pattern.




http://meganvanderpoel.blogspot.com/2012/09/pattern-precedents.html

http://gointothestory.blcklst.com/2012/02/doodling-in-math-spirals-fibonacci-and-being-a-plant-1-of-3.html

Have you ever used patterns in simplifying mathematical expressions? What advantages have you gained in doing such? Let us see how patterns are used to simplify mathematical expressions by doing the activity below. Try to multiply the following numerical expressions. Can you solve the following numerical expressions mentally?

$97 \times 103 =$
$25 \times 25 =$
$99 \times 99 \times 99 =$

Now, answer the following questions:

1. What do you notice about the given expressions?
2. Did you solve them easily? Did you notice some patterns in finding their answers?
3. What technique/s did you use? What difficulties did you encounter?

The indicated products can be solved easily using different patterns.

Post the topical questions and allow the students to write their answer on the Initial portion of the IRF worksheets in their journal notebook, this will enable you to know if they have idea on the lesson. (1) What makes a product special? and (2) What patterns are involved in multiplying expression?

The next activity is provided to review the multiplication skills of the students in polynomials, which is an important skill in completing this module. Allow the students to do the activity by pair. (Note: If most of the students have not yet attained the level of mastery in this skill, bridge on the topic multiplying polynomials)

Answers Key

Activity 3

1 a^2	6a	2 9		3 $9b^2$
- 81		a^2		- 24ab
		4 b^2	8a	$16a^2$
	5 $-16a^2$	6a		
	6 b^2	- 6b	7 8	
8 $16a^6$		9 - 2ab	- 6a	10 $4a^2$
11 $-25b^4$	$49a^{14}$		12 a^2	$-36b^2$

Teacher's Note and Reminders

Are your solutions different from your classmates? What was used in order to find the products easily?

*The problems you have answered are examples of the many situations where we can apply knowledge of special products. In this lesson, you will do varied activities which will help you answer the question, "**How can unknown quantities in geometric problems be solved**?"*

Let's begin by answering the "I" portion of the IRF Worksheet shown below. Fill it up by writing your initial answer to the topical focus question:

Activity 1 IRF WORKSHEET

Description: Below is the IRF worksheet which will determine your prior knowledge about the topical question.

Direction: Answer the topical questions: **(1) What makes a product special**? and **(2) What patterns are involved in multiplying algebraic expressions**? Write your answer in the initial part of the IRF worksheet.

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

Activity 2 COMPLETE ME!

Description: This activity will help you review multiplication of polynomials, the pre-requisite skill to complete this module.

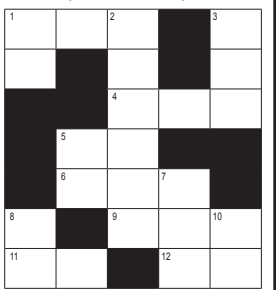
Directions: Complete the crossword polynomial by finding the indicated products below. After completing the puzzle, discuss with a partner the questions that follow.

Across

1. $(a + 3)(a + 3)$
4. $(b + 4a)^2$
5. $2a\,(-8a + 3a)$
6. $(b - 2)(b - 4)$
9. $-2a(b + 3a - 2)$
11. $(5b^2 + 7a^2)(-5b^2 + 7a^2)$
12. $(a - 6b)(a + 6b)$

Down

1. $(a + 9)(a - 9)$
2. $(3 + a + b)^2$
3. $(3b - 4a)(3b - 4a)$
5. $(-4a + b)(4a + b)$
7. $(2 - a)(4 - a)$
8. $(4a^3 - 5b^2)(4a^3 + 5b^2)$
10. $(2a + 6b)(2a - 6b)$



Activity 3 is provided to reinforce activity 2, but this time students should see the different patterns and initially will have an idea on the different special products. Provide station for each group where they will post their outputs. Allow the students to roam around and observe the different answers of the other groups. (Note: if you are handling a big class, cases may be given to more than 1 group to accommodate the class)

CASE 1:

$(x + 5)(x - 5) = x^2 - 25$
$(a - b)(a + b) = a^2 - b^2$
$(x + y)(x - y) = x^2 - y^2$
$(x - 8)(x + 8) = x^2 - 64$
$(2x + 5)(2x - 5) = 4x^2 - 25$

CASE 2:

$(x + 5)(x + 5) = x^2 + 10x + 25$
$(a - b)^2 = a2 - 2ab + b^2$
$(x + y)(x + y) = x^2 + 2xy + y^2$
$(x - 8)^2 = 16x + 64$
$(2x + 5)(2x + 5) = 4x^2 - 20x + 25$

CASE 3:

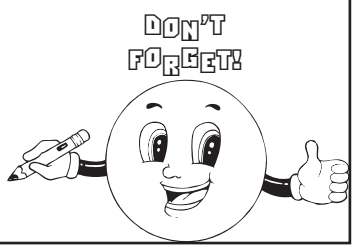
$(x + 5)^3 = x^3 + 15x^2 + 75x + 125$
$(a - b)(a - b)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$
$(x + y)^3 = x3 + 3x2y + 3xy2 + y3$
$(x + 4)(x + 4)(x + 4) = x^3 + 12x^2 + 48x + 64$
$(x + 2y)^3 = x^3 + 3x^2y + 12xy^2 + 8y^3$

CASE 4:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$
$(m + 2n - 3f)^2 = m^2 + 4n^2 + 9f^2 + 4mn - 6fm - 12fn$

You can process their answers after the activity; allow their outputs to be posted on their stations all throughout the lesson, so that they can revisit it on the course of the discussion.

Teacher's Note and Reminders



QUESTIONS ?

1. How did you find each indicated product?
2. Did you encounter any difficulty in finding the products? Why?
3. What concept did you apply in finding the product?

Activity 3 GALLERY WALK

Description: This activity will enable you to review multiplication of polynomials.
Direction: Find the indicated product of the expressions that will be handed to your group. Post your answers on your group station. Your teacher will give you time to walk around the classroom and observe the answers of the other groups. Answer the questions that follow.

CASE 1:

$(x + 5)(x - 5) =$
$(a - b)(a + b) =$
$(x + y)(x - y) =$
$(x - 8)(x + 8) =$
$(2x + 5)(2x - 5) =$

CASE 2:

$(x + 5)(x + 5) =$
$(a - b)^2 =$
$(x + y)(x + y) =$
$(x - 8)^2 =$
$(2x + 5)(2x + 5) =$

CASE 3:

$(x + 5)^3 =$
$(a - b)(a - b)(a - b) =$
$(x + y)^3 =$
$(x + 4)(x + 4)(x + 4) =$
$(x + 2y)^3 =$

CASE 4:

$(a + b + c)(a + b + c) =$
$(x + y + z)(x + y + z) =$
$(m + 2n - 3f)^2 =$

QUESTIONS ?

1. How many terms do the products contain?
2. Compare the product with its factors. What is the relationship between the factors and the terms of their product?
3. Do you see any pattern in the product?
4. How did this pattern help you in finding the product?

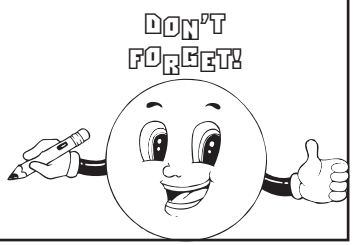
What to Process

These are the enabling activities / experiences that the learner will have to go through to validate their observations in the previous section. Interactive activities are provided for the students to check their understanding on the lesson.

Before performing activity 4 give first a short introduction on what is a square of binomial and how it is written mathematically. Ask them how they simplify such expressions.

Activity 4 can be performed by pair or as a group. Roam around to observe whether the students are doing the activity correctly. Use process questions to guide the students.

Teacher's Note and Reminders



You just tried finding the indicated products through the use of patterns. Are the techniques applicable to all multiplication problems? When is it applicable and when is it not?

Let us now find out what the answer is by doing the next part. What you will learn in the next sections will enable you to do the final project which involves making of a packaging box using the concepts of special products and factoring.

Let us start by doing the next activity.

What to Process

Your goal in this section is to learn and understand key concepts related to finding special products. There are special forms of algebraic expressions whose products are readily seen and these are called **special products**. There are certain conditions which would make a polynomial special. Discovering these conditions will help you find the product of algebraic expressions easily. Let us start in squaring a binomial

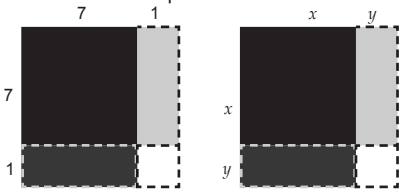
The square of a binomial which is expressed as $(x + y)^2$ or $(x + y)(x + y)$ and $(x - y)^2$ or $(x - y)(x - y)$ respectively. In your previous grade you did this by applying the FOIL method, which is sometimes tedious to do. There is an easier way in finding the desired product and that is what are we going to consider here.

Activity 4 FOLD TO SQUARE

Description: In this activity, you will model square of a binomial through paper folding. Investigate the pattern that can be produced in this activity. This pattern will help you find the square of a binomial easily. You can do this individually or with a partner.

Directions: Get a square paper measuring 8" × 8"

1. Fold the square paper 1" to an edge and make a crease.
2. Fold the upper right corner by 1" and make a crease.
3. Unfold the paper.
4. Continue the activity by creating another model for squaring a binomial by changing the measures of the folds to 2 in. and 3 in. Then answer the questions below.



	FIRST TERM	SECOND TERM	LAST TERM
$(x + 1)2$	x^2	$2x$	1
$(x + 2)2$	x^2	$4x$	4
$(x + 3)2$	x^2	$6x$	9
$(x + y)2$	x^2	$2xy$	y^2

Let them complete the table and emphasize that the first terms are the area of big squares, second terms are the total areas of the rectangles and the last terms are the areas of the small squares.

Note: Use process questions to guide the students in completing the table and recognized the pattern that exists in squaring binomials. Provide opportunity to the students to create their rule in this special product.

After completing the activity, process their answers and lead them in the discovery of the rule. Give more examples to the students to firm their understanding of the lesson. You can use video lessons, if available, in the discussion of this topic. URL's are provided in the students learning modules.

Challenge the students to ponder on the equation $(a + b)^2 = a^2 + b^2$. Let the students realize that the two expressions are not equal and that the product of squaring a binomial is a perfect square trinomial.

Teacher's Note and Reminders

QUESTIONS ?

1. How many different regions are formed? What geometric figures are formed? Give the dimensions of each region?
2. What is the area of each region?
3. If the longer part is represented by x, what will be its area? by x and 1?
4. What is the sum of the areas? Write the sum of areas in the box below.
5. If 1 is replaced by y, what will be the area?

	FIRST TERM	SECOND TERM	LAST TERM
$(x + 1)^2$			
$(x + 2)^2$			
$(x + 3)^2$			
$(x + y)^2$			

Did you find any pattern? What pattern is it?

1. How is the first term of the product related to the first term of the given binomial?
2. How is the last term of the product related to the last term of the given binomial?
3. What observation do you have about the middle term of the product and the product of the first and last terms of the binomial?

Observe the following examples:

a. $(x - 3)^2 = (x)^2 - 3x - 3x + (3)^2$
$= x^2 - 2(3x) + 9$
$= x^2 - 6x + 9$

b. $(x + 5)^2 = (x)^2 + 5x + 5x + (5)^2$
$= x^2 + 2(5x) + 25$
$= x^2 + 10x + 25$

c. $(3x + 4y)^2 = (3x)^2 + 12xy + 12xy + (4y)^2$
$= 9x^2 + 2(12xy) + 16y^2$
$= 9x^2 + 24xy + 16y^2$

Remember:

- Product rule
  $(a^m)(a^n) = a^{m+n}$
- Raising a power to a power
  $(a^m)^n = a^{mn}$

You can give this one as drill to the students.

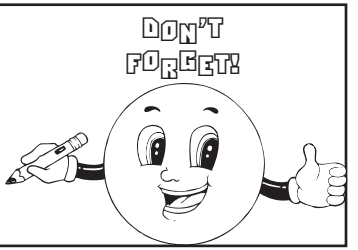
1. $(s + 4)^2 = s^2 + 8s + 16$
2. $(w - 5)^2 = w^2 - 10w + 25$
3. $(e - 7)^2 = e2 - 14e + 49$
4. $(2q - 4)^2 = 4q^2 - 16q + 16$
5. $(3z + 2k)^2 = 9z^2 + 12zk + 4k^2$
6. $(5d - 7d^2t)^2 = 25d^2 - 70d^3t + 49d^4t^2$
7. $(7q^2w^2 - 4w^2)^2 = 49q^4w^4 - 56q2w^4 + 16w^4$
8. $(\frac{2}{3}e - 6)^2 = \frac{4}{9}e^2 - 8e+36$
9. $(\frac{4}{5}kj - 6)^2 = \frac{16}{25}k^2j^2 - \frac{48}{5}kj + 36$
10. $[(x + 3) - 5]^2 = x^2 - 7x + 64$

You can use problem no. 10 in the learning module to link the next topic from the previous lesson to model squaring trinomials, allow them to do as a group the paper cutting activity.

Provide hooking questions to the students before performing this activity. Ask them how they will square a trinomial even without going to the process of FOIL method. Activity 5 may be given as a group activity or by pair. Remind the students that the folding they performed in creating vertical creases should be equal to the folds that they will do to create horizontal creases. After folding they can cut the creases to form different figures. They can use the measurement found in their learning modules.

Use process questions for the students to realize that the square of trinomial can be modelled by $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$. Provide more examples to generate rules in squaring trinomials.

Teacher's Note and Reminders



The square of binomial, consists of:

a. the square of the first term;
b. twice the product of the first and last terms; and
c. the square of the last term.

Remember that the square of a binomial is called a ***perfect square trinomial****.*

LET'S PRACTICE!

Square the following binomials using the pattern you have learned.

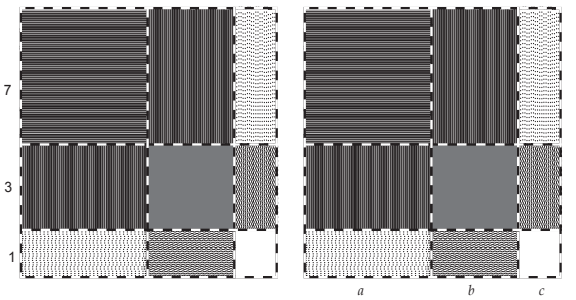
1. $(s + 4)^2$
2. $(w - 5)^2$
3. $(e - 7)^2$
4. $(2q - 4)^2$
5. $(3z + 2k)^2$
6. $(5d - 7d^2t)^2$
7. $(7q^2w^2 - 4w^2)^2$
8. $(\frac{2}{2}e - 6)^2$
9. $(\frac{4}{5}kj - 6)^2$
10. $[(x + 3) - 5]^2$

The square of a binomial is just one example of special products. Do the next activity to discover another type of special product, that is squaring a trinomial.

Activity 5 DISCOVER ME AFTER! (PAPER FOLDING AND CUTTING)

Description: In this activity you will model and discover the pattern on how a trinomial is squared that is $(a + b + c)^2$. Investigate and observe the figure that will be formed.

Directions: Get a 10" × 10" square paper. Fold the sides 7", 3" and 1" vertically and make crease. Using the same measures, fold horizontally and make creases. The resulting figure should be the same as the figure below.



Teacher's Note and Reminders

Answer:

1. $r^2 + t^2 + n^2 - 2rt + 2rn - 2tn$
2. $e^2 + 4a^2 + q^2 + 4ae + 2eq + 4q$
3. $m^2 + a^2 + y^2 + 2ma - 2my - 2ay$
4. $4s^2 + o^2 + 16n^2 + 4so - 16sn - 8on$
5. $4i^4 + 9a^2 + 25n^2 + 12i^2a - 10i^2n - 30an$
6. $225a^2 + 16n^2 + 36 - 120an + 48n - 180a$
7. $16a^2 + 16b^2 + 16c^2 + 24ab + 24ac + 24bc$
8. $81a^4 + 16b^4 + 9c^4 + 72a^2b^2 - 54a^2c^2 - 24a^2c^2$
9. $2.25a^4 + 5.29b^2 + 1 - 6.9a^2b + 3a^2 - 4.6b$
10. $\frac{9}{16}x^2 + \frac{16}{9}y^2 - 36 + 2xy + 9x - 16y$

QUESTIONS ?

1. How many regions are formed? What are the dimensions of each region in the figure?
2. What are the areas of each region?
3. If the side of the biggest square is replaced by a, how will you represent its area?
4. If one of the dimensions of the biggest rectangle is replaced by b, how will you represent its area?
5. If the side of the smaller square is replaced by c, how will you represent its area?
6. What is the sum of the areas of all regions? Do you observe any pattern in the sum of their areas?

Observe the following examples and take note of your observation.

a. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
b. $(m + n - d)^2 = m^2 + n^2 + d^2 + 2mn - 2md - 2nd$
c. $(3d + 2e + f)^2 = 9d^2 + 4e^2 + f^2 + 12de + 6df + 4ef$

The square of a trinomial, consists of:

a. the sum of the squares of the first, second and last terms;
b. twice the product of the first and the second terms;
c. twice the product of the first and the last terms; and
d. twice the product of the second and the last terms.

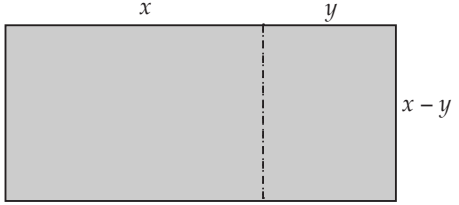
LET'S PRACTICE!

Square the following trinomials using the pattern you have learned.

1. $(r - t + n)^2$
2. $(e + 2a + q)^2$
3. $(m + a - y)(m + a - y)$
4. $(2s + o - 4n)^2$
5. $(2i^2 + 3a - 5n)^2$
6. $(15a - 4n - 6)^2$
7. $(4a + 4b + 4c)^2$
8. $(9a^2 + 4b^2 - 3c^2)^2$
9. $(1.5a^2 - 2.3b + 1)^2$
10. $(\frac{3x}{4} + \frac{4y}{3} - 6)^2$

Emphasize to the students that the activity will help them model the product of sum and difference of two terms. Make them realize that the area of the resulting figure is the product of the sum and difference of two terms. The resulting figure should be the same as the figure found below.

After doing the above activity, provide for more examples for the students to see the pattern in finding the product of the sum and difference of binomials. (Note: In the above activity, help the students realize that the dimension of the rectangle is $(x + y)$ by $(x - y)$ and its area is $x^2 - y^2$.)



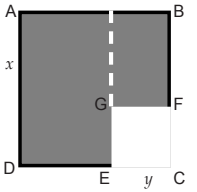
Answers on lets practice.

1. $w^2 - 36$
2. $a^2 - 16c^2$
3. $16y^2 - 25d^2$
4. $16f^2 - 9s^2d^2$
5. $144x^2 - 9$
6. $9s^4r^4 - 49a^2$
7. $L^6o^8v^{10} - 36e^6$
8. $\frac{25}{36}g^4a^4 - \frac{4}{9}d^4$
9. $4s^2nq^2m - 9d^6k$
10. $(s + 2)^2 - 16 = s^2 + 4s - 12$

Activity 6 TRANSFORMERS

Description: This activity will help us model the product of the sum and difference of two terms $(x - y)$ $(x + y)$ and observe patterns to solve it easily.

Directions: Prepare a square of any measure; label its side as x. Cut a small square of side y from any of its corner (as shown below). Answer the questions that follow.



In terms of x and y, answer the following:

1. What is the area of the original big square (**ABCD**)?
2. What is the area of the small square (**GFCE**)?
3. How are you going to represent the area of the new figure?

Cut along the broken line as shown and rearrange the pieces to form a rectangle.

1. What are the dimensions of the rectangle formed?
2. How will you get the area of the rectangle?
3. Represent the area of the rectangle that was formed. Do you see any pattern in the product of the sum and difference of two terms?

Study the relationship that exists between the product of the sum and difference of two terms and the factors and take note of the pattern formed.

a. $(x + y)(x - y) = x^2 - y^2$
b. $(a - b)(a + b) = a^2 - b^2$
c. $(m + 3)(m - 3) = m^2 - 9$
d. $(w - 5)(w + 5) = w^2 - 25$
e. $(2x - 5)(2x + 5) = 4x^2 - 25$

*The product of the sum and difference of two terms is the **difference of the squares of the terms**. In symbols, **(x + y)(x − y) = x² − y²**. Notice that the product is always a binomial.*

LET'S PRACTICE!

Multiply the following binomials using the patterns you have learned.

1. $(w - 6)(w + 6)$
2. $(a + 4c)(a - 4c)$
3. $(4y - 5d)(4y + 5d)$
4. $(3sd + 4f)(4f - 3sd)$

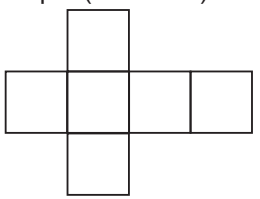
In activity no. 7, ask the students to do the solid figures the day before the activity. Cubes must have the following sizes:
Solid figures:

1. One 3 in. x 3 in. cube using card board.
2. One 1 in. x 1 in. cube using card board.
3. Three prisms whose square base is 3 in. and height of 1 in.
4. Three prisms whose square base is 1 in. and height of 3 in.

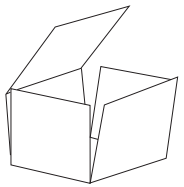
Ask the students to calculate the volume of each solid figure.

Note: The following are the patterns in creating the solid figures:

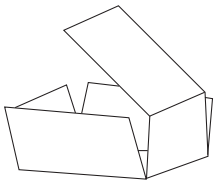
A. Step 1 (for cubes)



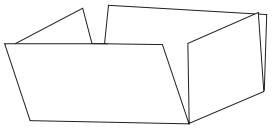
B.



A. Step 1 (for prism)



B.



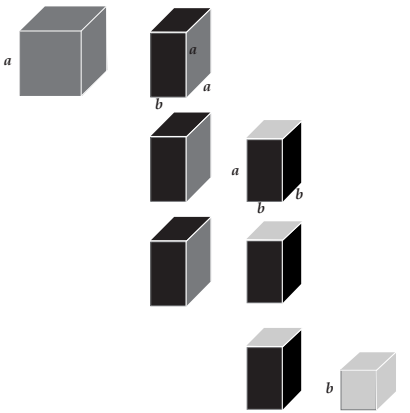
Give more exercises to the students regarding the lesson, allow the students to state in their own words the rule in cubing binomials based on the activity and examples. After the discussion, **have a short summary of all types of special products** the students have encountered.

5. $(12x - 3)(12x + 3)$
6. $(3s^2r^2 + 7q)(3s^2r^2 - 7q)$
7. $(l^3o^4v^5 - 6e^3)\,(l^3o^4v^5 + 6e^3)$
8. $(\frac{5}{6}g^2a^2 - \frac{2}{3}d^2)(\frac{5}{6}g^2a^2 + \frac{2}{3}d^2)$
9. $(2s^nq^m + 3d^{3k})\,(2s^nq^m - 3d^{3k})$
10. $[(s + 2) - 4][(s + 2) + 4]$

The previous activity taught you on how to find the product of sum and difference of two terms using patterns. Perform the next activity to discover another pattern in simplifying expressions of polynomials.

Activity 7 CUBRA CUBE

Description: A cubra cube is a set of cubes and prisms connected by nylon. The task is to form a bigger cube using all the figures provided. Your teacher will help you how to form a cubra cube. After performing the activity, answer the questions that follow.



QUESTIONS ?

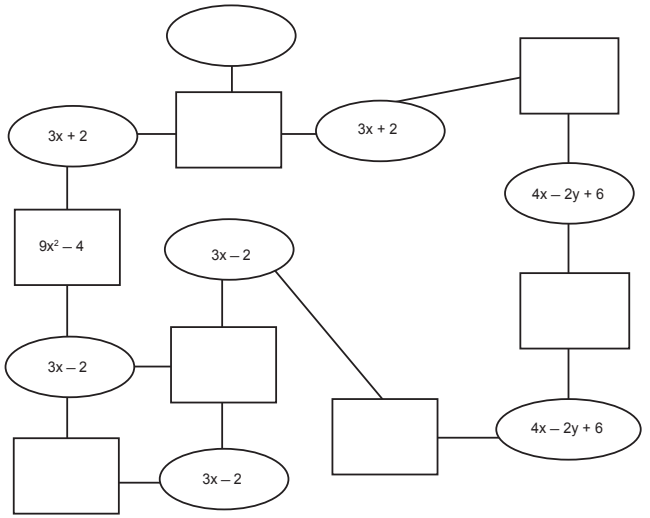
1. How many big cubes did you use? Small cubes?
2. How many different prisms do you have?
3. How many prisms are contained in the new cube?
4. What is the total volume of the new cube formed?
5. If the side of the big cube is marked as a and the smaller cube is marked as b, what is the volume of each figure?
6. What will be the total volume of the new cube?
7. What are the dimensions of the new cube?

Let the students complete the Revised part of the IRF worksheet, but this time they must have already realize and be able to correct the mistakes they have on initial part.

To reinforce students understanding, let them do the web – based exercises / games or you can have them the linking base game by group (found below). The students will write the product of the branches where the rectangle is attached. You can modify the example below to suit it your learners.

To include valuing, relate the activity to an organization, by asking the following questions:

1. What will happen to the web if one of your groupmate wrote the wrong product in one box?
2. What will happen in an organization if one of the members failed to do his job?



Note: You can use the video lessons found in the learning modules for the discussion of different types of special products.

Activity 9 (3 – 2- 1 chart) should be completed for you to know if there are still some confusions about the lesson. This activity should be served as bring home activity.

This time let us go back to the gallery walk activity and focus to case 3 which is an example of a cube of binomial $(x + y)^3$ or **$(x + y)(x + y)(x + y)$ and** $(x - y)^3$ or **$(x - y)(x - y)(x - y)$**.

To find the cube of a binomial of the form $(x + y)^3$:

a. Find the cube of each term to get the first and the last terms.
   $(x)^3$, $(y)^3$
b. The second term is three times the product of the square of the first term and the second term. **$3(x)^2(y)$**
c. The third term is three times the product of the first term and the square of the second term. **$3(x)(y)^2$**

Hence, $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

To find the cube of a binomial of the form $(x - y)^3$:

a. Find the cube of each term to get the first and the last terms.
   $(x)^3$, $(-y)^3$
b. The second term is three times the product of the square of the first term and the second term. **$3(x)^2(-y)$**
c. The third term is three times the product of the first term and the square of the second term. **$3(x)(-y)^2$**

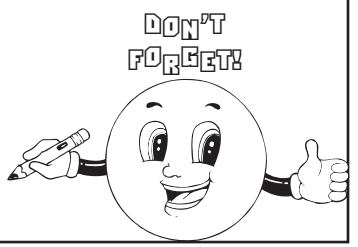
Hence, $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

Activity 8 IRF WORKSHEET

Description: Now that you have learned how to find the different special products, using the "R" portion of the IRF Worksheet, answer the topical focus question: **What makes a product special? What patterns are involved in multiplying algebraic expression?**

Initial Answer
Revised Answer
Final Answer

Teacher's Note and Reminders



WEB – BASED ACTIVITY: DRAG AND DROP

Description: Now, that you have learned the various special products, you will now do an interactive activity which will allow you to drag sets of factors and drop them beside special products. The activity is available in this website: http://www.media.pearson.com.au/schools/cw/au_sch_bull_gm12_1/dnd/2_spec.html.

QUESTIONS:

1. What special products did you use in the activity?
2. Name some techniques which you used to make the work easier.
3. What generalizations can you draw out of the examples shown?
4. Given the time constraint, how could you do the task quickly and accurately?

Activity 9 3-2-1 CHART

Description: In this activity, you will be asked to complete the 3-2-1 Chart regarding the special products that you have discovered.

3-2-1 Chart
Three things I found out:
1. _____ 2. _____ 3. _____
Two interesting things:
1. _____ 2. _____
One question I still have:
1. _____

As concluding activity for process, ask the students to complete the chart above by giving the different types of special products and state its step, they can add box if necessary. This activity may be given as bring home activity.

Teacher's Note and Reminders

What to Understand

This part provides learners activities to further validate and deepen their understanding on the applications of special products.

Answers:

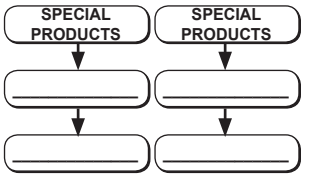
1. a. 10 m by 6 m
   b. $(8 + 2)(8 - 2)m^2 = (10)(6) = 60\ m^2$, difference of two squares
   c. $(x + 2)(x - 2)m^2$
   d. No, the area will be decreased by 4 m^2

2. a. 64 in^2, 256 in^2
   b. (74)(128) = 9472 ÷ 64 = 148 tiles
   c. 37
   d. Either, because he will spend the same amount whichever tile he will use.

Activity 10 WHAT'S THE WAY, THAT'S THE WAY!

Description: This activity will test if you really have understood our lesson by giving the steps in simplifying expressions containing special products in your own words.

Directions: Give the different types of special products and write the steps/process of simplifying it. You may add boxes if necessary.



Now that you know the important ideas about how patterns on special products were used to find the product of an algebraic expressions, let's go deeper by moving on to the next section.

What to Understand

Now that you have already learned and identified the different polynomials and their special products. You will now take a closer look at some aspects of the topic and check if you still have misconceptions about special products.

Activity 11 DECISION, DECISION, DECISION!

Directions: Help each person decide what to do by applying your knowledge on special products on each situation.

1. Jem Boy wants to make his 8 *meters* square pool into a rectangular one by increasing its length by 2 m and decreasing its width by 2 m. Jem Boy asked your expertise to help him decide on certain matters.

http://www.oyster.com/las-vegas/hotels/luxor-hotel-and-casino/photos/square-pool-north-luxor-hotel-casino-v169561/#

To check if the students still has difficulty in this topic, as bring home activity ask the students to do the muddiest point **(Activity 12)**. Process their answers the next day for your assessment if they are ready for the next topic

Teacher's Note and Reminders

a. What will be the new dimensions of Jem Boy's pool?
b. What will be the new area of Jem Boy's pool? What special product will be use?
c. If the sides of the square pool is unknown, how will you represent its area?
d. If Jem Boy does not want the area of his pool to decrease, will he pursue his plan? Explain your answer.

1. Emmanuel wants to tile his rectangular floor. He has two kinds of tiles to choose from, one of which is larger than the other. Emmanuel hired your services to help him decide which tile to use.
   a. What area will be covered by the 8" x 8" tile? 16" x 16" tile?
   b. If the rectangular floor has dimensions of 74" x 128", how many small square tiles are needed to cover it?
   c. How many square big tiles are needed to cover the rectangular floor?
   d. If each small tile costs Php 15.00 and each big tiles costs Php 60.00, which tile should Emmanuel use to economize in tiling his floor? Explain why.

Activity 12 AM I STILL IN DISTRESS?

Description: This activity will help you reflect about things that may still confuse you in this lesson.

Directions: Complete the phrase below and write it on your journal.

The part of the lesson that I still find confusing is _____ because _____.

Let us see if by your problem will be solved doing the next activity, where the muddiest point will be clarified.

Note: Before **activity 13** should be provided, prepare the students by letting them watch a video on solving composite areas or giving them examples of this type of problems.
In this activity, you should let the students realize that unknown quantities can be represented by any variable.

Answers:

1. a. x^2 sq. units, y^2 sq. units, xy sq. units
   b. $(x^2 + 2xy + y^2)$ sq. units
   c. Squaring binomial

2. a. x^2 sq. unit, y^2 sq. units
   b. $9y^2$ sq. units
   c. $(x^2 – 9y^2)$sq. units

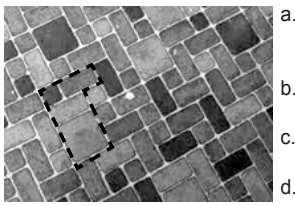
For the portfolio entry by the students, have the students form as group and do **Activity 14**, this must be done outside the class. Tell the students that they can sketch the figure if they have no devices to use.

Teacher's Note and Reminders

Activity 13 BEAUTY IN MY TILE!

Description: See different tile patterns on the flooring of a building and calculate the area of the region bounded by the broken lines, then answer the questions below.

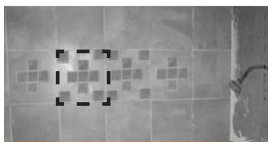
1.



http://www.apartmenttherapy.com/tile-vault-midcentury-rec-room-39808

a. What is the area represented by the big square? small square? rectangles?
b. What is the total area bounded by the region?
c. What special product is present in this tile design?
d. Why do you think the designer of this room designed it as such?

2.



a. What is the area represented by the big square? Small square?
b. What is the sum of all areas of small squares?
c. If the small squares were to be removed, how are you going to represent the area that will be left?

QUESTIONS ?

1. What difficulties did you experience in doing the activity?
2. How did you use special products in this activity?
3. What new insights did you gain?
4. How can unknown quantities in geometric problems be solved?

Activity 14 WHERE IS THE PATTERN?

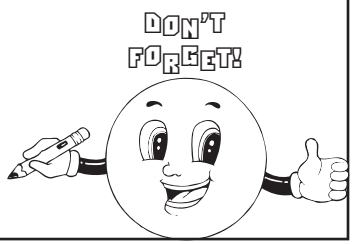
Descriptions: Take a picture/sketch of a figure that makes use of special products. Paste it in a piece of paper.

QUESTIONS ?

1. Did you find difficulty in looking for patterns where the concept of special products were applied?
2. What special products were applied in your illustration?
3. What realization do you have in this activity?

As part of the concluding activity for process, have the student's debate on the answers of the questions found in **Activity 15**. You can have the class form the different rules for debate.
As culminating activity of the students in this section, ask them to fill – up the final part of the IRF worksheet, this may be assigned as bring home activity. Tell them to compare their answers with the other and ask them their realization in this topic.

Teacher's Note and Reminders



What to Transfer

Provide learners opportunities to apply their understanding on special products activities that reflect meaningful and relevant problems/situations. Students will have the opportunity to see the relevance of their discussions and its possible application in real – life.

Activity 15 LET'S DEBATE!

Description: Form a team of 4 members with your classmates and debate on the two questions below. The team that can convince the other wins the game.

- "Which is better to use in finding products, patterns or long multiplication?"
- "Which will give us more benefit in life, taking the short – cuts or going the long way?

Activity 16 IRF WORKSHEET

Description: Now that you have learned the different special products, using the "F" portion of the IRF Worksheet, answer the topical focus question: **What makes a product special? What patterns are involved in multiplying algebraic expressions?**

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Let us now apply your learning to real – life situations. You will be given a practical task which will demonstrate your understanding.

Before doing the activity, ask the students' to do a research on the uses and importance of genetics in the study of human life. And give the following definition and small discussions on genetics especially the heterozygous and homozygous traits.

- **Genetics** is the area of biological study concerned with heredity and with the variations between organisms that result from it.
- **Homozygous** refers to having identical alleles (group of genes) for a single trait. **(SS)**
- **Heterozygous** refers to having two different alleles (group of genes) for a single trait. **(Ss)**

Note: Capital letter denotes dominant traits, while small letter denotes recessive traits. Dominant traits will come out in heterozygous.

Teacher's Note and Reminders

Activity 17 MAKE A WISH

Description: Concept of squaring binomials is used in the field of Genetics through PUNNET squares. PUNNETT SQUARES are used in genetics to model the possible combinations of parents' genes in offspring. In this activity you will discover how it will be used.

Direction: Investigate how squaring trinomials are applied in PUNNET squares and answer the following questions.

One cat carries heterozygous, long-haired traits (Ss), and its mate carries heterozygous, long-haired traits (Ss). To determine the chances of one of their offspring having short hair we can use PUNNET Squares.

	S	s
S	SS	Ss
s	Ss	ss

1. What are the chances that the offspring is a long – haired cat? A short – haired cat?
2. What are the different possible offsprings of the mates?
3. How many homozygous offsprings will they have? Heterozygous?
4. How is the concept of squaring binomials used in this process?
5. Do you think it is possible to use the process of squaring trinomials in the field of genetics?
6. Create another model of PUNNET square using a human genetic component. Explain the possible distribution of offsprings and how squaring trinomials help you in looking for its solution.
7. Create your own PUNNET square using the concept of squaring trinomials, using your dream genes.

Now that you have seen the different patterns that can be used in simplifying polynomial expressions, you are now ready to move to the next lesson which is factoring. Observe the different patterns in factoring that are related to special products so that you can do your final project, the making of packaging box.

Punnett square is named after **Reginald C. Punnett**, who devised the approach, and is used by biologists to determine the chances of an offspring's having a particular genotype. The Punnett square is a tabular summary of every possible combination of one maternal allele with one paternal allele for each gene being studied in the cross.

Lesson 2: Factoring

What to Know

Initially, begin with some interesting and challenging activities that will enable the students to see the association of products to factors and activate their prior knowledge on factoring.

As a review on basic concepts of factoring, allow the students to give the different dimensions of rectangle they can create out of a square whose area is 36 units squared (e.g. 18 and 2, 9 and 4), with this they will realize that the different factors of 36 are the dimensions of rectangle. Ask the topical question to the students and the essential question.

"What expressions can be factored? How are patterns used in finding the factors of an expression? How can unknown quantities in geometric problems be solved?

To start the lesson perform **Activity 1** by distributing thumbs up icon to the students and allow them to paste it under the response column. Thumbs up means a student has little mastery on the skills described and a thumbs down signifies that the student has already mastered the skills. This will serve as your guidance into the skills students still needed in this lesson. You can add another row for skills if necessary

Teacher's Note and Reminders

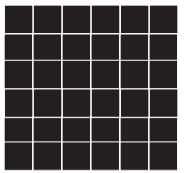
Lesson 2 Factoring

What to Know

Your goal in this section is to see the association of products to factors by doing the activities that follows.

Before you start doing the activities in this lesson, first do this challenge.

The figure below is a square made up of 36 tiles. Rearrange the tiles to create a rectangle, having the same area as the original square. How many such rectangles can you create? What are your considerations in looking for the other dimensions? What mathematical concepts did you consider in forming different dimensions? Why? Suppose the length of one side is increased by unknown quantities (e.g. x) how could you possibly represent the dimensions?



This module will help us break an expression into different factors and answer the topical questions, "**What algebraic expressions can be factored**? **How are patterns used in finding the factors of algebraic expression**? **How can unknown quantities in geometric problems be solved**?

To start with this lesson, perform the activities that follow.

Activity 1 LIKE! UNLIKE!

Description: This activity will help your teacher gauge how ready you are for this lesson through your responses.

Directions: Answer all the questions below honestly by pasting the *like* or *unlike* thumb that your teacher will provide you. *Like* means that you are the one being referred and *unlike* thumb means that you have no or little idea about what is being asked.

For conceptual map, ask the students to complete the I part of the IRF sheet. Ask the group to keep their answer so that they can revisit it after discussions.

To activate prior knowledge of the students on the skills they will use in this lesson perform **Activity 3**, at this point students should realize the association of factors and products, and observe the different pattern that will exist. Use the questions found after the activity..

Teacher's Note and Reminders

SKILLS ACQUIRED	RESPONSES
1. Can factor numerical expressions easily	
2. Can divide polynomials	
3. Can apply the quotient rule of exponents	
4. Can add and subtract polynomials	
5. Can work with special products	
6. Can multiply polynomials	

Before you proceed to the next topic, answer first the IRF form to determine how much you know in this topic and see your progress.

Activity 2 IRF WORKSHEETS

Description: Complete the table by filling first the initial column of the chart. This activity will determine how much you know about this topic and your progress.

	Initial	Revise	Final
Express the following as product of factors. 1. $4x^2 - 12x$ = _____ 2. $9m^2 - 16n^2$ = _____ 3. $4a^2 + 12a + 9$ = _____ 4. $2x^2 + 9x - 5$ = _____ 5. $27x^3 - 8y^3$ = _____ 6. $a^3 + 125b^3$ = _____ 7. $xm + hm - xn - hn$ = _____			

Answers Key

Activity 3
"FACTORING IS THE REVERSE OF MULTIPLICATION"

Teacher's Note and Reminders

Activity 3 MESSAGE FROM THE KING (Product – Factor Association)

Description: This activity will give you an idea on how factors is associated with products. You will match the factors in column A with the products in column B and decode the secret message.

COLUMN A		COLUMN B	
1.	$4x\ (3x - 5)$	A.	$6x^2y^2 + 3xy^3 - 3xy^2$
2.	$3xy^2(2x + y - 1)$	F.	$x^3 - 27$
3.	$(x + y)(x - y)$	G.	$4x^2 - 9$
4.	$(2x + 3)(2x - 3)$	R.	$4x^2 + 12x + 9$
5.	$(x - 5y)(x + 5y)$	U.	$12x^2 - 20x$
6.	$(x + y)^2$	E.	$6x^2 + x - 2$
7.	$(2x + 3)^2$	T.	$ac - ad + bc - bd$
8.	$(x - 5y)^2$	S.	$mr - nr + ms - ns$
9.	$(x + 4)(x - 3)$	C.	$x^2 - y^2$
10.	$(2x - 1)(3x + 2)$	I.	$2x^2 - x - 10$
11.	$(x + 2)(2x - 5)$	O.	$x^2 - 10xy + 25y^2$
12.	$(x - 3)(x^2 + 3x + 9)$	N.	$x^2 + x - 12$
13.	$(x + 3)(x^2 - 3x + 9)$	H.	$x^3 + 27$
14.	$(a + b)(c - d)$	M.	$x^2 + 2xy + y^2$
15.	$(m - n)(r + s)$	L.	$x^2 - 25y^2$
16.	$(3x + 4)(3x - 4)$	P.	$9x^2 - 16$
17.	$(3x - 4)^2$	V.	$9x^2 - 24x + 16$

12 2 3 14 8 7 11 9 4 — 11 15 — 14 13 10

7 10 17 10 7 15 10 — 8 12

6 1 5 14 11 16 5 11 3 2 14 11 8 9

QUESTIONS ?

1. What are your observations on the expression in column A? Compare them with those in column B.
2. Do you see any pattern?
3. Are the two expressions related?
4. Why is it important to know the reverse process of multiplication?

What to Process

These are the enabling activities / experience that the learner will have to go through to validate their observations in the previous activity. Interactive activities are provided for the students to check their understanding on the lesson.

Start the discussion by defining factoring, you can use the questions found in learning modules.
Allow the students to perform Activity 4, and use this as springboard in the discussion of factoring by greatest common monomial factor. The students will present their outputs in front. Use the questions to process the answer of the students. Tell the students when to use and not to use this type of factoring. Emphasize that this type of factoring should be use first before applying any type of factoring.
Give examples of this type factoring after the activity. You can use the examples found in learning module.

Teacher's Note and Reminders

What did you discover between the relationship of products and its factors? You have just tried finding out the relationship between factors and their product. You can use this idea to do the next activities.

What you will learn in the next session will also enable you to do the final project which involves model and lay – out making of a packaging box.

What to Process

The activity that you did in the previous section will help you understand the different lessons and activities you will encounter here.

The process of finding the factors of an expression is called **factoring**, which is the reverse process of multiplication. A **prime number** is a number greater than 1 which has only two positive factors: 1 and itself. Can you give examples of prime numbers? Is it possible to have a prime that is a polynomial? If so, give examples.

The first type of factoring that you will encounter is **Factoring the Greatest Common Monomial Factor**. To understand this let us do some picture analysis.

Activity 4 FINDING COMMON

Description: Your task in this activity is to identify common things that are present in the three pictures.

http://blog.ningin.com/2011/09/04/10-idols-and-groups-pigging-out/

http://k-pop-love.tumblr.com/post/31067024715/eating-sushi

QUESTIONS ?

1. What are the things common to these pictures?
2. Are there things that make them different?
3. Can you spot things that are found on one picture but not on the other two?
4. What are the things common to two pictures but not on the other?

Note to the teacher: Emphasize that the greatest common monomial factor should be divided to all terms of the expression and not only to its first term and that the number of terms of the other factor is equal to the number of terms the polynomial contains.

Teacher's Note and Reminders

Polynomial	Greatest Common Monomial Factor (CMF)	Quotient of Polynomial and CMF	Factored Form
$6m + 8$	2	$3m + 4$	$2(3m + 4)$
$12m^2o^2 + 4mo^2$	$4mo^2$	$3m + o$	$4mo^2(3m + o)$
$27d^4o^5t^3a^6 - 18d^2o^3t^6 - 15d^6o^4$	$3d^2o^3$	$9d^2o^2t^3a^6 - 6t^6 - 5d^4$	
$4(12) + 4(8)$	4	$(12 + 8)$	$4(12 + 8)$
$12WI^3N^5 - 16WIN + 20WINNER$	4WIN		

The above activity gave us the idea about the **Greatest Common Monomial Factor** that appears in every term of the polynomial. Study the illustrative examples on how factoring the Greatest Common Monomial Factor is being done.

Factor $12x^3y^5 - 20x^5y^2z$

a. Find the greatest common factor of the numerical coefficients.
The GCF of 12 and 20 is 4.

b. Find the variable with the least exponent that appears in each term of the polynomial.
*x and y are both common to all terms and 3 is the smallest exponent for x and 2 is the smallest exponent of y, thus, x^3y^2 is the **GCF** of the variables.*

c. The product of the **greatest common factor** in **(a)** and **(b)** is the GCF of the polynomial.
Hence, $4x^3y^2$ is the GCF of $12x^3y^5 - 20x^5y^2z$.

d. To completely factor the given polynomial, divide the polynomial by its GCF, the resulting quotient is the other factor.

Thus, the factored form of $12x^3y^5 - 20x^5y^2z$ is $4x^3y^2(3y^3 - 5x^2z)$

Below are other examples of Factoring the Greatest Monomial Factor.

a. $8x^2 + 16x \Rightarrow 8x$ is the **greatest monomial factor**. Divide the polynomial by $8x$ to get the other factor.
$8x(x + 2)$ is the factored form of $8x^2 + 16x$.

b. $12x^5y^4 - 16x^3y^4 + 28x^6 \Rightarrow 4x^3$ is the greatest monomial factor. Divide the given expression by the greatest monomial factor to get the other factor.

Thus, $4x^3(3x^2y^4 - 4y^4 + 7x^3)$ is the factored form of the given expression.

Complete the table to practice this type of factoring.

Polynomial	Greatest Common Monomial Factor (CMF)	Quotient of Polynomial and CMF	Factored Form
$6m + 8$	2	$3m + 4$	$2(3m + 4)$
	$4mo^2$		$4mo^2(3m + o)$
$27d^4o^5t^3a^6 - 18d^2o^3t^6 - 15d^6o^4$		$9d^2o^2t^3a^6 - 6t^6 - 5d^4$	
$4(12) + 4(8)$	4		
$12WI^3N^5 - 16WIN + 20WINNER$			

Before doing the activity for factoring difference of two squares, ask the students why the difference of two squares was given such name.

To start the discussion you can use number pattern to see the relationship of factors to product. You may bring back the students to multiplying sum and difference of binomials in special product to see how factors may be obtained. Students should realize that factors of difference of two squares are sum and difference of binomials.

Ask students to generate rule in factoring difference of two squares.

For paper cutting, students must realize that the area of the new figure formed is the difference of the area of the two squares, which is **($a^2 - b^2$)** and that the dimensions of the rectangle formed are **($a + b$) x ($a - b$)**.
This activity may be done by pair or as a group.

Teacher's Note and Reminders

Now that you have learned how to factor polynomials using their greatest common factor we can move to the next type of factoring, which is the difference of two squares. ***Why do you think it was given such name?*** *To model it, let's try doing the activity that follows.*

Activity 5 INVESTIGATION IN THE CLASSROOM

Description: This activity will help you understand the concepts of difference of two squares and how this pattern is used to solve numerical expressions. Investigate the number pattern by comparing the products then write your generalizations afterwards.

NUMBER PATTERN:

a. (11)(9) = (10 + 1)(10 – 1) = 100 – 1 =
b. (5)(3) = (4 + 1)(4 – 1) = 16 – 1 =
c. (101)(99) = (100 + 1)(100 – 1) = 10000 – 1 =
d. (95)(85) = (90 + 5)(90 – 5) = 8100 – 25 =
e. (n – 5)(n + 5) =

How do you think products are obtained? What are the different techniques used to solve for the products?

What is the relationship of the product to its factor? Have you seen any pattern in this activity?

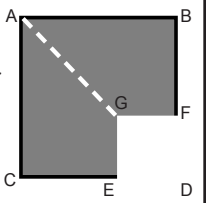
For you to have a clearer view of this type of factoring, let us have paper folding activity again.

Activity 6 INVESTIGATION IN PAPER FOLDING

Description: This activity will help you visualize the pattern of difference of two squares.

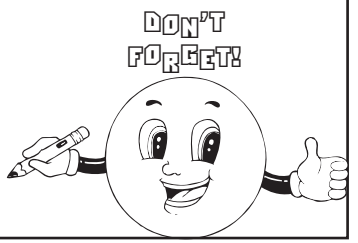
Directions:

1. Get a square paper and label the sides as *a*.
2. Cut – out a small square in any of its corner and label the side of the small square as *b*.
3. Cut the remaining figure in half.
4. Form a rectangle



You can use the examples found in learning module for the discussion. Give more examples if necessary.
(**Note**: Remind students to use first factoring greatest common monomial factor if applicable before factoring it through difference of two squares)

Teacher's Note and Reminders



QUESTIONS ?

1. What is the area of square ABDC?
2. What is the area of the cut – out square GFDE?
3. What is the area of the new figure formed?
4. What is the dimension of the new figure formed?
5. What pattern can you create in the given activity?

For you to have a better understanding about this lesson, observe how the expressions below are factored and observe the relationships of the term with each other.

a. $x^2 - y^2 = (x + y)(x - y)$

b. $4x^2 - 36 = (2x + 6)(2x - 6)$

c. $a^2b^4 - 81 = (ab^2 - 9)(ab^2 + 9)$

d. $16a^6 - 25b^2 = (4a^3 - 5b)(4a^3 + 5b)$

e. $(\frac{9}{16}r^4 - \frac{1}{25}t^2 n^6) = (\frac{3}{4}r^2 + \frac{1}{5}tn^3)(\frac{3}{4}r^2 - \frac{1}{5}tn^3)$

QUESTIONS ?

1. What is the first term of each polynomial?
2. What is the last term of each polynomial?
3. What is the middle sign of the polynomial?
4. How was the polynomial factored?
5. What pattern is seen in the factors of the difference of two terms?
6. Can all expressions be factored using difference of two squares? Why or why not?
7. When can you factor expressions using difference of two squares?

Remember the factored form of a polynomial that is a difference of two squares is the sum and difference of the square roots of the first and last terms.

- *$4x^2 - 36y^2$ ➔ the square root of $4x^2$ is $2x$ and the square root of $36y^2$ is $6y$. To write their factors write the product of the sum and difference of the square roots of $4x^2 - 36y^2$, that is $(2x + 6y)(2x - 6y)$.*

To check students understanding on factoring difference of two squares, ask them to make pairs of square terms and factor it after. Students can give as many pairs of difference of two square as they can create.
(Note: Teachers must see to it that students must form difference of two squares)

Example Answer: $81m^4 - 121c^4 = (9m^2 - 11c^2)(9m^2 + 11c^2)$

To start with factoring sum or difference of two cubes, allow students to multiply $(a + b)(a^2 + ab + b^2)$ and $(a - b)(a^2 + ab + b^2)$. They should get $(a^3 + b^3)$ and $(a^3 - b^3)$ respectively as the product. Ask the process question to the students and help them see the pattern in factoring sum or difference of two cubes. Guide them to generate the rule in factoring sum or difference of two cubes.

Teacher's Note and Reminders

Activity 7 PAIR MO KO NYAN!

Description: This game will help you develop your factoring skills by formulating your problem based on the given expressions. You can integrate other factoring techniques in creating expressions. Create as many factors as you can.

Directions: Form difference of two squares problems by pairing two squared quantities then find their factors. (Hint: You can create expressions that may require the use of the greatest common monomial factor)

x^2y^3	$16s^2$	25	$81m^4$	k^2	$4m^6$
$24p^2$	$100n^8$	w^6d^{18}	k^6u^{12}	$36h^{10}$	9
$20a^4$	$25a^2b^3$	144	$121c^4$	$88m^4$	
$49x^2y^8$	1	$(x + 3)$	$\frac{16}{64}$	$36z^4x^{16}$	$121h^{18}$
169	$225v^{22}$	$(x - 7)^2$		$30o^4p^6$	$196d^{18}$

You have learned from the previous activity how factoring the difference of two squares is done and what expression is considered as the difference of two squares. We are now ready to find the factors of the sum or difference of two cubes. To answer this question, find the indicated product and observe what pattern is evident.

a. $(a + b)(a^2 - ab + b^2)$
b. $(a - b)(a^2 + ab + b^2)$

What are the resulting products? How are the terms of the products related to the terms of the factors? What if the process was reversed and you were asked to find the factors of the products? How are you going to get the factor? Do you see any common pattern?

Use **Activity 8** (Road Map to Factor) as guide in factoring sum or cubes of binomials, this will give the students steps in factoring such expression. Give more examples of sum or difference of two cubes and factor it to firm – up the understanding of the students in factoring this expression.

Note: Remind the students to use first factoring by greatest common monomial factor before applying this type of factoring if necessary.

Answers to problem:

1. $(x^3 - y^3)$ unit cube $=(x - y)(x^2 + xy + y^2)$
2. $(x^3 + y^3)$ unit cube $= (x + y)(x^2 - xy + y^2)$

Teacher's Note and Reminders

Activity 8 ROAD MAP TO FACTOR

Answer the following problems by using the map as your guide.

Is the given expression a sum or difference of two cubes?

If No → Use other factoring technique/method

If Yes → Are the binomials sums or differences of two cubes?

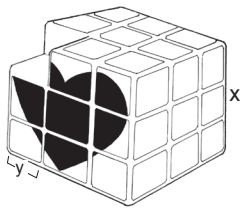
If DIFFERENCE:

1. What are the cube roots of the first and last terms?
2. Write their **difference** as the first factor. $(x - y)$.
3. For the second factor, get the trinomial factor by:
   a. Squaring the first term of the first factor;
   b. Adding the product of the first and second terms of the first factor.
   c. Squaring the last term of the first factor
4. Write them in factored form. $(x - y)(x^2 + xy + y^2)$

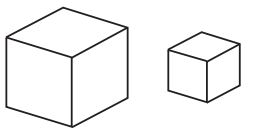
If Sum:

1. What are the cube roots of the first and last terms?
2. Write their sum as the first factor. $(x + y)$.
3. For the second factor, get the trinomial factor by:
   a. Squaring the first term of the first factor;
   b. Subtracting the product of the first and second terms of the first factor.
   c. Squaring the last term of the first factor
4. Write them in factored form. $(x + y)(x^2 - xy + y^2)$

1. Represent the volume of this figure. What is the factored form of the volume of given figure?



2. What are the volumes of the cubes? If the cubes are to be joined to create platform for a statue, what will be the volume of the platform? What are the factors of the volume of the platform?



To start factoring perfect square trinomials, use algebra tiles to model it. This activity will give the students picture of perfect square trinomials. See to it that the students will produce a square.

Discuss the answers of the students on process questions. Point out that the result of squaring binomial is a perfect square trinomial. At this point students should see the pattern of factoring perfect square trinomials and be able to generate the rule in factoring such polynomials.

Discuss when an expression is a perfect square. Do the perfect hunt activity to check the students understanding in identifying perfect square trinomials.

Teacher's Note and Reminders

Activity 9 Let's tile it up!

Directions: Prepare the following:

1. 4 big squares measuring 4" × 4" and represent each square as x^2.
2. 8 rectangular tiles with measures of 4" × 1" and represent it as x.
3. 16 small squares whose measures is 1" × 1" and represent this as 1.

Form squares using:

- 1 big square tile, 2 rectangular tiles and 1 small square.
- 1 big square tile, 4 rectangular tiles and 4 small squares.
- 1 big square tile, 6 rectangular tiles and 9 small squares.
- 4 big square tiles, 4 rectangular tiles and 1 small square.
- 4 big square tiles, 8 rectangular tiles and 4 small squares.

QUESTIONS ?

1. How will you represent the total area of each figure?
2. Using the sides of the tiles, write all the dimensions of the squares.
3. What did you notice about the dimensions of the squares?
4. Did you find any pattern in their dimensions? If yes, what are those?
5. How can unknown quantities in geometric problems be solved?

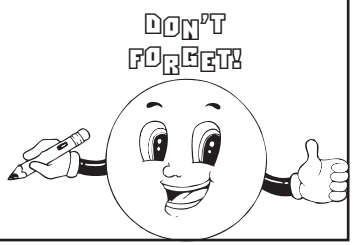
*The polynomials formed are called **perfect square trinomials.***

***Perfect square trinomial** is the result of squaring a binomial. A perfect square trinomial has first and last terms which are perfect squares and a middle term which is twice the product of the square root of first and last terms.*

Answer to Activity 10

$25x^2$	10x	81	18x	x^2	4
15x	$16x^2$	- 24x	9	10x	28x
$4x^2$	- 16x	16	15x	25	$49x^2$
$16x^2$	49	8x	16	$24x^2$	9
25	14x	8x	40x	30x	10x
7x	x^2	12x	$25x^2$	40	$12x^2$

Teacher's Note and Reminders



Activity 10 PERFECT HUNT

Description: Look for the different perfect square trinomials found in the box. Answers might be in diagonal, horizontal or vertical in form.

	$10x$	81	$18x$	x^2	4
$15x$	$16x^2$	$-24x$	9	$10x$	$28x$
$4x^2$	$-16x$	16	$15x$	25	$49x^2$
$16x^2$	49	$8x$	16	$24x^2$	9
25	$14x$	$8x$	$40x$	$30x$	$10x$
$7x$	x^2	$12x$	$25x^2$	40	$12x^2$

To factor perfect square trinomials:

a. Get the square root of the first and last terms.
b. List down the square root as sum/difference of two terms as the case may be.

You can use the following relationships to factor perfect square trinomials:

(First term)² + 2(First term)(Last term) + (Last term)² = (First term + Last term)²
(First term)² – 2(First term)(Last term) + (Last term)² = (First term – Last term)²

Remember to factor out first the greatest common monomial factor before factoring the perfect square trinomial.

Ex. 1. Factor $n^2 + 16n + 64$

Solution:

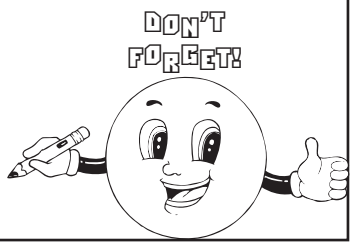
a. Since $n^2 = (n)^2$ and $64 = (8)^2$, then both the first and last terms are perfect squares. And $2(n)(8) = 16n$, then the given expression is a perfect square polynomial.
b. The square root of the first term is n and the square root of the last term is 8, then the polynomial is factored as $(n + 8)^2$.

After the above, the rule in factoring perfect square trinomial may be discuss you can use the examples in learning module. Examples of factoring perfect square trinomials should be given to ensure mastery.

Answers on exercise:

a. $(m + 6)^2$	e. $(7a - 6)^2$	i. $2(3h + 1)^2$
b. $(4d - 3)^2$	*f*. $(11c^2 + 3)^2$	j. $5f^2(2f - 3)^2$
c. $(a^2b - 3)^2$	g. $(5r + 8)^2$	
d. $(3n + 5d)^2$	*h*. $(\frac{1}{4}z + \frac{2}{3})^2$	

Teacher's Note and Reminders



Ex. 2. Factor **$4r^2 - 12r + 9$**

Solution:

a. Since $4r^2 = (2r)^2$ and $9 = (3)^2$, and since $-12r = 2(2r)(3)$ then it follows the given expression is a perfect square trinomial.

b. The square root of the first term is $2r$ and the square root of the last term is 3 so that its factored form is $(2r - 3)^2$.

Ex. 3. Factor **$75t^3 + 30t^2 + 3t$**

Solution:

a. Observe that $3t$ is common to all terms, thus, factoring it out first we have:
$3t\,(25t^2 + 10t + 1)$

b. Notice that $25t^2 = (5t)^2$ and $1 = (1)^2$, and $10t = 2(5t)(1)$, then $25t^2 + 10t + 1$ is a perfect square trinomial.

c. Factoring $25t^2 + 10t + 1$ is $(5t + 1)^2$, thus, the factors of the given expression are $3t\,(5t + 1)^2$.

Explain why in **Example 3**, $(5t + 1)^2$ is not the only factor. What is the effect of removing $3t$?

Exercises

Supply the missing term to make a true statement.

a.	$m^2 + 12m + 36$	=	$(m + \_\_\_)^2$
b.	$16d^2 - 24d + 9$	=	$(4d - \_\_\_)^2$
c.	$a4b^2 - 6abc + 9c^2$	=	$(a^2b \_\_\_ \_\_\_)^2$
d.	$9n^2 + 30nd + 25d^2$	=	$(\_\_\_ \_\_ 5d)^2$
e.	$49^{s2} - 84g + 36$	=	$(\_\_\_ \_\_ \_\_\_)^2$
f.	$121c^4 + 66c^2 + 9$	=	$(\_\_\_ \_\_ \_\_\_)^2$
g.	$25r^2 + 40rn + 16n^2$	=	$(\_\_\_ \_\_ \_\_\_)^2$
h.	$\frac{1}{16}x^2 + \frac{1}{3}x + \frac{4}{9}$	=	$(\_\_ \_\_ \_\_)^2$
i.	$18h^2 + 12h + 2$	=	$2\,(\_\_\_ \_\_ \_\_\_)^2$
j.	$20f^4 - 60f^3 + 45f^2$	=	$\_\_\_\,(2f \_\_ \_\_\_)^2$

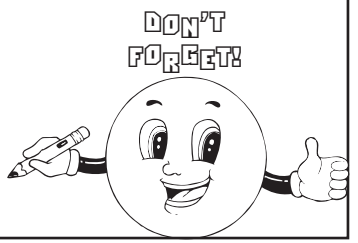
Is $q^2 + q - 12$ a perfect square trinomial? Why?

Are all trinomials perfect squares? How do we factor trinomials that are not perfect squares?

In the next activity, you will see how trinomials that are not perfect squares are factored.

Give examples of quadratic trinomials that are not perfect square. Ask them to factor it. This will make the students realize that there are some trinomials that are not factorable using perfect square trinomials. Use this as springboard before proceeding to activity 11.
Note: Make sure to it that the students will form rectangle as their figure. Ask them to compare the dimensions of the figure formed in this activity and activity 9.

Teacher's Note and Reminders



Activity 11 TILE ONCE MORE!!

Description: You will arrange the tiles according to the instructions given to form a polygon and find its dimensions afterwards.

Directions:
1. Cut – out 4 pieces of 3 in. by 3 in. card board and label each as x^2 representing its area.
2. Cut – out 8 pieces of rectangular cardboard with dimensions of 3 in. by 1 in. and label each as x representing its area.
3. Cut – out another square measuring 1 in. by 1 in. and label each as 1 to represent its area.

Form rectangles using the algebra tiles that you prepared. Use only tiles that are required in each item below.

a. 1 big square tile, 5 rectangular tile and 6 small square tiles.
b. 1 big square tile, 6 rectangular tiles and 8 small square tiles.
c. 2 big square tiles, 7 rectangular tiles and 5 small square tiles.
d. 3 big square tiles, 7 rectangular tiles and 4 small square tiles.
e. 4 big square tiles, 7 rectangular tiles and 3 small square tiles.

QUESTIONS ?

What is the total area of each figure?
2. Using the sides of the tiles, write all the dimensions of the rectangles.
3. How did you get the dimensions of the rectangles?
4. Did you find difficulty in getting the dimensions?

Based on the previous activity, **how can the unknown quantities in geometric problems be solved?**

If you have noticed there are two trinomials that were formed in the preceding activity, trinomials that contains numerical coefficient greater than 1 in its highest degree and trinomials whose numerical coefficient in its highest degree is exactly 1.

Let us study first how factoring trinomials whose leading coefficient is 1 being factored.

Ex. Factor $p^2 + 5p + 6$
Solution: a. List all the possible factors of 6.

Factors of 6	
2	3
6	1
-2	-3
-6	-1

Give examples of general trinomials whose leading coefficient is 1. You can use trial and error in factoring these examples. Use the examples found in learning module. Giving more examples is highly suggested. You can ask the students to generalize how factoring of this trinomial is attained.
Remind them again that they should use factoring by greatest common monomial factor using this type of factoring, if applicable.

Teacher's Note and Reminders

b. Find factors of 6 whose sum is 5.
- **2 + 3 = 5**
- 6 + 1 = 7
- (-2) + (-3) = -5
- (-6) + (-1) = -7

c. Thus, the factor of $p^2 + 5p + 6$ = **$(p + 2)(p + 3)$**

Ex. Factor **$v^2 + 4v - 21$**
Solution: a. List all the factors of **– 21**

Factors of -21	
-3	7
-7	3
-21	1
-1	21

b. Find factors of -21 whose sum is 4.
- **-3 + 7 = 4**
- -7 + 3 = -4
- -21 + 1 = -20
- -1 + 20 = 19

c. Hence, the factors of $v^2 + 4v - 21$ = **$(v - 3)(+ 7)$**

Factor **$2q^3 - 6q^2 - 36q$**, since there is a common monomial factor, begin by factoring out $2q$ first, rewriting it, you have $2q\ (q^2 - 3q - 18)$.

a. Listing all the factors of – 18.

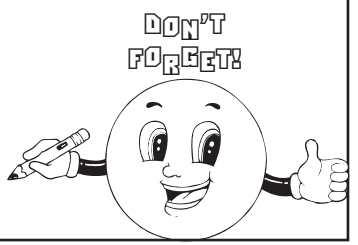
Factors of -18	
-1	18
-2	9
-3	6
-18	1
-9	2
-6	3

b. Since – 6 and 3 are the factors whose sum is – 18, then the binomial factors of $q^2 - 3q - 18$ are **$(q - 6)(q + 3)$**.
c. Therefore, the factors of $2q^3 - 6q - 36q$ are $2q(q - 6)(q + 3)$.

To check students understanding on this factoring technique, you can do the bingo game. Write on a strip the polynomials below and place them on container. Draw the strip and read it in class, give the students time to factor the polynomials.

1. $n^2 - n - 20$
2. $n^2 + 5n + 6$
3. $n^2 - 4n - 32$
4. $n^2 - n - 42$
5. $n^2 + 9n + 18$
6. $n^2 + 11n + 18$
7. $n^2 + 17n + 72$
8. $n^2 - 12n + 35$
9. $n^2 - 8n - 48$
10. $n^2 + 14n - 32$
11. $n^2 - 17n + 72$
12. $n^2 + 9n + 8$
13. $n^2 + 10n + 24$
14. $n^2 - 2n - 48$
15. $n^2 + 11n + 24$

Teacher's Note and Reminders



Remember:

To factor trinomials with 1 as the numerical coefficient of the leading term:

a. factor the leading term of the trinomial and write these as the leading term of the factors;
b. list down all the factors of the last term;
c. identify which factor pair sums up to the middle term; then
d. write factor pairs as the last term of the binomial factors.

NOTE: ***ALWAYS PERFORM FACTORING USING GREATEST COMMON MONOMIAL FACTOR FIRST BEFORE APPLYING ANY TYPE OF FACTORING.***

Activity 12 FACTOR BINGO GAME!

Description: Bingo game is an activity to practice your factoring skills with speed and accuracy.

Instruction: On a clean sheet of paper, draw a 3 by 3 grid square and mark the center as **FACTOR**. Pick **8** different factors from the table below and write the in the grid. As your teacher reads the trinomial, you will locate its factors and marked it x. The first one who makes the x pattern wins.

$(n + 4)(n - 5)$	$(n + 2)(n + 9)$	$(n - 8)(n - 9)$	$(n + 2)(n + 3)$
$(n + 9)(n + 8)$	$(n + 1)(n + 8)$	$(n - 8)(n + 4)$	$(n - 7)(n - 5)$
$(n + 6)(n + 4)$	$(n - 7)(n + 6)$	$(n - 12)(n + 4)$	$(n - 8)(n + 6)$
$(n + 3)(n + 6)$	$(n - 2)(n + 16)$	$(n + 3)(n + 8)$	

QUESTIONS ?

1. How did you factor the trinomials?
2. What did you do to factor the trinomials easily?
3. Did you find any difficulty in factoring the trinomials? Why?
4. What are your difficulties? How will you address those difficulties?

Give polynomials whose numerical coefficient of the leading term is not 1. Factor this using trial and error. Allow the students to stress out the disadvantages that they have encountered in using this technique.
Introduce the factoring by grouping or the AC method after. Ask them to compare the process.

Teacher's Note and Reminders

What if the numerical coefficient of the leading term of the trinomial is not 1, can you still factor it? Are trinomials of that form factorable? Why?

Trinomials on this form are written on the form $ax^2 + bx + c$, where a and b are the numerical coefficients of the variables and c as the constant term. There are many ways of factoring these types of polynomials, one of which is by inspection.

Trial and error are being utilized in factoring this type of trinomials. Here is an example:

Factor $\mathbf{6z^2 - 5z - 6}$ through trial and error.
Give all the factors of $6z^2$ and – 6

Factors of:	
$6z^2$	-6
$(3z)(2z)$	(3)(-2)
$(6z)(z)$	(-3)(2)
	(1)(-6)
	(-1)(6)

Write the all possible factors using the values above and determine the middle term by multiplying the factors.

Possible Factors	Sum of the product of the outer terms and the product of the inner terms
$(3z - 2)(2z + 3)$	$9z - 4z = 5z$
$(3z + 3)(2z - 2)$	$-6z + 6z = 0$
$(3z - 3)(2z + 2)$	$6z - 6z = 0$
$\mathbf{(3z + 2)(2z - 3)}$	$\mathbf{-9z + 4z = -5z}$
$(3z + 1)(2z - 6)$	$-18z + 2z = -16z$
$(3z - 6)(2z + 1)$	$3z - 12z = -9z$
$(6z + 3)(z - 2)$	$-18z + 3z = -15z$
$(6z - 2)(z + 3)$	$18z - 2z = 16z$
$(6z - 3)((z + 2)$	$12z - 3z = 9z$
$(6z + 2)(z - 3)$	$-18z + 2z = -16z$
$(6z + 1)(z - 6)$	$-36z + z = -35z$
$(6z - 6)(z + 1)$	$6z - 6z = 0$

In the given factors, $\mathbf{(3z + 2)(2z - 3)}$ gives the sum of $-5z$, thus, making it as the factors of the trinomial $6z^2 - 5z - 36$.

How was inspection used in factoring? What do you think is the disadvantage of using this?

Give the example you have used above and solve it through factoring by grouping. Provide for more examples.

Teacher's Note and Reminders

Factoring through inspection is a tedious and a long process, thus, knowing another way of factoring trinomial would be very beneficial in your study of this module.

Another way of factoring is through grouping or AC method. Closely look at the given steps and compare it with trial and error.

Factor $6z^2 - 5z - 6$

1. Find the product of the leading term and the last term.

   $6z^2 - 5z - 6$

   $(6z^2)(-6) = -36z^2$

2. Find the factor of $-36z^2$ whose sum is $-5z$.
   $-9z + 4z = -5z$
3. Rewrite the trinomial as four – term expressions by replacing the middle term by the sum factor.
   $6z^2 - 9z + 4z - 6$
4. Group terms with common factors.
   $(6z^2 - 9z) + (4z - 6)$
5. Factor the groups using greatest common monomial factor.
   $3z\,(2z - 3) + 2(2z - 3)$
6. Factor out the common binomial and write the remaining factor as sum or difference of binomial.
   $(2z - 3)(3z + 2)$

Factor $2k^2 - 11k + 12$

1. Multiply the first and last terms.
   $(2k^2)(12) = 24k^2$
2. Find the factors of $24k^2$ whose sum is $-11k$.
   $(-3k) + (-8k) = -11k$
3. Rewrite the trinomial as four – term expressions by replacing the middle term by the sum factor.
   $2k^2 - 3k - 8k + 12$
4. Group terms with common factor
   $(2k^2 - 8k) + (-3k + 12)$
5. Factor the groups using greatest common monomial factor.
   $2k(k - 4) - 3(k - 4)$
6. Factor out the common binomial and write the remaining factor as sum or difference of binomial.
   $(k - 4)(2k - 3)$

To practice the factoring skills of the students, do Activity 13 in class. Group the students and distribute flaglet on each group. Ask one group to give a factorable polynomial then let the other group factor it.

Extend the concept of factoring by grouping by applying it to polynomials with four terms. You can use the examples on learning module. Perform Activity 14 as a group after.

Teacher's Note and Reminders

Factor $6h^2 - h - 2$

1. Multiply the first and last terms.
   $(6h^2)(-2) = -12h^2$
2. Find the factors of $12h^2$ whose sum is h.
   $(-4h) + (3h) = -h$
3. Rewrite the trinomial as four – term expressions by replacing the middle term by the sum factor.
   $6h^2 - 4h - 3h - 2$
4. Group terms with common factor
   $(6h^2 - 3h) + (-4h - 2)$
5. Factor the groups using greatest common monomial factor.
   $3h(2h - 1) - 2(2h - 1)$
6. Factor out the common binomial, and write the remaining factor as sum or difference of binomial.
   $(3h - 2)(2h - 1)$

Activity 13 WE HAVE!

Description: This game will help you practice your factoring skills through a game.
Instruction: Form a group of 5. Your task as a group is to factor the trinomial that the other group will give. Raise a flaglet if you have already factored the trinomial and shout, We have it! The first group to get 10 correct answers wins the game.

QUESTIONS ?

1. Do you find difficulty in playing the game? Why?
2. What hindered you to factor the trinomial?
3. What plan do you have to address these difficulties?

Let's extend!!

We can use factoring by grouping technique in finding the factors of a polynomial with more than three terms.

Let's try factoring **$8mt - 12at - 10mh - 15ah$**

Solution:
1. Group terms with common factor.
   $(8mt - 12at) + (-10mh - 15ah)$
2. Factor out the greatest common monomial factor in each group.
   $4t(2m - 3a) - 5h(2m - 3a)$ ➔ Why?
3. Factor out the common binomial and write the remaining factor as sum or difference of binomial.
   $(2m - 3a)(4t - 5h)$

Teacher's Note and Reminders

Answers for Activity 14

1.	$4wt + 2wh + 6it + 3ih$	=	WITH
2.	$15te - 12he + 10ty - 8hy$	=	THEY
3.	$hv + av + he + ae$	=	HAVE
4.	$10ti - 8ts - 15hi + 12hs$	=	THIS
5.	$88fo + 16ro - 99fm - 18rm$	=	FROM
6.	$7s + 35om + 9se + 45oe$	=	SOME
7.	$42wa + 54wt + 56ha + 72ht$	=	WHAT
8.	$36yu - 24ro + 12ou - 72yr$	=	YOUR
9.	$72he + 16we + 27hn + 6wh$	=	WHEN
10.	$26wr - 91or + 35od - 10wd$	=	WORD

- Make a wrap – up of all the factoring that you discussed, ask the students to differentiate it. You can use the graphic organizer for this activity. This may serve as bring home activity.
- Peer mentoring maybe done to help students understand better the topic. Make sure to it that in every group there is one responsible student. You may facilitate the mentoring or you can give the group free hand in doing this activity. Have this activity
- To practice the factoring skills of the students with speed and accuracy you can do flashcard drill by pair or as a team.

Factor $\mathbf{18lv + 6le + 24ov + 8oe}$

Solution:

1. Group terms with common factor.
   $\mathbf{(18lv + 6le) + (24ov + 8oe)}$ ➔ Why?

2. Factor out the greatest common monomial factor in each group.
   $\mathbf{6l(3v + e) + 8o(3v + 3)}$ ➔ Why?

3. Factor out the common binomial and write the remaining factor as sum or difference of two terms.
   $\mathbf{(3v + e)(6l + 8o)}$

Activity 14 FAMOUS FOUR WORDS

Description: This activity will reveal the most frequently used four - letter word (no letter is repeated) according to world – English.org through the use of factoring.

Instruction: With your groupmates factor the following expressions by grouping and write a four - letter word using the variable of the factors to reveal the 10 most frequently used four - letter word.

1. $4wt + 2wh + 6it + 3ih$
2. $15te - 12he + 10ty - 8hy$
3. $hv + av + he + ae$
4. $10ti - 8ts - 15hi + 12hs$
5. $88fo + 16ro - 99fm - 18rm$
6. $7s + 35om + 9se + 45oe$
7. $42wa + 54wt + 56ha + 72ht$
8. $36yu - 24ro + 12ou - 72yr$
9. $72he + 16we + 27hn + 6wh$
10. $26wr - 91or + 35od - 10wd$

Activity 15 TEACH ME HOW TO FACTOR (GROUP DISCUSSION /PEER MENTORING)

Description: This activity is intended for you to clear your queries about factoring with the help of your group mates.

Direction: Together with your group mates, discuss your thoughts and queries regarding factoring. Figure out the solution to each others' questions, you may ask other groups or your teacher for help.

Ask the students to go back to IRF sheet and answer the Revise part. Discuss the answer of the students in class.

Teacher's Note and Reminders

QUESTIONS ?

1. What different types of factoring have you encountered?
2. What are your difficulties in each factoring technique?
3. Why did you face such difficulties?
4. How are you going to address these difficulties?

Activity 16 WITH A BLINK!

Description: This is a flash card drill activity to help you practice with speed and accuracy your factoring technique.

Instruction: As a group you will factor the expressions flashed by your teacher, each correct answer is given a point. The group with the most number of points wins the game.

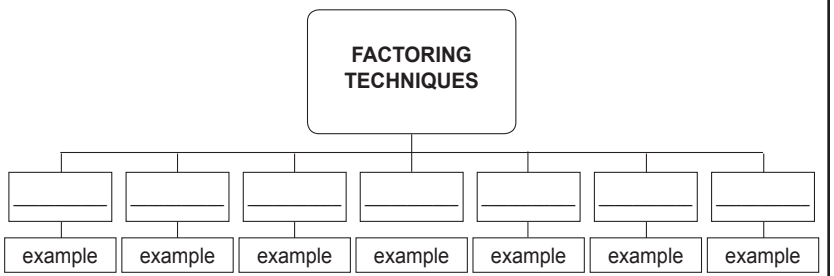
QUESTIONS ?

1. What techniques have you used to answer the questions?
2. What things did you consider in factoring?
3. Did you find difficulty in the factoring the polynomials? Why?

Now that we have already discussed the different types of factoring, let us summarize our learning by completing the graphic organizer below.

Activity 17 GRAPHIC ORGANIZER

Description: To summarize the things you have learned, as a group, complete the chart below. You may add box if necessary.



Teacher's Note and Reminders

What to Understand

This part provides learners activities to further validate and deepen their understanding on the applications of factoring and to check their knowledge against misconception.

Answers for Activity 19

1. No, the factors of $x^2 - 4x - 12$ are $(x - 6)(x + 2)$, while $12 - 4x - x^2$ has $(2 - x)(6 + x)$ as its factors.
2. Yes, $3x(x^2 - 4)$, thus, $3x^3 - 12x = 3x\,(x - 2)(x + 2)$.
3. Difference of two squares is only applied if the middle operation is minus.
4. a. $(x + 2)(x + 2) = x^2 + 4x + 4$
   b. $(0.4x - 3)(0.4x + 3) = (0.16x^2 - 9)$
   c. $4x^2y^5 - 12x^3y^6 + 2y^2 = 2y^2\,(2x^2y^3 - 6x^3y^4 + 1)$
   d. $3x^2 - 27 = 3\,(x^2 - 9) = 3\,(x + 3)(x - 3)$

Activity 18 IRF REVISIT

Revisit your IRF sheet and revise your answer by filing in column 2 under REVISE.

Initial	Revise	Final
	Express the following as product of factors. 1. $4x^2 - 12x =$ _____ 2. $9m^2 - 16n^2 =$ _____ 3. $4a^2 + 12a + 9 =$ _____ 4. $2x^2 + 9x - 5 =$ _____ 5. $27x^3 - 8y^3 =$ _____ 6. $a^3 + 125b^3 =$ _____ 7. $xm + hm - xn - hn =$ _____	

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand

Your goal in this section is to take a closer look at some aspects of the topic and to correct some misconceptions that might have developed.

The following activities will check your mastery in factoring polynomials.

Activity 19 SPOTTING ERRORS

Description: This activity will check how well you can associate between product and its factors.

Instructions: Do as directed.

1. Your classmate asserted that $x^2 - 4x - 12$ and $12 - 4x - x^2$ has the same factors. Is your classmate correct? Prove by showing your solution.
2. Can the difference of two squares be applicable to $3x^3 - 12x$? If yes, how? If no, why?
3. Your classmate factored $x^2 + 36$ using the difference of two squares, how will you make him realize that his answer is not correct?
4. Make a generalization for the errors found in the following polynomials.
   a. $x^2 + 4 = (x + 2)(x + 2)$
   b. $1.6x^2 - 9 = (0.4x - 3)(0.4x + 3)$

After performing activity 19 allow the students to revisit **IRF worksheets** and discuss their answers as a group. You can ask them their thoughts in this lesson.

Teacher's Note and Reminders

Challenge the students by doing **Activity 21**. Guide them in doing this activity and help them realize that there is an error in this process. After the activity make the students realize that **2 = 1 is not possible.** Cite the mistake in the activity given. As **bring home activity**, allow the students to complete their journal to reflect their experiences in this module/lesson.

c. $4x^2y^5 - 12x^3y^6 + 2y^2 = 2y^2\,(2x^2y^3 - 6x^3y^4)$
d. $3x^2 - 27$ is not factorable or prime

5. Are all polynomial expressions factorable? Cite examples to defend your answer.

Activity 20 IRF REVISIT

Revisit your IRF sheet and revise your answer by filing in column 3 under FINAL column.

Initial	Revise	Final
		Express the following as product of factors. 1. $4x^2 - 12x$ = ___ 2. $9m^2 - 16n^2$ = ___ 3. $4a^2 + 12a + 9$ = ___ 4. $2x^2 + 9x - 5$ = ___ 5. $27x^3 - 8y^3$ = ___ 6. $a^3 + 125b^3$ = ___ 7. $xm + hm - xn - hn$ = ___

QUESTIONS ?

1. What have you observed from your answers in your initial column? Is there a big difference?
2. What realization can you make with regard to the relationship of special products and factors?

Activity 21 MATHEMAGIC! 2 = 1 POSSIBLE TO ME

Description: This activity will enable you to apply factoring to prove if 2 = 1.
Instruction: Prove that 2 = 1 by applying your knowledge of factoring. You will need the guidance of your teacher in doing such.

If $a = b$, Is 2 = 1?

a. Were you able to prove that 2 = 1?
b. What different factoring techniques have you used to arrive at the solution?
c. What error can you pinpoint to disprove that 2 = 1?
d. What was your realization in this activity?

Guide the students in doing this activity, and help them realize that there is an error in this process. After the activity make the students realize that **2 = 1 is not possible**. Cite the mistake in the activity given. As **bring home activity**, allow the students to complete their journal to reflect their experiences in this module / lesson.

$a = b$

$(a)^2 = (b)^2 \longrightarrow$ this is possible because the principle of equality was still followed.

$a^2 = ab \longrightarrow$ b^2 was rewritten as (b)(b) and since a = b, then a can be substituted to b.

$a^2 - b^2 = ab - b^2 \longrightarrow$ subtracting b^2 to both side

$(a - b)(a + b) = b(a - b) \longrightarrow$ Factoring the expressions

$\frac{(a-b)(a+b)}{a-b} = \frac{b(a-b)}{a-b} \longrightarrow$ to remove a – b

$b + b = b \longrightarrow$ since a = b, substitute a by b.

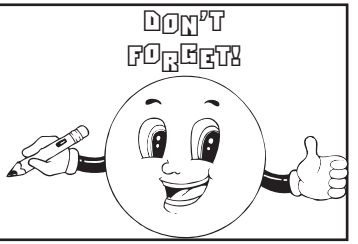
$2b = b \longrightarrow$ simplifying expression

$\frac{2b}{b} = \frac{b}{b} \longrightarrow$ divide both sides by b

$2 = 1$ (Note: This is not possible because when both sides were divided by a – b, the expression becomes undefined for $a - b = 0$)

To prepare the students for their final project do scaffold 1 and 2. This must be a guided activity. Allow them to answer the process questions after and discuss it in class.

Teacher's Note and Reminders



Activity 22 JOURNAL WRITING

Description: This activity will enable you to reflect about the topic and activities you underwent.

Instruction: Reflect on the activities you have done in this lesson by completing the following statements. Write your answers on your journal notebook.

A. Reflect on your participation in doing all the activities in this lesson and complete the following statements:
- I learned that I...
- I was surprised that I...
- I noticed that I...
- I discovered that I...
- I was pleased that I...

Activity 23 LET'S SCALE TO DRAW!

Description: In this activity you will draw plane figures to help you do the final project after this module.

Directions: Using the skills you have acquired in the previous activity, follow your teacher's instruction.

1. Drawing Plane Figures
   a. a square with a side which measures 10 cm.
   b. a rectangle with a length 3 cm more than its width.
   c. draw any geometric figure and label its dimensions algebraically.

2. A discussion on scale drawing will follow. After the discussion, the teacher will demonstrate the steps on how to do the following:
   a. A tree is five meters tall. Using a scale of 1m:2cm, draw the tree on paper.
   b. The school's flag pole is 10 m high. Using a scale of 2.5m:1 dm, draw a smaller version of the tree. Give the height of your drawing.

3. The teacher will demonstrate how a cube can be made using a square paper. Follow what your teacher did.

Teacher's Note and Reminders

What to Transfer

Perform activity 25 to prepare the students for the final project.

For students culminating project ask them to do a packaging box and they will assume the role of member of a design department of a packaging company. Present to the students the rubrics on how they will be graded in this project. Students will present their output in front of the class. You may involve the class in evaluating their works. This culminating project must be done by group. You can have a display of their outputs.

Activity 24 Model Making

Description: Creation of a solid figure out of a given plane figure and expressing it in terms of factors of a polynomial.

1. Create a solid figure from the rectangular figure that was provided by following the steps given.
   a. Cut – out a 2 in by 2 in squares in all edges of a 12 in by 6 in rectangle.
   b. Fold all the sides upward.
   c. Paste/tape the edges of the new figure.

QUESTIONS ?

a. What is the area of the original rectangle if its side is x units?
b. If the sides of the small squares is y, what expression represents its area?
c. How will you express the area of the new figure in terms of the variables stated in letters a and b?
d. What is the dimension of the new figure formed? How about the volume of the solid?
e. If the value of x = 4 cm and the value of y = 1 cm, what will be the dimension of the new figure? Its area? Its volume?
f. How did factoring help you find the dimensions of the new figure formed? the area? the volume?
g. What did you learn from this activity?

How can unknown quantities in geometric problems be solved?

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding in special products and factoring.

SAMPLE RUBRIC FOR THE TRANSFER ACTIVITY

How did you find the performance task? How did the task help you see the real world application of the topic?

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Explanation of the Proposal (20%)	Explanations and presentation of the lay-out is detailed and clear.	Explanations and presentation of the lay-out is clear.	Explanations and presentation of the lay-out is a little difficult to understand but includes critical components.	Explanations and presentation of the lay –out is difficult to understand and is missing several components.	
Accuracy of Computations (30%)	The computations done are accurate and show understanding of the concepts of special products and factoring. There is an explanation for every computation made.	The computations done are accurate and show a wise use of the concepts of special products and factoring.	The computations done are erroneous and show some use of the concepts of special products and factoring.	The computations done are erroneous and do not show wise use of the concepts of special products and factoring.	
Utilization of Resources (20%)	Resources are efficiently utilized with less than 10% excess.	Resources are fully utilized with less than 10%-25% excess.	Resources are utilized but a lot of excess.	Resources are not utilized properly.	
Appropriateness of the Model (30%)	The models are well-crafted and useful for understanding the design proposal. They showcase the desired product and are artistically done.	The models are well-crafted and useful for understanding the design proposal. They showcase the desired product.	The diagrams and models are less useful in understanding the design proposal	The diagrams and models are not useful in understanding the design proposal.	
				OVERALL RATING	

Activity 25 I BRING MY TRASH HOME

Description: Perform the activity in preparation for your final output in this module.

In response to the school's environmental advocacy, you are required to make cylindrical containers for your trash. This is in support of the "I BRING MY TRASH HOME!" project of our school. You will present your output to your teacher and it will be graded according to the following criteria: explanation of the proposal, accuracy of computations, and utilization of the resources and appropriateness of the models.

Activity 26 PACKAGING ACTIVITY

Description: This activity will showcase your learning in this module. You will assume the role of a member of a designing team that will present your proposal to a packaging company.

The RER packaging company is in search for the best packaging for a new dairy product that they will introduce to the market. You are a member of the design department of RER Packaging Company. Your company is tapped to create the best packaging box that will contain two identical cylindrical containers with the box's volume set at 100 in^3. The box has an open top. The cover will just be designed in reference to the box's dimensions. You are to present the design proposal for the box and cylinder to the Chief Executive Officer of the dairy company and head of the RER Packaging department. The design proposal is evaluated according to the following: explanation of the proposal, accuracy of computations, utilization of the resources and appropriateness of the models.

The first commercial paperboard (not corrugated) box was produced in England in 1817

POST - TEST

1. Which statement is true?

   a. The square of a binomial is also a binomial.
   b. The product of a sum and difference of two terms is a binomial.
   c. The product of a binomial and a trinomial is the square of a trinomial.
   d. The terms of the cube of a binomial are all positive.

   Answer: B

2. Which of the following is NOT a difference of two squares?

   a. $\frac{1}{4}x^4 - 1$

   b. $x^2 - 0.0001y^4$

   c. $1.6(x - 1)^2 - 49$

   d. $(x + 1)4 - 4x^6$

   Answer: C, it is the only binomial that is not a difference of two squares

3. Which of the following can be factored?

   a. $0.08x^3 - 27y^3$
   b. $1.44(x^2 + 1) - 0.09$
   c. $24xy(x - y) + 5\ (x + y)$
   d. $0.027(x^2 + 1)^3 - 8$

   Answer: D, it is factorable by difference of two cubes

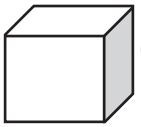
4. Which of the following values of k will make $x^2 - 5x + k$ factorable?

   a. 5
   b. 12
   c. -10
   d. -14

   Answer: D

5. If a square pool is to be made a rectangle such that the length is increased by 6 units and the width is decreased by 6 units, what will happen to its area?
   a. The area will increase by 12
   b. The area will decrease by 12
   c. The area will increase by 36.
   d. The area will decrease by 36
   Answer: D

6. What is the surface area of the given cube below?



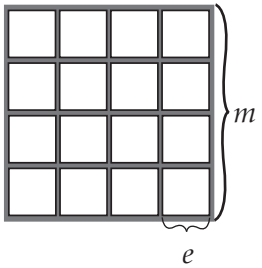
$(x + 3y)$ cm

   A. $(6x + 18y)$ cm^2
   B. $(x^2 + 6xy + 9y^2)$cm^2
   C. $(6x^2 + 26xy + 54y^2)$cm^2
   D. $(x^3 + 9x^2y + 27xy^2 + 27y^3)$cm^2
   Answer: C

7. Factor $16x^4 – 625y^{16}$ completely.
   a. $(4x^2 – 25y^4)(4x^2 + 25y^4)$
   b. $(4x^4 – 25y^8)(4x^4 + 25y^8)$
   c. $(2x^2 + 5y^4)(2x^2 – 5y^4)\ (4x^4 + 25y^8)$
   d. $(2x^2 + 5y^4)(2x^2 – 5y^4)(2x^2 + 5y^4)(2x^2 + 5y^4)$
   Answer: C, by factoring completely

8. The area of a rectangular garden is $(12x^2 – 8x – 15)$m^2, what are its dimensions?
   a. $(3x – 5)$m by $(4x + 3)$m
   b. $(6x + 5)$m by $(2x – 3)$m
   c. $(6x – 3)$m by $(2x – 5)$m **Answer: B**
   d. $(12x – 15)$m by $(x + 1)$

9. How much wood is needed in the window frame illustrated below?



a. $(m + e)^2$ square units
b. $(m^2 + e^2)$ square units
c. $(m – e)(m – e)$ square units
d. $(m + e)(m – e)$ square units

Answer: D

10. If the area of a square garden is $(4x^2 – 12x + 9)$ square units, is it possible to solve its sides?
a. Yes, using factoring difference of two squares.
b. No, one of the sides must be given
c. Yes, the area is a perfect square trinomial
d. No, the area is not factorable

Answer: C

11. Which of the following is a possible base of a triangle whose area is $(2x^2 – 6x + 9)$ square meters?
A. $(2x – 9)$ meters
B. $(4x – 6)$ meters
C. $(2x + 1)$ meters
D. $(4x – 3)$ meters

Answer: B

12. Liza factored the expression $15x^2y^3 + 10x^4y + 5xy$ as $5xy\ (3xy^2 + 2x^3)$. Did Liza factor it correctly?
    a. No, because $5xy$ is not the common factor.
    b. Yes, because the last term is cancelled out
    c. Yes, there exist a common factor on all terms
    d. No, because the last term when factored is 1 and should not be omitted
    Answer: D

13. Anne squared $3x + 4y$ as $9x^2 + 16y^2$, which of the following statement is correct with the answer of your classmate?
    a. The answer is correct, because to square a binomial distribute the exponent
    b. The answer is wrong, because the product of squaring a binomial is a trinomial.
    c. The answer is correct, because the product of squaring a binomial is another binomial
    d. The answer is wrong, because to square a binomial is to multiply the expression by 2.
    Answer: B

14. Below is the solution of Rogelio in factoring $3x^4 - 243$:
    $3(x^4 - 81)$
    $(x^2 - 9)(x^2 + 9)$
    $(x + 3)(x - 3)$
    Is the solution of Rogelio correct?

    A. No, because the other factors was omitted.
    B. No, because it lacks 3 as its factor.
    C. Yes, because $3x^4 - 243$ is divisible by $x + 3$.
    D. Yes, because the complete factorization of the expression is $(x + 3)(x - 3)$
    Answer: A

USE THE DATA BELOW TO ANSWER THE QUESTIONS THAT FOLLOW.

15. A driver asked you to create a utility box with no top from a 12 in by 10 in piece of metal by cutting identical squares from each corner and turning up the sides. The box must have a capacity of 96 in^3.
    If you are the driver, what are the standards you will look into the box?
    I. Appropriateness of the dimension
    II. Artistic

III. Durability
IV. Innovations

a. I & II
b. III & IV
c. I & III
d. II & IV

Answer: C

16. Which of the following is the appropriate thing to do to ensure that the correct dimension of the box will be obtained?
    a. Find a model for the box and measure it.
    b. Measure the sides of the squares thoroughly to create a box.
    c. Make a trial and error until the desired capacity is obtained.
    d. Find the dimension of the square to be cut through factoring and scale drawing.

    Answer: D

17. Marie Fe ask your advice on what to do so that her heterozygous blue eyed dog will have a big chance of having a blue eyed offspring, what advice could you give?
    a. Bring her dog to an ob – gyne
    b. Pair it with another blue eyed dog
    c. Pair it with a homozygous blue eyed dog
    d. Pair it with a heterozygous blue eyed dog.

    Answer: C, using the concept of Punnet square, a homozygous creature paired with another homozygous will have a high chance of resulting into the desired genes.

18. As finance officer of RTN plantation, you were asked by the company to prepare a budget to fence 120 hectares of your company's lot. What will you do to minimize the use of fencing materials, knowing that the length is 1 hectare less than twice the width?
    a. Estimate the dimension of the lot.
    b. Measure the dimension of the lot manually.
    c. Solve the dimension of the lot.
    d. Hire an engineer to survey the lot.

    Answer: C

19. Your friend an event organizer approach you to seek for your help to arrange 80 chairs in a weeding and suit it in the demand of the couple that the number of chairs in each rows is two less than the number of rows. How will you help your friend as to not to consume too much time in arranging?
    a. Make a trial and error of arrangement.
    b. Make a plan of arrangement of the chairs.
    c. Ask chair renting company to resolve the problem
    d. Used the data given and make an appropriate plan.
    Answer: D

20. As the principal of a school, you asked an architect to prepare a blue print for new classroom that you plan to build. The square classroom should have different areas for utilities (lavatory, CR, storage room and locker). What criteria will you use to approve the blue print?
    I. Maximizing the area
    II. Appropriateness of the location utilities.
    III. Dimensions of classroom utilities.
    IV. Uniqueness of design

    a. I, II & III
    b. I, II & IV
    c. I, III & IV
    d. II, III & IV
    Answer: A

SUMMARY

After completion of this module the students should have learned that products of some polynomials are obtained using the different patterns, and these products are called **special products**. They must also learn the different examples of special products, such as, the square of binomials, sum and difference of two terms, squaring trinomials, and cubing a binomial.

Students must have also realized that factor of different products can be obtained through the use of different patterns and rules. They should already learned the different types of factoring such as: (1) Factoring by greatest common monomial factor, (2) Factoring difference of two squares, (3) Factoring perfect square trinomials, (4) Factoring general trinomials, (5) Factoring the sum or difference of two cubes, and (6) Factoring by grouping.

And at this point student must already understand and used the concepts of special products and factoring in the context of real – life situations.

GLOSSARY OF TERMS USED IN THIS LESSON:

AREA – the amount of surface contained by a figure
COMPOSITE FIGURE – a figure that is made from two or more geometric figures
FACTOR – an exact divisor of a number.
GENETICS – is the area of biological study concerned with heredity and with the variations between organisms that result from it.
GEOMETRY – the branch of mathematics that deals with the nature of space and the size, shape, and other properties of figures as well as the transformations that preserve these properties.
GREATEST COMMON MONOMIAL FACTOR – is the greatest factor contained in every term of an algebraic expression.
HETEROZYGOUS – refers to having two different alleles (group of genes) for a single trait.
HOMOZYGOUS – refers to having identical alleles (group of genes) for a single trait.
PATTERN – constitutes a set of numbers or objects in which all the members are related with each other by a specific rule.
PERFECT SQUARE TRINOMIAL – result of squaring a binomial.
PERIMETER – the distance around a polygon.
POLYNOMIAL – is a finite sum of terms each of which is a real number or the product of a numerical factor and one or more variable factor raised to a whole – number powers.
PRODUCT – the answer of multiplication
PUNNET SQUARE - is a diagram that is used to predict an outcome of a particular cross or breeding experiment. And is used by

biologist to determine the chance of an offspring's having a particular genotype.
SCALE DRAWING – a reduced or enlarged drawing whose shape is the same as an actual object that it represents.
VOLUME – the measure of space occupied by a solid body

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