TEACHING GUIDE

Module 3: RELATIONS AND FUNCTIONS

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of linear functions.

Performance Standard:

The learner is able to formulate real-life problems involving linear functions and solve these with utmost accuracy using variety of strategies.

UNPACKING THE STANDARDS FOR UNDERSTANDING

	LEARNING COMPETENCIES
SUBJECT: Grade 8 Mathematics **QUARTER:** Second Quarter **STRAND:** Algebra **TOPIC:** Relations and Functions **LESSONS:** 1. Rectangular Coordinate System 2. Representations of Relations and Functions 3. Linear Function and Its Application	1. describe and illustrate the Rectangular Coordinate System and its uses; 2. describe and plot positions on the coordinate plane using the coordinate axes; 3. define relation and function; 4. illustrate a relation and a function; 5. determine if a given relation is a function using ordered pairs, graphs and equations; 6. differentiate between dependent and independent variables; 7. describe the domain and range of a function. 8. define linear function; 9. describe a linear function using its points, equation and graph;

	10. identify the domain and range of a linear function; 11. illustrate the meaning of the slope of a line; 12. find the slope of a line given two points, equation and graph; 13. determine whether a function is linear given the table; 14. write the linear equation $Ax + By = C$ in the form $y = mx + b$ and vice-versa; 15. graph a linear equation given (a) any two points, (b) the x-intercept and y-intercept, (c) the slope and a point on the line, (d) the slope and y-intercept; 16. describe the graph of a linear equation in terms of its intercepts and slope; 17. find the equation of a line given (a) two points; (b) the slope and a point; (c) the slope and its intercept; and 18. solve real-life problems involving linear functions and patterns.	
	ESSENTIAL UNDERSTANDING: Students will understand that problems involving constant rate of change can be solved using linear function.	**ESSENTIAL QUESTION:** How can the value of a quantity given the rate of change be predicted?
	TRANSFER GOAL: Students will on their own formulate and make representations of quantitative relationships in real-life situations and use these to solve problems.	

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to come up with in this module:

a. CoordinArt and Constellation Art making where Rectangular Coordinate System is applied by locating significant points in xy-plane;
b. A gallery walk of informative and creative leaflets whose contents are representations of relations and functions; and
c. A creative leaflet illustrating that electricity bill is a function of its power consumption.

Assessment Matrix

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Pre–assessment/ Diagnostic	**Pre-test** →	→	→	
	Oral Questioning Explanation Self-Knowledge	**Written Exercises** Explanation Application Self-Knowledge	**IRF Worksheet** Self-Knowledge	
Formative	**Written Exercises** Explanation Application Self-Knowledge		**IRF Worksheet** Self-Knowledge	**Gallery Walk (Making Informative Leaflets)** Application Explanation Self-Knowledge
	Quiz Explanation Perspective Self-Knowledge	**Quiz** Explanation Perspective Self-Knowledge	**Spotting Erroneous Coordinates** Explanation Perspective Self-Knowledge Empathy	**Making a Relation** Application Explanation Perspective
		Graph Analysis Explanation Interpretation Perspective	**Quiz** Explanation Perspective Self-Knowledge	**Finding Steepness of an Inclined Object (Steep Up!)** Explanation Perspective Self-Knowledge
		Flash Card Drill Interpretation Self-Knowledge	**Graph Analysis** Explanation Interpretation Perspective	

			Story Telling Interpretation Application Self-Knowledge Perspective	
Summative	**Unit Test** Explanation Interpretation Application Self-Knowledge	**Unit Test** Explanation Interpretation Application Self-Knowledge	**Unit Test** Explanation Interpretation Application Self-Knowledge	**CoordinArt Making** Explanation Application Interpretation Perspective
		Writing the Steps of Graphing Linear Equations Self-Knowledge Explanation		**Constellation Art Making (Optional)** Explanation Application Interpretation Perspective
	Periodical Examination Explanation Application Self-Knowledge Perspective Interpretation	**Periodical Examination** Explanation Application Self-Knowledge Perspective Interpretation	**Periodical Examination** Explanation Application Self-Knowledge Perspective Interpretation	**Periodical Examination** Explanation Application Self-Knowledge Perspective Interpretation
Self-assessment			**IRF Worksheet** Self-Knowledge	

Legend:
Six Facets of Understanding: Explanation, Interpretation, Application, Perspective, Empathy, Self-Knowledge

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	• describe and illustrate the Rectangular Coordinate System and its uses; • describe and plot points on the coordinate plane using the coordinate axes; • define relation and function; • illustrate relation and function; • determine if a given relation is a function using ordered pairs, graphs and equations; • differentiate between dependent and independent variables; • describe the domain and range of a function; • define linear function; • describe a linear function using its points, equation and graph; • identify the domain and range of a linear function; • illustrate the meaning of the slope of a line; • find the slope of a line given two points, equation and graph; • determine whether a function is linear given the table; • write the linear equation $Ax + By = C$ in the form $y = mx + b$ and vice-versa; • graph a linear equation given (a) any two points, (b) the x-intercept and y-intercept, (c) the slope and a point on the line, and (d) the slope and y-intercept; • describe the graph of a linear equation in terms of its intercepts and slope; • find the equation of a line given (a) two points; (b) the slope and a point; (c) the slope and its intercept; and • solve real-life problems involving linear functions and patterns.	Paper and Pen Test (Refer to attached post-assessment) Items 1, 2 and 3	1 point for every correct response
Process/Skills 25%		Paper and Pen Test (Refer to attached post-assessment) Items 4, 5, 6, 7 and 8	1 point for every correct response
Understanding 30%	• Students will understand that problems involving constant rate of change can be solved using linear function; and • The value of y increases as the value of x increases. (Misconception)	Paper and Pen Test (Refer to attached post-assessment) Items 9, 10, 11, 12, 13 and 14	1 point for every correct response

Product 30%	**GRASPS** Students will, on their own, formulate and make representations of quantitative relationships in real life situations and use these to solve problems.	Paper and Pen Test (Refer to attached post-assessment) Items 15, 16, 17, 18, 19 and 20	1 point for every correct response
	The learner is able to formulate real-life problems involving linear functions and solve these with utmost accuracy using a variety of strategies.	Your student is a *barangay* councilor in San Sebastian. Every month he conducts information drive on the different issues that concern every member in the community through the use of leaflets. For the next month, his focus is on electricity consumption of every household. He is tasked to prepare a leaflet design which will clearly explain about electricity bill and consumption. Include recommendations that will help lessen electricity utilization. He is expected to orally present your design to the other officials in your *barangay*. He will be assessed according to the following criteria: (1) use of appropriate mathematical concepts and accuracy, (2) organization, (3) quality of presentation, and (4) practicality of recommendations.	**Rubric on Problem Posing / Formulation and Problem Solving** Criteria: Relevant Creative Insightful Authentic Clear **Rubric on CoordinArt Making** Criteria: Creative Accurate Authentic Neatness **Rubric on Leaflet Design** Criteria: Use of mathematical concepts and accuracy Organization Quality of presentation Practicality of recommendations

C. Planning for Teaching-Learning

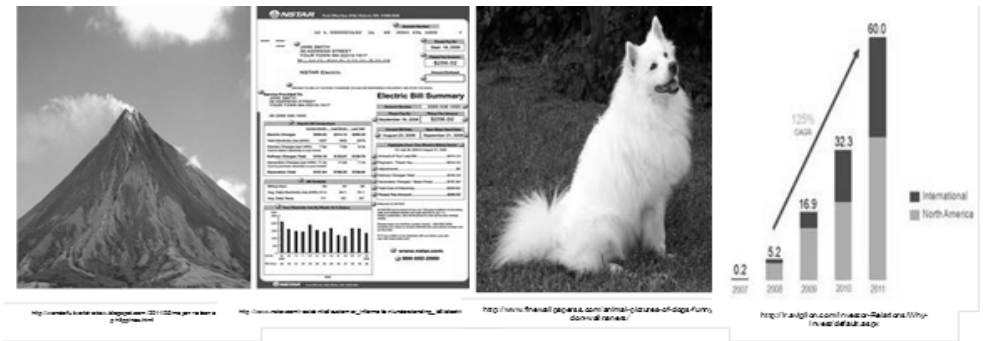
Introduction:

This module covers the key concepts of linear functions. It is divided into three lessons, namely: Rectangular Coordinate System, Representations of Relations and Functions and Linear Function and its Applications. In Lesson 1, the students will plot points on the xy-plane. The students will also describe and illustrate the Rectangular Coordinate System and its uses. In Lesson 2, the students will illustrate the difference of relations and functions, and of independent and dependent variables, then give the domain and the range of a function. In Lesson 3, the students will describe a linear function using its points, equation and graph, illustrate the meaning of slope, find the slope, write the linear equation in any form, draw the graph of the linear equation, and solve real-life problems involving linear functions and patterns.

In most lessons, students are encouraged to visit the links provided in the module. They are also encouraged to use software such as GeoGebra to graph the linear equation easily. They are also allowed to use any graphing materials, sharp edge and other tools. They are provided with varied activities to process the knowledge and skills acquired, deepen their understanding and transfer it to new context.

As an introduction to the main lesson, ask the students the following questions:

Have you ever asked yourself how the steepness of the mountain affects the speed of a mountaineer? How does the family's power consumption affect the amount of the electric bill? How is a dog's weight affected by its food consumption? How is the revenue of the company related to number of items produced and sold? How is the grade of a student affected by the number of hours spent in studying?



Hook the students to find out the answers to these questions leading to the essential question: **"How can the value of a quantity given the rate of change be predicted?"**

Objectives:

After the learners have gone through the lessons contained in this module, they are expected to:

1. describe and illustrate the Rectangular Coordinate System and its uses;
2. describe and plot positions on the coordinate plane using the coordinate axes;
3. define relation and function;
4. illustrate a relation and a function;
5. determine if a given relation is a function using ordered pairs, graphs and equations;
6. differentiate between dependent and independent variables;
7. describe the domain and range of a function.
8. define linear function;
9. describe a linear function using its points, equation and graph;
10. identify the domain and range of a linear function;
11. illustrate the meaning of the slope of a line;
12. find the slope of a line given two points, equation and graph;
13. determine whether a function is linear given the table;
14. write the linear equation $Ax + By = C$ in the form $y = mx + b$ and vice-versa;
15. graph a linear equation given (a) any two points, (b) the x-intercept and y-intercept, (c) the slope and a point on the line, (d) the slope and y-intercept;
16. describe the graph of a linear equation in terms of its intercepts and slope;
17. find the equation of a line given (a) two points; (b) the slope and a point; (c) the slope and its intercept; and
18. solve real-life problems involving linear functions and patterns.

Pre–test

Direction: Read the questions carefully. Write the letter that corresponds to your answer on a separate sheet of paper.

1. What is the Rectangular Coordinate System?
   a. It is used for naming points in a plane.
   b. It is a plane used for graphing linear functions.
   c. It is used to determine the location of a point by using a single number.
   d. It is a two-dimensional plane which is divided by the axes into four regions called quadrants.
   Answer: D

2. Which of the following is true about the points in Figure 1?
   a. J is located in Quadrant III.
   b. C is located in Quadant II.
   c. B is located in Quadrant IV.
   d. G is located in Quadrant III.
   Answer: D

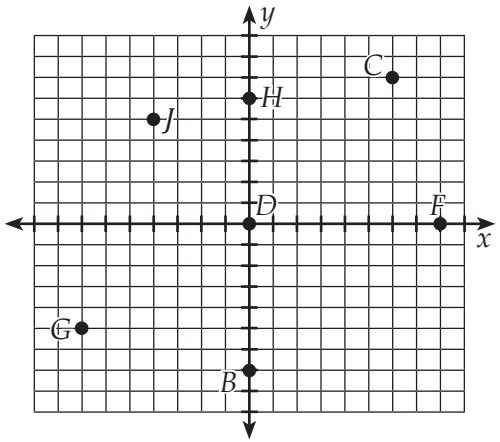


Figure 1

3. Which of the following sets of ordered pairs does NOT define a function?
   a. {(3, 2), (-3, 6), (3, -2), (-3, -6)}
   b. {(1, 2), (2, 6), (3, -2), (4, -6)}
   c. {(2, 2), (2, 3), (2, 4), (2, -9)}
   d. {(4, 4), (-3, 4), (4, -4), (-3, -4)}
   Answer: B

4. What is the domain of the relation shown in Figure 2?
   a. $\{x|x \in \Re\}$ c. $\{x|x > -2\}$
   b. $\{x|x \geq 0\}$ d. $\{x|x \geq -2\}$
   Answer: D

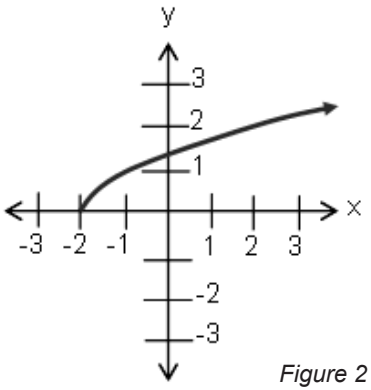


Figure 2

5. Determine the slope of the line $3x + y = 7$.

a. 3 c. $\frac{1}{3}$

b. -3 d. $-\frac{1}{3}$

Answer: B

6. Rewrite $2x + 5y = 10$ in the slope-intercept form.

a. $y = -\frac{2}{5}x + 2$ c. $y = -\frac{2}{5}x + 10$

b. $y = \frac{2}{5}x + 2$ d. $y = \frac{2}{5}x + 10$

Answer: A

7. Find the equation of the line with the slope -2 and passing through (5, 4).

a. $y = 2x + 1$ c. $y = 2x + 14$

b. $y = -2x + 1$ d. $y = -2x + 14$

Answer: D

8. Which line passes through the points (3, 4) and (8, -1)?

a. $y = -x + 7$ c. $y = x + 7$

b. $y = -x - 1$ d. $y = x - 1$

Answer: A

9. Jonathan has a job mowing lawns in his neighborhood. He works up to 10 hours per week and gets paid Php 25 per hour. Identify the independent variable.

a. the job c. the lawn mowing

b. the total pay d. the number of hours worked

Answer: D

10. Some ordered pairs for a linear function of x are given in the table below.

x	1	3	5	7	9
y	-1	5	11	17	23

Which of the following equations was used to generate the table above?

a. $y = 3x - 4$ b. $y = 3x + 4$ c. $y = -3x - 4$ d. $y = -3x + 4$

Answer: A

11. As x increases in the equation $5x + y = 7$, the value of y

a. increases.
b. decreases.
c. does not change.
d. cannot be determined.

Answer: B

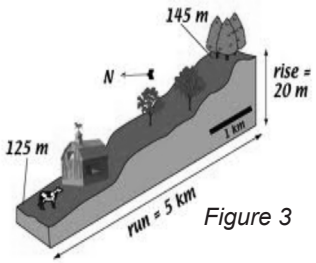


Figure 3

12. What is the slope of the hill illustrated in Figure 3? (Hint: Convert 5 km to m.)

a. 4 c. $\frac{1}{4}$

b. 125 d. $\frac{1}{125}$

Answer: D

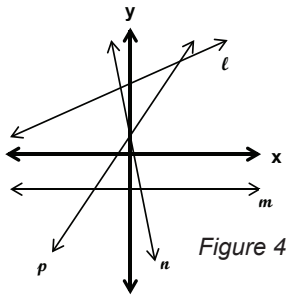


Figure 4

13. Which line in Figure 4 is the steepest?

a. line l b. line m c. line n d. line p

Answer: C

14. Joshua resides in a certain city, but he starts a new job in the neighboring city. Every Monday, he drives his new car 90 kilometers from his residence to the office and spends the week in a company apartment. He drives back home every Friday. After 4 weeks of this routinary activity, his car's odometer shows that he has travelled 870 kilometers since he bought the car. Write a linear model which gives the distance y covered by the car as a function of x number of weeks since he used the car.

a. $y = 180x + 150$ b. $y = 90x + 510$ c. $y = 180x + 510$ d. $y = 90x + 150$

Answer: A

For item numbers 15 to 17, refer to the situation below.

A survey of out-of-school youth in your *barangay* was conducted. From year 2008 to 2012, the number of out-of-school youths was tallied and was observed to increase at a constant rate as shown in the table below.

Year	2008	2009	2010	2011	2012
Number of out-of-school youth, y	30	37	44	51	58

15. If the number of years after 2008 is represented by x, what mathematical model can you make to represent the data above?
    a. $y = -7x + 30$ b. $y = -7x + 23$ c. $y = 7x + 30$ d. $y = 7x + 23$
    Answer: C

16. If the pattern continues, can you predict the number of out-of-school youths by year 2020?
    a. Yes, the number of out-of-school youths by year 2020 is 107.
    b. Yes, the number of out-of-school youths by year 2020 is 114.
    c. No, because it is not stipulated in the problem.
    d. No, because the data is insufficient.
    Answer: B

17. The number of out-of-school youths has continued to increase. If you are the SK Chairman, what would be the best action to minimize the growing number of out-of-school youths?
    a. Conduct a job fair.
    b. Create a sports project.
    c. Let them work in your *barangay*.
    d. Encourage them to enrol in Alternative Learning System.
    Answer: D

18. You are a Math teacher. You gave a task to each group of students to make a mathematical model, a table of values, and a graph about the situation below.

A boy rents a bicycle in the park. He has to pay a fixed amount of Php 10 and an additional cost of Php 15 per hour or a fraction of an hour, thereafter.

What criteria will you consider so that your students can attain a good output?

I. Accuracy
II. Intervals in the Axes
III. Completeness of the Label
IV. Appropriateness of the Mathematical Model

a. I and II only
b. I, II and III only
c. II, III and IV only
d. I, II, III and IV

Answer: D

19. If y refers to the cost and x refers to the number of hours, what is the correct mathematical model of the situation given in item 18?

a. $y = 15x + 10$ b. $y = 10x + 15$ c. $y = 15x - 10$ d. $y = 10x - 15$

Answer: A

20. You are one of the trainers of a certain TV program on weight loss. You notice that when the trainees run, the number of calories c burned is a function of time t in minutes as indicated below:

t	1	2	3	4	5
$c(t)$	13	26	39	52	65

As a trainer, what best piece of advice could you give to the trainees to maximize weight loss?

a. Spend more time for running and eat as much as you can.
b. Spend more time for running and eat nutritious foods.
c. Spend less time for running.
d. Sleep very late at night.

Answer: A

What to Know

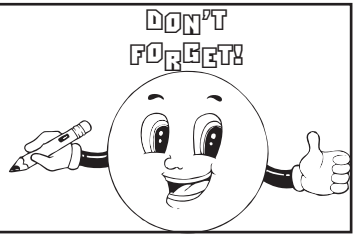
Provide students the opportunity to recall the binary operations and the cardinality of sets as well as the number line. Ask them to perform Activities 1 and 2. In Activity 1, you may use colorful chips, cartolinas, or any tangible objects to represent colors of each set. This is an opening activity so allow them to be motivated for them to be eager to learn more in this module. You may modify the activity based on the availability of materials. As you go through with the activities, let them realize the importance of Rectangular Coordinate System in real life. Pose the topical Essential Question**: How can the Rectangular Coordinate System be used in real life?**

Answers Key

Activity 1

1. $A \cup B$ = {red, blue, orange, violet, white}
2. $A \cap B$ = {red}
3. $A \cup A \cup C$ = {red, blue, orange, violet, white, black}
4. $n(A \cup A) = 5$
5. $n(A \cap B) = 1$
6. $A \cap C$ = {blue}
7. $A \cap B \cap C$ = { }
8. $A \cap (B \cup C)$ = {red, blue}
9. $n(A \cap (B \cup C)) = 2$

Teacher's Note and Reminders



Lesson 1 Rectangular Coordinate System

What to Know

Let's start this module by reviewing the important lessons on "Sets." As you go through this part, keep on thinking about this question: **How can the Rectangular Coordinate System be used in real life?**

Activity 1 RECALLING SETS

Description: This activity will help you recall the concept of sets and the basic operations on sets.

Direction: Let A = {red, blue, orange}, B = {red, violet, white} and C = {black, blue}. Find the following.

1. $A \cup B$	4. $n(A \cup B)$	7. $A \cap B \cap C$
2. $A \cap B$	5. $n(A \cap B)$	8. $A \cap (B \cup C)$
3. $A \cup B \cup C$	6. $A \cap C$	9. $n(A \cap (B \cup C))$

QUESTIONS ?

Have you encountered difficulty in this lesson? If yes, what is it?

Activity 2 BOWOWOW!

Description: This activity is in the form of a game which will help you recall the concept of number line.

Direction: Do as directed the given tasks.

1. Group yourselves into 9 or 11 members.
2. Form a line facing your classmates.
3. Assign integers which are arranged from least to greatest to each group member from left to right.
4. Assign zero to the group member at the middle.
5. Recite the number assigned to you.

Teacher's Note and Reminders

Elicit students' present knowledge of Rectangular Coordinate System by answering the "Initial Answer" column in the IRF Worksheet.

What to Process

Provide students enabling activities/experiences that they will have to go through to validate understanding of Rectangular Coordinate System. These would correct some of their misconceptions on this topic that have been encountered in real-life situations. After letting the students give their initial answers to the questions in the IRF Worksheet, tell them that at the end of the lesson, they are expected to do the CoordinArt Making as a demonstration of their understanding about the Rectangular Coordinate System.

Let the students read and understand some important notes on Rectangular Coordinate System before they perform the succeeding activities. Tell them to study carefully the examples provided.

6. **Bow as you recite and when the last member is done reciting, all of you bow together and say Bowowow!**

QUESTIONS ?

1. What is the number line composed of?
2. Where is zero found on the number line?
3. What integers can be seen in the left side of zero? What about on the right side of zero?
4. Can you draw a number line?

Activity 3 IRF WORKSHEET

Description: Below is the IRF Worksheet in which you will give your present knowledge about the concept.

Direction: Give your initial answers of the questions provided in the first column and write them in the second column.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is a rectangular coordinate system?			
2. What are the different parts of the rectangular coordinate system?			
3. How are points plotted on the Cartesian plane?			
4. How can the Rectangular Coordinate System be used in real life?			

You just tried answering the initial column of the IRF Sheet. The next section will enable you to understand what a Rectangular Coordinate System is all about and do a CoordinArt to demonstrate your understanding.

What to Process

Your goal in this section is to learn and understand the key concepts of Rectangular Coordinate System.

Teacher's Note and Reminders

Rectangular Coordinate System is introduced using the concept of sets. You have learned the binary operations of sets: **union** and **intersection**. Recall that $A \cup B$ and $A \cap B$ are defined as follows:

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ $\qquad$ $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

The **product set** or **Cartesian product** of nonempty sets A and B, written as $A \times B$ and read "A cross B," is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$. In symbols,

.

Illustrative Examples:

Let $A = \{2, 3, 5\}$ and $B = \{0, 5\}$. Find (a) $A \times B$ and (b) $B \times A$.

Solution:

$A \times B = \{(2, 0), (2, 5), (3, 0), (3, 5), (5, 0), (5, 5)\}$
$B \times A = \{(0, 2), (5, 2), (0, 3), (5, 3), (0, 5), (5, 5)\}$

The cardinality of set A is 3, symbolized as $n(A) = 3$. The cardinality of a set is the number of elements in the set. The cardinality of $A \times B$, written as $n(A \times B)$, can be determined by multiplying the cardinality of A and the cardinality of B. That is,

$$n(A \times B) = n(A) \cdot n(B)$$

Illustrative Examples:

Let $A = \{2, 3, 5\}$ and $B = \{0, 5\}$. Find (a) $n(A \times B)$, and (b) $n(B \times A)$.

Questions:

Is $n(A \times B) = n(B \times A)$?
Why?

Solution:

$n(A \times B) = 3 \cdot 2 = 6$
$n(B \times A) = 2 \cdot 3 = 6$

Answers to the Questions:

Yes, $n(A \times B) = n(B \times A)$.

It is because $n(A \times B) = n(B \times A)$ implies $n(A) \cdot n(B) = n(B) \cdot n(A)$ and it holds by Multiplication Property of Equality.

QUESTIONS ?

What can you conclude?

Answer
For any nonempty sets A and B,
1. $n(A \times B) = n(B \times A)$.
2. $A \times B \neq B \times A$.

Answers to the Exercises:

Exercise 1. Given that A = {4, 7, 8} and B = {5, 6}, find the following:
1. $A \times B$ = **{(4, 5), (4, 6), (7, 5), (7, 6), (8, 5), (8, 6)}**
2. $B \times A$ = **{(5, 4), (6, 4), (5, 7), (6, 7), (5, 8), (6, 8)}**
3. $n(A \times B)$ = **6**
4. $n(B \times A)$ = **6**

Exercise 2. Find (a) $X \times Y$, (b) $Y \times X$, (c) $n(X \times Y)$, and (d) $n(Y \times X)$ given the following sets X and Y:
1. X = {2, 3} and Y = {8, 3}
   (a) $X \times Y$ **= {(2, 8), {2, 3), (3, 8), (3, 3)}**
   (b) $Y \times X$ **= {(8, 2), {3, 2), (8, 3), (3, 3)}**
   (c) $n(X \times Y)$ **= 4**
   (d) $n(Y \times X)$ **= 4**
2. X = {1, 3, 6} and Y = {1, 5}
   (a) $X \times Y$ **= {(1, 1), {1, 5), (3, 1), (3, 5), (6, 1), (6, 5)}**
   (b) $Y \times X$ **= {(1, 1), {5, 1), (1, 3), (5, 3), (1, 6), (5, 6)}**
   (c) $n(X \times Y)$ **= 6**
   (d) $n(Y \times X)$ **= 6**
3. X = {2, 5, 8, 9} and Y = {0, 8}
   (a) $X \times Y$ **= {(2, 0), {2, 8), (5, 0), (5, 8), (8, 0), (8, 8), (9, 0), (9, 8)}**
   (b) $X \times Y$ **= {(0, 2), {8, 2), (0, 5), (8, 5), (0, 8), (8, 8), (0, 9), (8, 9)}**
   (c) $n(X \times Y)$ **= 8**
   (d) $n(Y \times X)$ **= 8**
4. X = {a, e, i, o, u} and Y = {$y \mid y$ is a letter of the word paper}.
   (a) $X \times Y$ **= {(a, p), {a, a), (a, e), (a, r), (e, p), (e, a), (e, e), (e, r), (i, p), (i, a), (i, e), (i, r), (o, p), (o, a), (o, e), (o, r), (u, p), (u, a), (u, e), (u, r)}**
   (b) $Y \times X$ **= {(p, a), {a, a), (e, a), (r, a), (p, e), (a, e), (e, e), (r, e), (p, i), (a, i), (e, i), (r, i), (p, o), (a, o), (e, o), (r, o), (p, u), (a, u), (e, u), (r, u)}**
   (c) $n(X \times Y)$ **= 20**
   (d) $n(Y \times X)$ **= 8**
5. X = {$x \mid 1 < x < 10$, x is a prime number} and Y = {$y \mid y \in N, 1 < y < 3$}
   (a) $X \times Y$ **= {(2, 2), {3, 2), (5, 2), (7, 2)}**
   (b) $X \times Y$ **= {(2, 2), {2, 3), (2, 5), (2, 7)}**
   (c) $n(X \times Y)$ **= 4**
   (d) $n(Y \times X)$ **= 4**

State your conclusions by competing the statements below using the correct relation symbol = or ≠.

For any nonempty sets A and B,
1. $n(A \times B)$ ___ $n(B \times A)$.
2. $A \times B$ ___ $B \times A$.

Exercise 1

Given that A = {4, 7, 8} and B = {5, 6}, find the following:

1.	$A \times B$	3.	$n(A \times B)$
2.	$B \times A$	4.	$n(B \times A)$

Exercise 2

Find (*a*) $X \times Y$, (*b*) $Y \times X$, (*c*) $n(X \times Y)$, and (*d*) $n(Y \times X)$ given the following sets X and Y:
1. X = {2, 3} and Y = {8, 3}
2. X = {1, 3, 6} and Y = {1, 5}
3. X = {2, 5, 8, 9} and Y = {0, 8}
4. X = {*a, e, i, o, u*} and Y = {$y \mid y$ is a letter of the word paper}.
5. X = {$x \mid 1 < x < 10$, x is a prime number} and Y = {$y \mid y \in N, 1 < y < 3$}

Let $\Re$ be the set of real numbers. The notation $\Re^2$ is the set of **ordered pairs** (x, y), where x and $y \in \Re$; that is,

$$\Re^2 = \Re \times \Re = \{(x, y) \mid x \in \Re, y \in \Re\}.$$

$\Re^2$ is also called the ***xy*-plane** or **Cartesian plane** in honor of the French mathematician René Descartes (1596 – 1650), who is known as the "Father of Modern Mathematics."

The **Cartesian plane** is composed of two perpendicular number lines that meet at the point of origin (0, 0) and divide the plane into four regions called **quadrants**. It is composed of infinitely many points. Each point in the coordinate system is defined by an **ordered pair** of the form (x, y), where x and $y \in \Re$. The first coordinate of a point is called the x-coordinate or abscissa and the second coordinate is called the y-coordinate or ordinate. We call (x, y) an ordered pair because it is different from (y, x). The horizontal and vertical lines, typically called the x-axis and the y-axis, respectively, intersect at the point of origin whose coordinates are (0, 0). The signs of the first and second coordinates of a point vary in the four quadrants as indicated below.

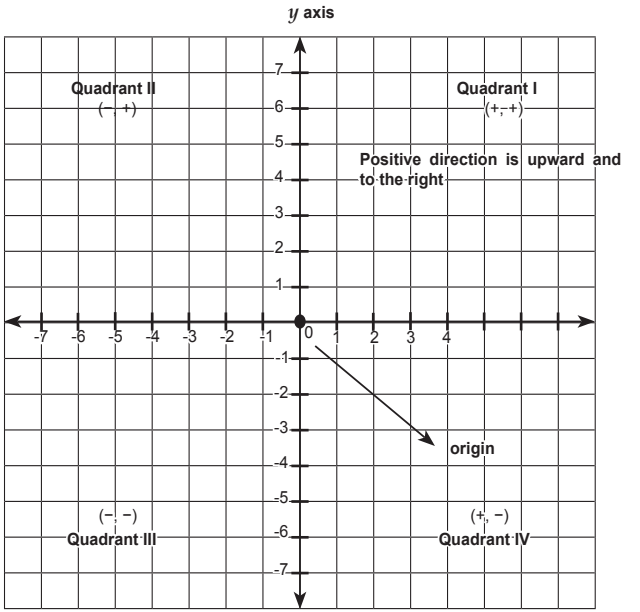
Quadrant I	$x > 0$, or x is positive	$y > 0$, or y is positive	or (+, +);
Quadrant II	$x < 0$, or x is negative	$y > 0$, or y is positive	or (−, +);
Quadrant III	$x < 0$, or x is negative	$y < 0$, or y is negative	or (−, −);
Quadrant IV	$x > 0$, or x is positive	$y < 0$, or y is negative	or (+, −).

There are also points which lie in the x- and y-axes. The points which lie in the x-axis have coordinates $(x, 0)$ and the points which lie in the y-axis have coordinates $(0, y)$, where x and y are real numbers.

Discuss the Rectangular Coordinate System connecting it with the concepts of sets. Start the discussion with the founder of the Cartesian plane, René Descartes followed by the different parts of the Cartesian plane such as axes, quadrants, origin, points, abscissa, and ordinate.

Teacher's Note and Reminders

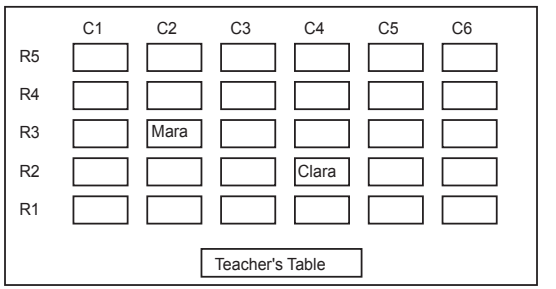
Illustrated below is a Cartesian plane.



How do you think can we apply this in real life? Let's try the next activity.

Example

Suppose Mara and Clara belong to a class with the following seating arrangement.



Teacher's Note and Reminders

Let the students locate seats of their classmates using rows and columns. Ask them to perform A ctivity 4. See to it that the chairs are arranged properly. You may also extend this activity outside the classroom by forming lines. See to it that each student is equidistant to one another.

Questions:

1. Using ordered pairs, how do we describe Mara's seat? How about Clara's seat?
2. Using ordered pairs, how do we locate the seat of any classmate of Mara and Clara?
3. Can we make a set of ordered pairs? If yes, state so.

Solutions:

1. Mara's seat is at the intersection of Column 2 and Row 3. Clara's seat is at the intersection of Column 4 and Row 2. In symbols, we can write (2, 3) and (4, 2), respectively, if we take the column as the x-axis and the row as y-axis.
2. We locate the seat of Mara's and Clara's classmates' by using column and row. We can use ordered pair (Column #, Row #) to locate it.
3. Here is the set of ordered pairs:
   {(C1, R1), (C2, R1), (C3, R1), (C4, R1), (C5, R1), (C6, R1),
   (C1, R2), (C2, R2), (C3, R2), (C4, R2), (C5, R2), (C6, R2),
   (C1, R3), (C2, R3), (C3, R3), (C4, R3), (C5, R3), (C6, R3),
   (C1, R4), (C2, R4), (C3, R4), (C4, R4), (C5, R4), (C6, R4),
   (C1, R5), (C2, R5), (C3, R5), (C4, R5), (C5, R5), (C6, R5)}

Activity 4 LOCATE YOUR CLASSMATE!

Description: This activity will enable you to locate the seat of your classmate in your classroom using ordered pairs. This can be done by groups of five members each.

Direction: Locate your seat and the seats of groupmates in the classroom. Complete the table below:

Name	Location

QUESTIONS ?

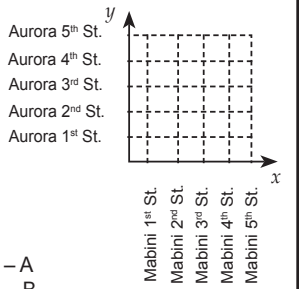
How do you locate the seat of your classmate in the classroom?

Teacher's Note and Reminders

Let the students experience decribing the coordinates of locations in real life by performing Activities 6 and 7.

Activity 5 MEET ME AT THIRDY'S RESIDENCE

Description: Finding a particular point such (1, 4) in the coordinate plane is similar to finding a particular place on the map. In this activity, you will learn how to plot points on the Cartesian plane.



Direction: With the figure at the right above, find the following locations and label each with letters as indicated.

a. Mabini 4th corner Aurora 1st Streets – A
b. Mabini 2nd corner Aurora 2nd Streets – B
c. Mabini 3rd corner Aurora 5th Streets – C
d. Mabini 5th corner Aurora 4th Streets – D
e. Mabini 1st corner Aurora 1st Streets – E

QUESTIONS ?

1. How do you find each location?
2. Which axis do you consider first? next?
3. If (1, 4) represents Mabini 1st Street corner Aurora 4th Street, then how could these points be represented?
   a. (3, 1) d. (4, 2)
   b. (4, 5) e. (5, 3)
   c. (1, 2)
4. If you are asked to plot those points mentioned in item number 3 in the Cartesian plane, can you do it? If yes, plot them.
5. How can Rectangular Coordinate System be used in real life?

Activity 6 HUMAN RECTANGULAR COORDINATE SYSTEM

Description: This activity is a form of a game which will enable you to learn the Rectangular Coordinate System.

Direction: Form two lines. 15 of you will form horizontal line (x-axis) and 14 for the vertical line (y-axis). These lines should intersect at the middle. Others may stay at any quadrant separated by the lines. You may sit down and will only stand when the coordinates of the point, the axis or the quadrant you belong is called.

Teacher's Note and Reminders

Parts of the Building	Coordinates	Quadrant
Example: Morning Room	(-11, 8)	II
1. Gilt Room	(-11, 5)	II
2. Terrace Hall	(12, -3)	IV
3. Old Kitchen	(12, -6)	IV
4. Billiard Room	(12, 8)	I
5. Salon	(6, 2)	I
6. Reception Hall	(-11, -4)	III
7. Grand Staircase	(1, -1)	IV

Parts of the Building	Coordinates	Quadrant
8. Marble Hall	(-5, 2)	II
9. Reception Office	(-11, -10)	III
10. Drawing Room	(2, 8)	I
11. Entrance	(-13, -2)	III
12. library	(-6, 7)	II
13. Spa	(7, -7)	IV
14. Harborough Room	(7, 7)	I

QUESTIONS ?

1. What is the Rectangular Coordinate System composed of?
2. Where do you see the origin?
3. What are the signs of coordinates of the points in each quadrant?
   a. Quadrant I
   b. Quadrant II
   c. Quadrant III
   d. Quadrant IV

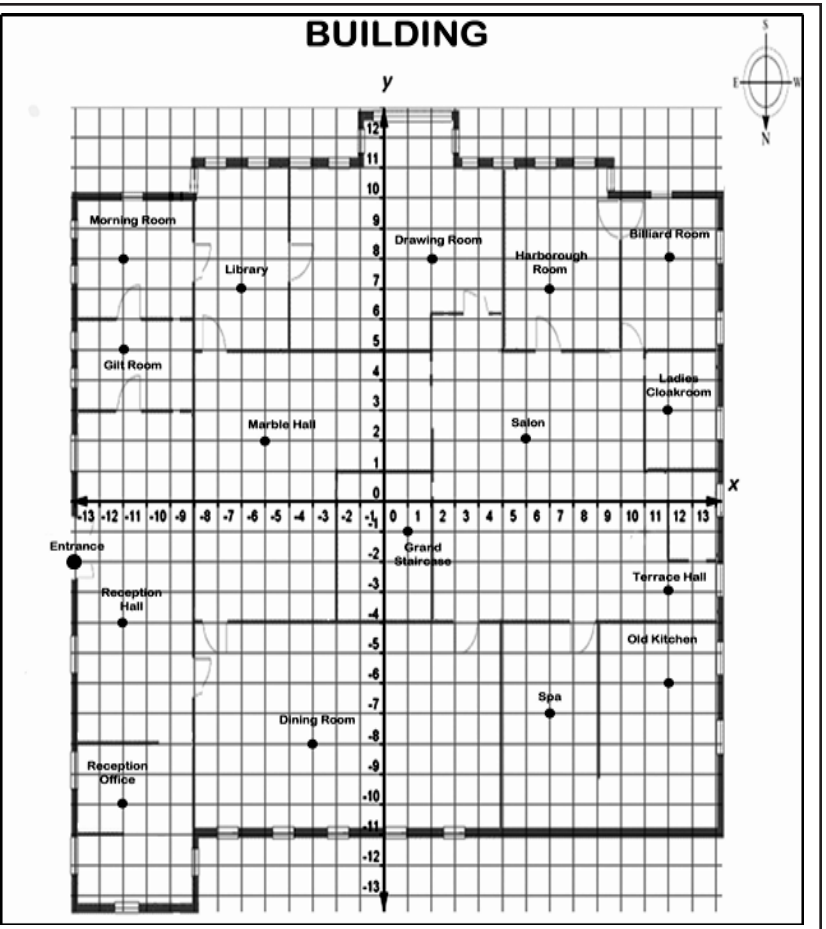
Activity 7 PARTS OF THE BUILDING

Description: This activity will enable you to give the coordinates of the part of building.
Direction: Describe the location of each point below by completing the following table. An example is done for. Note that the point indicates the center of the given part of the building.

Parts of the Building	Coordinates	Quadrant
Example: Morning Room	(-11, 8)	II
1. Gilt Room		
2. Terrace Hall		
3. Old Kitchen		
4. Billiard Room		
5. Salon		
6. Reception Hall		
7. Grand Staircase		

Parts of the Building	Coordinates	Quadrant
8. Marble Hall		
9. Reception Office		
10. Drawing Room		
11. Entrance		
12. library		
13. Spa		
14. Harborough Room		

Teacher's Note and Reminders



QUESTIONS ?

1. What is the Rectangular Coordinate System composed of?
2. How can the Rectangular Coordinate System be used in real life?

Let the students find the coordinates of the point and identify the quadrant/ axis where it is located by performing Activity 8.

Answers to the Activity 8:

Object	Coordinates	Quadrant/Axis
Example: ball	(4, 2)	I
1. spoon	(6, -5)	IV
2. television set	(-5, 6)	II
3. laptop	(2, -4)	IV
4. bag	(-4, -3)	III
5. pillow	(1, 5)	I
6. camera	(0, 0)	x-axis and y-axis
7. table	(-2, 2)	II

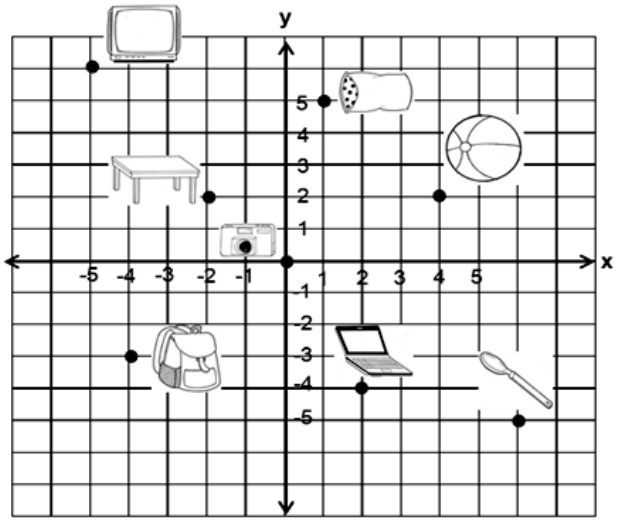
Teacher's Note and Reminders

Activity 8 OBJECTS' POSITION

Description: This activity will enable you to give the coordinates of the point where the object is located.

Direction: Describe the location of each point below by the completing the following table. An example is done for you.

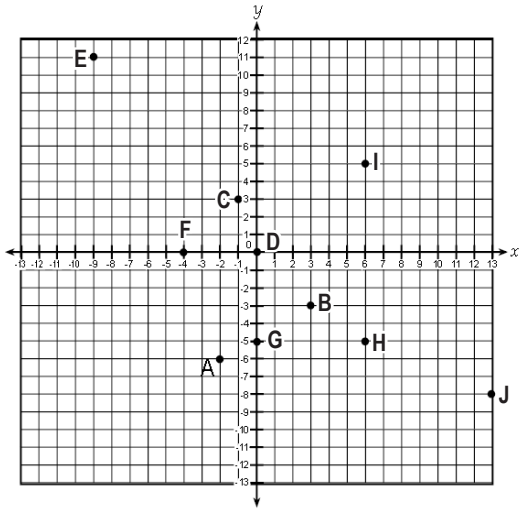
Object	Coordinates	Quadrant/Axis
Example: ball	(4, 2)	I
1. spoon		
2. television set		
3. laptop		
4. bag		
5. pillow		
6. camera		
7. table		



QUESTIONS ?

How can the Rectangular Coordinate System be used in real life?

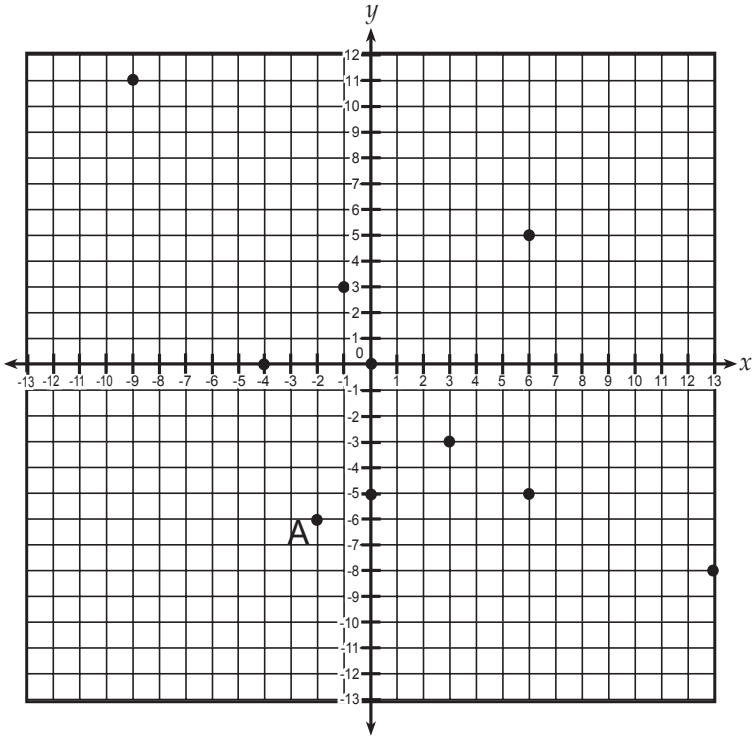
Answer to Exercise 3



Teacher's Note and Reminders

Exercise 3

Indicate the name of each point in the Cartesian plane. Name each point by writing the letter beside it. The coordinates are provided in the box below. An example is done for you.



1. A(-2, -6)	6. F(-4, 0)
2. B(3, -3)	7. G(0, -5)
3. C(-1, 3)	8. H(6, -5)
4. D(0, 0)	9. I(6, 5)
5. E(-9, 11)	10. J(13, -8)

Answer to Exercise 4

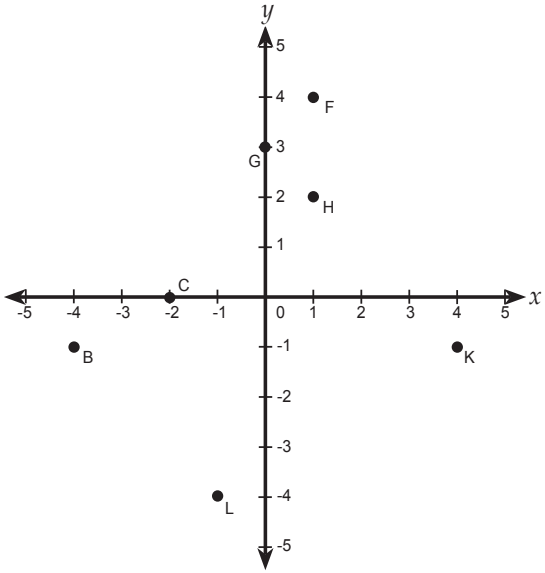
Coordinates	Quadrant / Axis
1. B(-4, -1)	Quadrant III
2. C(-2, 0)	x-axis
3. F(1, 4)	Quadrant I
4. G(0, 3)	y-axis
5. H(1, 2)	Quadrant I
6. L(1, -4)	Quadrant III
7. K(4, -1)	Quadrant IV

Teacher's Note and Reminders

Exercise 4

Write the coordinates of each point. Identify the quadrant/axis where each point lies. Complete the table below.

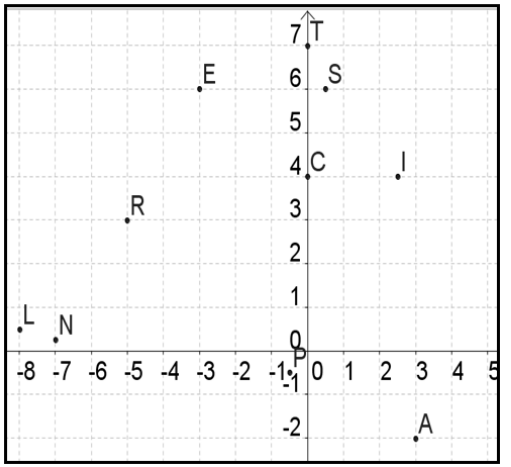
Coordinates	Quadrant / Axis
1. B(__ , __)	
2. C(__ , __)	
3. F(__ , __)	
4. G(__ , __)	
5. H(__ , __)	
6. L(__ , __)	
7. K(__ , __)	



QUESTIONS ?

1. In what quadrant/axis does a point lie?
2. How do you locate points on the Cartesian plane?
3. Have you had an experience in your daily life where a Rectangular Coordinate System is applied? If yes, cite it.
4. How can the Rectangular Coordinate System be used in real life?

Answer to Exercise 5



Let the students perform Activity 9 by revisiting the IRF Worksheet. Consider this activity as part of a formative assessment. Compare their revised answers to their initial answers. Pose again the topical Essential Question: **How can the Rectangular Coordinate System be used in real life?**

Teacher's Note and Reminders

Now, make a Cartesian plane and plot points on it. Can you do it? Try the next exercise.

Exercise 5

Draw a Cartesian plane. Plot and label the following points.

Web Links

Kindly click this link http://www.onlinemathlearning.com/rectangular-coordinate-system.html and watch the video provided for your reference.

1. C(0, 4)
2. A(3, -2)
3. R(-5, 3)
4. T(0, 7)
5. E(-3, 6)
6. S($\frac{1}{2}$, 6)
7. I($\frac{5}{2}$, 4)
8. N(-7, $\frac{1}{4}$)
9. P(-$\frac{1}{2}$, -$\frac{1}{2}$)
10. L(-8, $\frac{1}{2}$)

Activity 9 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will give your present knowledge about the concept.

Direction: Give your revised answers of the questions in the first column and write them in the third column. Compare your revised answers from your initial answers.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is a rectangular coordinate system?			
2. What are the different parts of the rectangular coordinate system?			
3. How do you locate points on the Cartesian plane?			
4. How can the Rectangular Coordinate System be used in real life?			

In this section, the discussion was all about the Rectangular Coordinate System. You have learned the important concepts of Rectangular Coordinate System. As you go through, keep on thinking of the answer of the question: ***How can the Rectangular Coordinate System be used in real life?***

What to Understand

Activities in this stage shall provide opportunity for the learners to reflect, revisit, and rethink on their experiences. Moreover, the learners shall express their understanding of Rectangular Coordinate System.

Answers to Activity 10

A. No, the correct coordinates of A are (4, 2), not (2, 4). She interchanged the x-coordinate and the y-coordinate.
B. No, the correct coordinates of B are (0, 4), not (4, 0) and that of D are (-4, 0), not (0, -4). He interchanged the x-coordinate and the y-coordinate.

Teacher's Note and Reminders

What to Understand

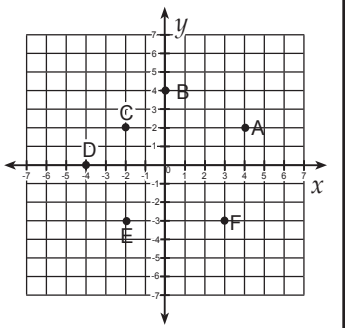
Your goal in this section is to take a closer look at some aspects of the topic.

Activity 10 SPOTTING ERRONEOUS COORDINATES

Description: This activity will enable you to correct erroneous coordinates of the point.
Direction: Do the task as directed

A. Susan indicated that A has coordinates (2, 4).
   1. Do you agree with Susan?
   2. What makes Susan wrong?
   3. How will you explain to her that she is wrong in a subtle way?

B. Angelo insisted that B has coordinates (4, 0) while D has coordinates (0, -4). If yes, why? If no, state the correct coordinates of points of B and D.



QUESTIONS ?

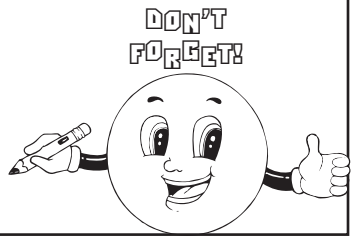
1. How did you find the activity?
2. How can the Rectangular Coordinate System be used in real life?

Challenge Questions:
Use graphing paper to answer the following questions:
1. What value of k will make the points (-4, -1), (-2, 1) and (0, k)?
2. What are the coordinates of the fourth vertex of the square if three of its vertices are at (4, 1), (-1, 1) and (-1, -4)?
3. What are the coordinates of the fourth vertex of the rectangle if three vertices are located at (-2, -7), (3, -7) and (3, 5)?

Introduce CoordinArt to the students in order for them to do well Activity 11. You may allow them to visit the links given below. You can give this as their group assignment.

1. bird - http://www.go2album.com/showAlbum/323639/coordinartiguana_macaw.
2. car - http://store.payloadz.com/details/800711-Other-Files-Documents-and-Forms-sports-car-.html.
3. statue - http://www.plottingcoordinates.com/coordinart_patriotic.html.

Teacher's Note and Reminders

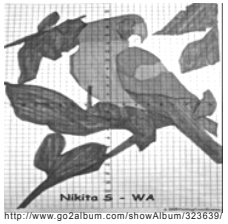


Have the students give their present knowledge about the concept. They will fill up the "Final Answer" column. Compare their final answers to their initial and revised answers. This is one way of assessing the their self-knowledge on the topic.

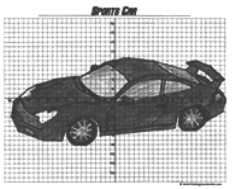
Activity 11 COORDINART

Description: This activity will give you some ideas on how Cartesian plane is used in drawing objects. Perform this activity in group of 5 to 10 students.

Direction: Select only one among the three coordinArts. Identify the ordered pairs of the significant points so that the figure below would be drawn.



http://www.go2album.com/showAlbum/323639/coordinartiguana_macaw



http://store.payloadz.com/details/800711-Other-Files-Documents-and-Forms-sports-car-.html

http://www.plottingcoordinates.com/coordinart_patriotic.html

The websites below are the sources of the images above. You may use these for more accurate answers.

1. bird - http://www.go2album.com/showAlbum/323639/coordinartiguana_macaw.
2. car - http://store.payloadz.com/details/800711-Other-Files-Documents-and-Forms-sports-car-.html.
3. statue - http://www.plottingcoordinates.com/coordinart_patriotic.html.

Activity 12 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will give your present knowledge about the concept.

Direction: Write in the fourth column your final answer to the questions provided in the first column. Compare your final answers with your initial and revised answers.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is a rectangular coordinate system?			
2. What are the different parts of the rectangular coordinate system?			
3. What are the uses of the rectangular coordinate system?			
4. How do you locate points on the Cartesian plane?			

What to Transfer

Give students the opportunity to demonstrate their understanding of Rectangular Coordinate System by doing a practical task. Let them perform Activity 13. You can ask them to work in groups. Show them the criteria to be used in evaluating their output. Use the rubric for CoordinArt Making.

Teacher's Note and Reminders

QUESTIONS

1. What have you learned about the first lesson in this module?
2. How meaningful is that learning to you?

Now that you have a deeper understanding of the topic, you are now ready to do the task in the next section.

What to Transfer

Give students the opportunity to demonstrate their understanding of representation of relations and functions by doing a practical task. Let them perform Activity 14. You can ask them to work in groups. Discuss to them the criteria to be used in evaluating their output.

Activity 13 COORDINART MAKING

Description: This activity will enable you to apply your knowledge in Rectangular Coordinate System to another context.

Materials: graphing paper
ruler
pencil and ballpen
coloring material

Direction: Group yourselves into 5 to 10 members. Make you own CoordinArt using graphing paper, ruler, pencil or ballpen, and any coloring material. Your output will be assessed using the rubric below:

RUBRIC: COORDINART MAKING

CRITERIA	Exemplary 4	Satisfactory 3	Developing 2	Beginning 1
Accuracy of Plot	All points are plotted correctly and are easy to see. The points are neatly connected.	All points are plotted correctly and are easy to see.	All points are plotted correctly.	Points are not plotted correctly.

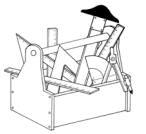
Teacher's Note and Reminders

Activity 14 is optional. You may or may not give this activity to your class. This is intended for advanced classes or special curricula. The same rubric in CoordinArt Making is used to score their output.

Finally, you may ask the students the topical Essential Question: **"How can the Rectangular Coordinate System be used in real life?"** Aside from what is specified, can you cite another area or context where this topic is applicable?

Originality	Product shows a large amount of original thought. Ideas are creative and inventive.	Product shows some original thought. Work shows new ideas and insights.	Uses other people's ideas and giving them credit but there is little evidence of original thinking.	Uses other people's ideas, but does not give them credit.
Neatness and Attractiveness	Exceptionally well designed, neat, and attractive. Colors that go well together are used to make the graph more readable. A ruler and graphing paper are used.	Neat and relatively attractive. A ruler and graphing paper are used to make the graph more readable.	Lines and curves are neatly drawn but the graph appears quite plain.	Appears messy and "thrown together" in a hurry. Lines and curves are visibly crooked.

Activity 14 CONSTELLATION ART



Description: This activity will enable you to apply your knowledge in Rectangular Coordinate System to another context.

Materials: graphing paper
pencil and ballpen
coloring material

Direction: Group yourselves into 5 to 10 members. Research constellations and their names. Choose the one that you like most. Make your own constellation using graphing paper, ruler, pencil or ballpen, and any coloring material.

How did you find the performance task? How did the task help you see the real-world use of the topic?

*You have completed this lesson. Before you go to the next lesson, answer the question: **"How can the Rectangular Coordinate System be used in real life?"** Aside from what is specified, can you cite another area or context where this topic is applicable?*

What to Know

Initially, begin with some interesting and challenging exploratory activities on representations of relations and functions that will make the learners aware of what is going to happen or where the said pre-activities would lead to through meaningful and relevant real-life context. Ask the students to perform Activity 1 which will lead to their understanding of relations. Pose the topical Essential Question: **How are the quantities related to each other?**

Answers to Activity 1

Kitchen Utensils	School Supplies	Gadgets
fork ladle pot grater knife	notebook liquid eraser paper ballpen pencil	iPod cellphone laptop table digital camera

a. Column 1: **(fork, kitchen utensil), (ladle, kitchen utensil), (pot, kitchen utensil), (grater, kitchen utensil), and (knife, kitchen utensil)**
b. Column 2: **(notebook, school supply), (liquid eraser, school supply), (paper, school supply), (ballpen, school supply), and (pencil, school supply)**
c. Column 3: **(iPod, gadget), (cellphone, gadget), (laptop, gadget), (tablet, gadget), and (digital camera, gadget)**

Lesson 2 Representations of Relations and Functions

What to Know

Let's start this lesson by looking at the relationship between two things or quantities. As you go through, keep on thinking about this question: **How are the quantities related to each other?**

Activity 1 CLASSIFY!

Description: This activity will enable you to write ordered pairs. Out of this activity, you can describe the relation of an object to its common name.

Direction: Group the following objects in such a way that they have common property/characteristics.

fork	liquid eraser	grater
pencil	knife	iPod
laptop	ballpen	pot
digital camera	tablet	cellphone
ladle	notebook	paper

Kitchen Utensils	School Supplies	Gadgets
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

This activity will provide students information for them to give their initial ideas about relations and functions. Let them do Activity 2 on their own.

Answer to Activity 2

{(narra, tree), (tulip, flower), (orchid, flower), (mahogany, tree), (rose, flower), (apricot, tree)}

Teacher's Note and Reminders

Form some ordered pairs using the format:
(object, common name).

a. Column 1: _____
b. Column 2: _____
c. Column 3: _____

QUESTIONS ?

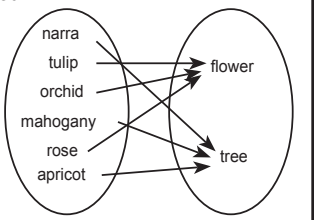
1. How many objects can be found in each column?
2. How did you classify the objects?
3. Based on the coordinates you have formulated, is there a repetition of the first coordinates? What about the second coordinates?

Activity 2 REPRESENTING A RELATION

Description: Given a diagram, you will be able to learn how to make a set of ordered pairs.

Direction: Describe the mapping diagram below by writing the set of ordered pairs. The first two coordinates are done for you.

Set of ordered pairs:
{(narra, tree), (tulip, flower),
(_____, _____), (_____, _____),
(_____, _____), (_____, _____)}

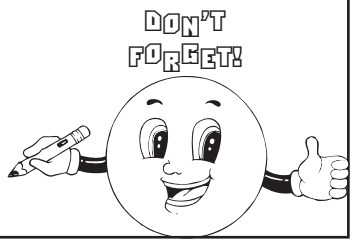


QUESTIONS ?

1. How did you make a set of ordered pairs?
2. How many elements are there in the set of ordered pairs you have made?
3. What elements belong to the first set? Second set?
4. Is there a repetition on the first coordinates? How about the second coordinates?
5. Does the set of ordered pairs represent a relation?
6. How is a relation represented?

Elicit present knowledge about relations and functions by answering the "Initial Answer" column in the IRF Worksheet.

Teacher's Note and Reminders



What to Process

After letting the students give their initial answers to the questions in the IRF Worksheet, tell them that at the end of the lesson, they are expected to make an informative leaflet of representations of relations and functions as a demonstration of their understanding.
Let the students read and understand important notes on relations and functions before they perform the succeeding activities. Tell them to study carefully the example provided.

Activity 3 IRF WORKSHEET

Description: Below is the IRF Worksheet that you will accomplish to record your present knowledge about the concept.

Direction: Write in the second column your initial answers to the questions provided in the first column.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is relation?			
2. What is function?			
3. What do you mean by domain of relation/ function?			
4. What do you mean by range of relation/ function?			
5. How are relations and functions represented?			
6. **How are the quantities related to each other?**			

You gave your initial ideas on representations of relations and functions. The next section will enable you to understand how a relation and a function represented and do a leaflet design to demonstrate your understanding.

What to Process

Your goal in this section is to learn and understand the key concepts of Representations of Relations and Functions.

A **relation** is any set of ordered pairs. The set of all first coordinates is called the **domain** of the relation. The set of all second coordinates is called the **range** of the relation.

Teacher's Note and Reminders

Ask the students to perform Activity 4. This activity will enable them to compose a correspondence of their height and weight which makes a relation. Afterwhich, allow the students to answer Exercises 1, 2, 3, 4 and 5.

Illustrative Example

Suppose you are working in a fast food company. You earn Php 40 per hour. Your earnings are related to the number of hours of work.

Questions:

1. How much will you earn if you work 4 hours a day? How about 5 hours? 6 hours? 7 hours? Or 8 hours?
2. Express each in an ordered pair.
3. Based on your answer in item 2, what is the domain? What is the range?

Solutions:

1. The earning depends on the number of hours worked. An amount of Php 160 is earned for working 4 hours a day, Php 200 for 5 hours, Php 240 for 6 hours, Php 280 for 7 hours and Php 320 in 8 hours.
2. (4, 160), (5, 200), (6, 240), (7, 280), and (8, 320)
3. The domain of the relation is {4, 5, 6, 7, 8}. The range of the relation is {160, 200, 240, 280, 320}.

Activity 4 MAKE YOUR OWN RELATION!

Description: This activity will enable you to make a relation, a correspondence of your height and weight.

Materials: tape measure or other measuring device
weighing device
ballpen
paper

Direction: Form groups of 5 to 10 members. Find your height and weight and of the other members of the group. Express your height in centimeters and weight in kilograms. Write the relation of height and weight in an ordered pair in the form (height, weight).

QUESTIONS ?

How are height and weight related to each other?

Answers to Exercise 1:
1. Php 20 for 1 hour, Php 40 for 2 hours and Php 60 for 3 hours.
2. (1, 20), (2, 40) and (3, 60). (Note to the Teacher: The correct ordered pair is (1, 20), not (20, 1) because the amount of bicycle rental is dependent on the numbers of hours rented.)
3. In the relation above, the domain is {1, 2, 3} while the range is {20, 40, 60}.
4. The amount of the bicycle rental is dependent on the rental time.

Answers to Exercise 2:
1. Php 5 for 3 minutes, Php 7 for 4 minutes and Php 9 for 5 minutes.
2. (3, 5), (4, 7) and (5, 9). (Note to the Teacher: The correct ordered pair is (3, 5), not (5, 3) because the charge is dependent on the number of minutes of call.)
3. In the relation above, the domain is {3, 4, 5} while the range is {5, 7, 9}.
4. The charge of the pay phone depends on the number of minutes calling.

Answers to Exercise 3:
1. John will pay Php 12 for 1 hour, Php 24 for 2 hours, Php 36 for 3 hours and Php 48 for 4 hours.
2. (1, 12), (2, 24), (3, 36) and (4, 48)
3. Yes
4. In the relation above, the domain is {1, 2, 3, 4}. However, the range is {12, 24, 36, 48}.
5. The amount John will have to pay depends on the time he played. The amount is 12 times the length of time.
6. Php 48 is the amount that John would have saved.

Answers to Exercise 4:
1. The perimeter of the square whose side is 1 cm long is 4 cm; for 2 cm is 8 cm; 3 cm, 12 cm; 4 cm, 16 cm; 5 cm, 20 cm; and 20 cm, 80 cm
2. (1, 4), (2, 8), (3, 12), (4, 16), (5, 20) and (20, 80).
3. Yes
4. In the relation above, the domain is {1, 2, 3, 4, 5, 20}. However, the range is {4, 8, 12, 16, 20, 80}.
5. The perimeter of the square is dependent on the length of its side. The perimeter of the square is 4 times the length of its side.

Answers to Exercise 5:
1. The person who weighs 26 lbs on the moon weighs 156 lbs on earth, 27 lbs on the moon weighs 162 lbs on earth, and 28 lbs on the moon weighs 168 lbs on earth.
2. The person who weighs 174 lbs on earth weighs 29 lbs on the moon, 180 lbs on earth is 30 lbs on the moon, and 186 lbs on earth is 31 lbs on the moon.
3. {(120, 20), (126, 21), (132, 22), (138, 23), (144, 24), (150, 25)}
4. Yes
5. Based on the given table, the domain is {120, 126, 132, 138, 144, 150}. However, the range is {20, 21, 22, 23, 24, 25}.
6. The person's weight on the moon is one-sixth of his weight on earth.

Exercise 1

Suppose the bicycle rental at the Rizal Park is worth Php 20 per hour. Your sister would like to rent a bicycle for amusement.
1. How much will your sister have to pay if she would like to rent a bicycle for 1 hour? 2 hours? 3 hours?
2. Based on your answers in item 1, write ordered pairs in the form (time, amount).
3. Based on your answers in item 2, what is the domain? What is the range?
4. How are rental time and cost of rental related to each other?

Exercise 2

Suppose you want to call your mother by phone. The charge of a pay phone call is Php 5 for the first 3 minutes and an additional charge of Php 2 for every additional minute or a fraction of it.
1. How much will you pay if you have called your mother in 1 minute? 2 minutes? 3 minutes? 4 minutes? 5 minutes?
2. Out of your answers in item 1, write ordered pairs in the form (time, charge).
3. Based on your answers in item 2, what is the domain? What is the range?
4. **How are time and charge related to each other?**

Exercise 3

John pays an amount Php 12 per hour for using the internet. During Saturdays and Sundays, he enjoys and spends most of his time playing a game especially if he is with his friends online. He plays the game almost 4 hours.
1. How much will John pay for using the internet for 1 hour? 2 hours? 3 hours? 4 hours?
2. Express each as an ordered pair.
3. Is it a relation? Explain.
4. Based on your answers in item 3, what is the domain? What is the range?
5. How are time and amount related to each other?
6. If John has decided not to play the game in the internet cafe this weekend, what is the maximum amount that he would have saved?

Exercise 4

The perimeter of a square depends on the length of its side. The formula of perimeter of a square is $P = 4s$, where P stands for perimeter and s stands for the side.
1. What is the perimeter of the square whose side is 1 cm long? How about 2 cm long? 3 cm long? 4 cm long? 5 cm long? 20 cm long?
2. Express each in an ordered pair.
3. Is it a relation? Why?
4. Based on your answers in item 3, what is the domain? What is the range?
5. **How are the perimeter and the side related to each other?**

Exercise 5

The weight of a person on earth and on the moon is given in the table as approximates.

Weight on earth (N)	120	126	132	138	144	150
Weight on the moon (N)	20	21	22	23	24	25

Source: You Min, Gladys Ng. (2008). GCE "O" Level Pure Physics Study Guide. Fairfield Book Publishers: Singapore.

Discuss the different ways of representing a relation. Provide examples and allow the students to give counterexamples.

Teacher's Note and Reminders

1. What is the weight of a person on earth if he weighs 26 N on the moon? 27 N? 28 N?
2. What is the weight of a person on the moon if he weighs 174 N on earth? 180 N? 186 N?
3. Write the set of ordered pairs using the given table.
4. Is it a relation? Why?
5. Based on your answer in item 3, what is the domain? What is the range? Explain.
6. How are the weight on the moon and the weight on earth related to each other?

Representations of Relations

Aside from ordered pairs, a relation may be represented in four other ways: (1) table, (2) mapping diagram, (3) graph, and (4) rule.

Table

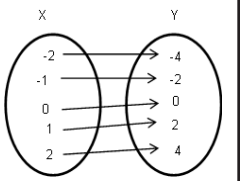
The table describes clearly the behavior of the value of y as the value of x changes. Tables can be generated based on the graph. Below is an example of a table of values presented horizontally. At the right is also a table of values that is presented vertically.

x	-2	-1	0	1	2
y	-4	-2	0	2	4

x	y
-2	-4
-1	-2
0	0
1	2
2	4

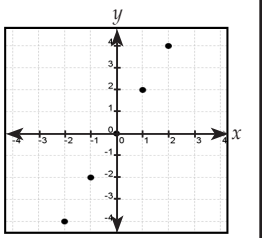
Mapping Diagram

Subsequently, a relation can be described by using a diagram as shown at the right. In this example, -2 is mapped to -4, -1 to -2, 0 to 0, 1 to 2, and 2 to 4.



Graph

At the right is an example of a graphical representation of a relation. It illustrates the relationship of the values of x and y.



Rule

Notice that the value of y is twice the value of x. In other words, this can be described by the equation $y = 2x$, where x is an integer from -2 to 2.

Consider this as an example of representations of a relation and function. For the set of ordered pairs, you may give only two pairs and allow the students to complete the set. For the table, give some values of x only, then let them complete the table. For the mapping diagram, allow them to complete it on their own. Ask them the process questions and give feedback immediately.

Teacher's Note and Reminders

Illustrative Example

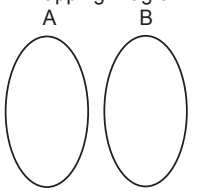
Given the graph, complete the set of ordered pairs and the table of values; draw the mapping diagram; and generate the rule.

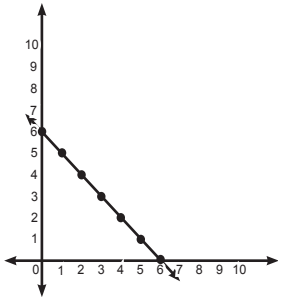
Set of ordered pairs:
{(0, 6), (1, 5), (__, __), (__, __), (__, __), (__, __), (__, __)}

Table

x	y
0	
1	
2	
3	
4	
5	
6	

Mapping Diagram





Rule: _____

Questions:

1. How did you complete the set of ordered pairs?
2. How did you make the table?
3. How did you make the mapping diagram?
4. What is the rule? How did you come up with the rule?

Answers:

The set of ordered pairs is {(0, 6), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)}. We use the set of ordered pairs in completing the table. The set of ordered pairs shows that 0 is mapped to 6, 1 to 5, 2 to 4, ..., and 6 to 0. Notice that the sum of x and y, which is 6, is constant. Thus, the rule can be written as $x + y = 6$. This can also be written in set notation as indicated below:

$$\{(x,y) \mid x + y = 6\}$$

Note that the graph does not start with (0, 6) nor it ends with (6, 0). Arrow heads indicate that we can extend it in both directions. Thus, it has no starting and ending points.

Answers to Exercise 6

Set of ordered pairs:

{(0, 0), (1, -1), (1, 1), (4, -2), (4, 2)}

Graph:

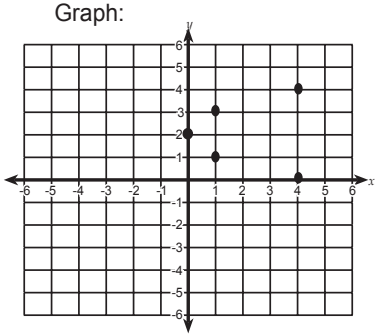


Table:

x	y
0	0
1	-1
1	1
4	-2
4	2

41. Page 38. Erase - in counter-examples.

After letting the students do Exercise 6, discuss the important notes. You may also do it in a form of oral questioning.

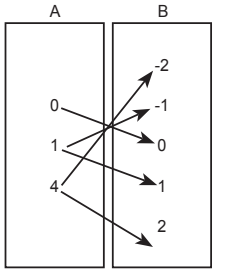
sir pag delete ko parang d na kumpleto ang sentence

Discuss the domain and range of the relation. Provide examples and ask the students to give counter-examples.

Teacher's Note and Reminders

Exercise 6

Given the mapping diagram below, make a table; write a set of ordered pairs; and draw its graph.



Set of ordered pairs:

{(__, __), (__, __), (__, __), (__, __), (__, __)}

Graph:

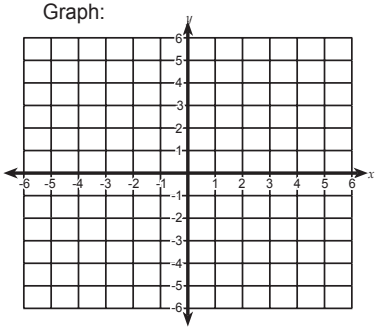


Table:

x	y

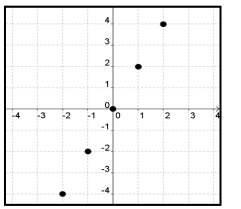
Questions:

1. How did you write the set of ordered pairs?
2. How did you make the table?
3. How did you graph?
4. Did you encounter any difficulty in making table, set of ordered pairs, and the graph? Why?
5. Can you generate a rule? Explain your answer.

Note that:

- {1, 2, 3, 4, 5} is not a relation because it is not a set of ordered pairs.
- {(1, 5), (2, 4), (-1, 8), (0, 10)} is a relation because it is a set of ordered pairs.
- The rule $x + y = 7$ represents a relation because this can be written in a set of ordered pairs {..., (0, 7), (1, 6), (2, 5), (3, 4), (4, 3), ...}
- If the ordered pairs are plotted in the Cartesian plane, then a graph can be drawn to describe the relation. The graph also illustrates a relation.

Domain and Range



It is noted that the domain of a relation is the set of first coordinates while the range is the set of second coordinates. Going back to the graph, the domain of the relation is {-2, -1, 0, 1, 2} and range is {-4, -2, 0, 2, 4}. Note that we write the same element in the domain or range once.

Answers to Exercise 7:

1. Domain: {0, 1, 2, 3, 4}; Range: {2, 3, 4, 5, 6}
2. Domain: {0}; Range: {2, 4, 6, 8, 10}
3. Domain: {-5, -2, 1, 4, 7}; Range: {-2, 0, 2}
4. Domain: {0, -1, -2, -3, -4}; Range: {2, 3, 4, 5, 6}
5. Domain: {0, 1, 2, 3, 4}; Range: {-2, -3, -4, -5, -6}

Answers to Exercise 8:

1. Domain: {-2, 0, 2}; Range: {-5, -1, 8, 9, 10}
2. Domain: {0}; Range: {1, 2, 3, 4}
3. Domain: {-2, 0, 1, 2}; Range: {-5, -1, 0, 6}
4. Domain: {0, 1, 2, 3, 4}; Range: {1, 2, 3}

Answers to Exercise 9:

1. Domain: {-1, 0, 1, 2, 3}; Range: {3, 6, 9, 12, 15}
2. Domain: {-2, -1, 0}; Range: {5, -5, 3, -3, -1}
3. Domain: {-2, -1, 0, 1, 2}; Range: {0, 1, 2}
4. Domain: {5}; Range: {-5, 0, 5, 10, 15 }

Teacher's Note and Reminders

Illustrative Example

Determine the domain and range of the mapping diagram given in **Exercise 6**.

Solution:

The domain of the relation is {0, 1, 4} while its range is {-2, -1, 0, 1, 2}.

Visit the websites below for enrichment.

1. http://www.youtube.com/watch?v=7Hg9JJceywA; and
2. http://www.youtube.com/watch?v=I0f9O7Y2xI4.

Exercise 7

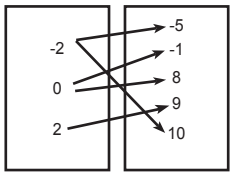
Determine the domain and the range of the relation given the set of ordered pairs.

1. {(0, 2), (1, 3), (2, 4), (3, 5), (4, 6)}
2. {(0, 2), (0, 4), (0, 6), (0, 8), (0, 10)}
3. {(-5, -2), (-2, -2), (1, 0), (4, 2), (7, 2)}
4. {(0, 2), (-1, 3), (-2, 4), (-3, 5), (-4, 6)}
5. {(0, -2), (1, -3), (2, -4), (3, -5), (4, -6)}

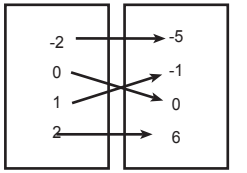
Exercise 8

Determine the domain and the range of each mapping diagram.

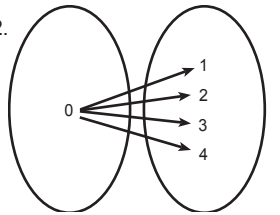
1.



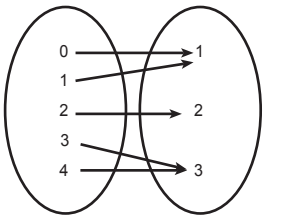
3.



2.



4.



Exercise 9

Determine the domain and the range of the table of values.

1.

x	-1	0	1	2	3
y	3	6	9	12	15

2.

x	-2	-2	-1	-1	0
y	5	-5	3	-3	-1

3.

x	-2	-1	0	1	2
y	2	1	0	1	2

4.

x	5	5	5	5	5
y	-5	0	5	10	15

Answers to Exercise 10:

1. Domain: {-2, -1, 0, 1, 2, 3 }; Range: {-2, 0, 1, 2, 3}
2. Domain: {-2, -1, 0, 1, 2, 3}; Range: {-2, -1, 0, 1, 2, 3}
3. Domain: {-3, -2, -1, 0, 1, 2, 3}; Range: {1}
4. Domain: $\{x - 2 \leq x \leq 2\}$; Range: $\{y|-2 \leq y \leq 2\}$

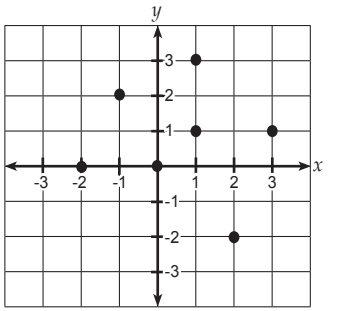
Teacher's Note and Reminders

Discuss the different types of correspondences. Show an example of each correspondence using the mapping diagram. Provide some mapping diagrams and let the students identify what type of correspondence is each.

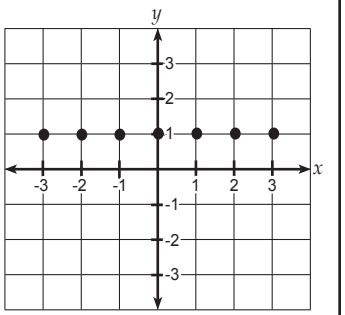
Exercise 10

Determine the domain and the range of the relation illustrated by each graph below.

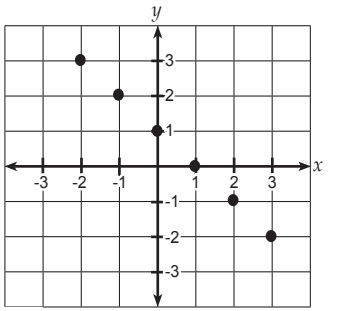
1.



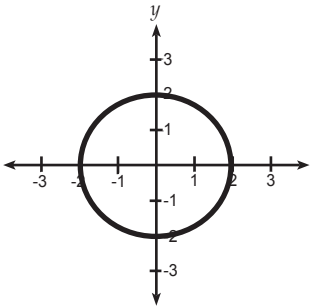
3.



2.



4.



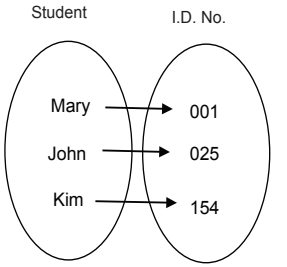
Note: The points in the graph are those points on the curve.

A correspondence may be classified as one-to-one, many-to-one or one-to-many. It is one-to-one if every element in the domain is mapped to a unique element in the range; many-to-one if any two or more elements of the domain are mapped to the same element in the range; or one-to-many if each element in the domain is mapped to any two or more elements in the range.

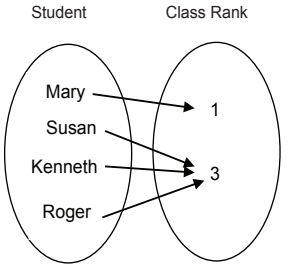
Teacher's Note and Reminders

Explain to the students Illustrative Example 1. Let the students identify what type of correspondence is the mapping diagram and the table.

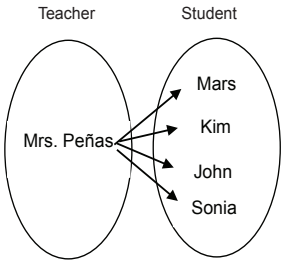
One-to-One Correspondence



Many-to-One Correspondence



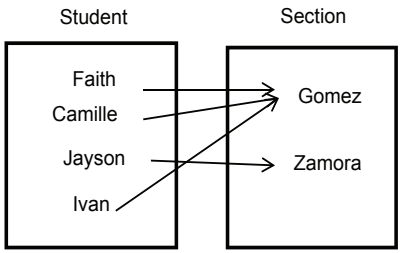
One-to-Many Correspondence



Illustrative Example 1

Consider the table and mapping diagram below.

Mapping Diagram



Table

Government Agency	Official Websites
DepEd	www.deped.gov.ph
DSWD	www.dswd.gov.ph
SSS	www.sss.gov.ph
PhilHealth	www.philhealth.gov.ph

Give a set of ordered pair and allow the students to write it in a mapping diagram and in a table. Then, give Illustrative Example 2.

Teacher's Note and Reminders

Introduce function as a special type of relations. Discuss the vast applications of functions in real life. Provide sets of ordered pairs and allow the students to identify which set represents functions. Let them generalize that all functions are relations. However, some relations are not functions. Allow students to give counterexamples of sets which represent functions.
Let them generalize that only one-to-one and many-to-one correspondences are functions.

Questions to Ponder

1. What type of correspondence is the mapping? Explain.
2. What type of correspondence is the table? Explain.

Solutions:

1. The mapping diagram is many-to-one because three students, namely: Faith, Camille and Ivan are classmates or belong to the same section Gomez.
2. The table is one-to-one correspondence because one element in the domain (government agency) is mapped to one and only one element in the range (official website).

Illustrative Example 2

Consider the sets of ordered pairs below.

Set A: {(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)}
Set B: {(2, 2), (2, -2), (3, 3), (3, -3), (4, 4), (4, -4)}
Set C: {(0, 1), (1, 1), (2, 1), (3, 1), (4, 1), (5, 1)}

Questions to Ponder

1. What is the domain of each set of ordered pairs?
2. What is the range of each set of ordered pairs?
3. What type of correspondence is each set of ordered pairs? Explain.
4. Which set/sets of ordered pairs is/are functions? Explain.

Solutions:

1. The domain of set A is {3, 4, 5, 6, 7}; set B is {2, 3, 4}; and set C is {0, 1, 2, 3, 4, 5}.
2. The range of set A is {4, 5, 6, 7, 8}; set B is {-4, -3, -2, 2, 3, 4}; and set C is {1}.
3. Correspondence in Set A is one-to-one; set B is one-to-many; and set C is many-to-one.
4. Sets A is a function because there exists a one-to-one correspondence between elements. For example, 3 corresponds to 4, 4 to 5, 5 to 6, 6 to 7, and 7 to 8. Similary, set C is a function because every element in the domain corresponds to one element in the range. However, set B is not a function because there are elements in the domain which corresponds to more than one element in the range. For example, 2 corresponds to both 2 and -2.

A **function** is a special type of relation. It is a relation in which every element in the domain is mapped to *exactly one* element in the range. Furthermore, a set of ordered pairs is a function if no two ordered pairs have equal abscissas.

Questions to Ponder

1. Among the types of correspondence, which ones are functions? Why?
2. Does one-to-one correspondence between elements always guarantee a function? How about many-to-one? Justify your answer.
3. Does one-to-many correspondence between elements always guarantee a function? Justify your answer.

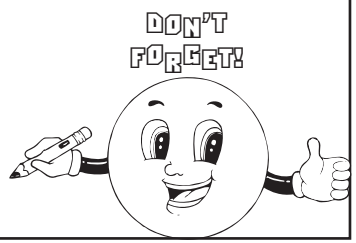
Answers to Exercise 11:

Exercise 7	Exercise 8
1. Function	1. Not function
2. Not function	2. Not function
3. Function	3. Function
4. Function	4. Function
5. Function	

Exercise 9	Exercise 10
1. Function	1. Not function
2. Not function	2. Function
3. Function	3. Function
4. Not function	4. Not function

Let the students perform Activity 5 by pair. Let them observe and process their answers using the guide questions through oral questioning to enable them to generalize the rule of Vertical Line Test.

Teacher's Note and Reminders



Exercise 11

Go back to Exercises 7 to 10, identify which ones are functions. Explain.

Note that all functions are relations but some relations are not functions.

Activity 5 PLOT IT!

Description: In the previous activities, you have learned that a set of ordered pairs is a function if no two ordered pairs have the same abscissas. Through plotting points, you will be able to generalize that a graph is that of a function if every vertical line intersects it in at most one point.

Direction: Determine whether each set of ordered pairs is a function or not. Plot each set of points on the Cartesian plane. Make some vertical lines in the graph. **(Hint: $\sqrt{3} = 1.73$)**

1. {(4, 0), (4, 1), (4, 2)}
2. {(0, -2), (1, 1), (3, 7), (2, 4)}
3. {(-2, 2), (-1, 1), (0, 0), (1, 1)}
4. {(-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8)}
5. {(3, 3), (0, 0), (-3, 3)}
6. {(-2, 0), (-1, $\sqrt{3}$), (-1, -$\sqrt{3}$), (0, 2), (0, -2), (1, $\sqrt{3}$), (1, -$\sqrt{3}$), (2, 0)}

Determine whether each set of ordered pairs represents a function or not. Put a tick mark on the appropriate column. Determine also the number of points that intersect any vertical line.

Set of Ordered Pairs	Function	Not Function	Number of Points that Intersect a Vertical Line
1. {(4, 0), (4, 1), (4, 2)}			
2. {(0, -2), (1, 1), (3, 7), (2, 4)}			
3. {(-2, 2), (-1, 1), (0, 0), (1, 1)}			
4. {(-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8)}			
5. {(3, 3), (0, 0), (-3, 3)}			
6. {(-2, 0), (-1, $\sqrt{3}$), (-1, -$\sqrt{3}$), (0, 2), (0, -2), (1, $\sqrt{3}$), (1, -$\sqrt{3}$), (2, 0)}			

Answers to Activity 6:

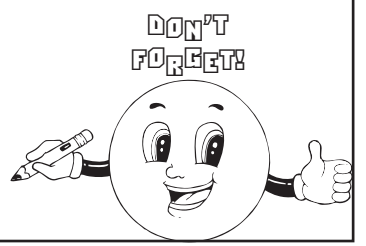
Set of Ordered Pairs	Function	Not Function	Number of Points that Intersect a Vertical Line
1. {(4, 0), (4, 1), (4, 2)}		/	3
2. {(0, -2), (1, 1), (3, 7), (2, 4)}	/		1
3. {(-2, 2), (-1, 1), (0, 0), (1, 1)}	/		1
4. {(-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8)}	/		1
5. {(3, 3), (0, 0), (-3, 3)}	/		1
6. {(-2, 0), (-1, $\sqrt{3}$), (-1, -$\sqrt{3}$), (0, 2), (0, -2), (1, $\sqrt{3}$), (1, -$\sqrt{3}$), (2, 0)}		/	2

Let the students do the Vertical Line Test to identify whether each graph represents a function or not. Supplemental video lessons are provided for students.

Answers to Exercises 12:

By Vertical Line Test, graphs in items 1 and 3 are functions while that in items 2 and 4 are not.

Teacher's Note and Reminders



QUESTIONS ?

1. Which set of ordered pairs define a function?
2. In each set of ordered pairs which defines a function, what is the maximum number of point/s that intersect every vertical line?
3. Which set of ordered pairs does not define a function?
4. In each set of ordered pairs which does not define a function, what is the maximum number of points that intersect every a vertical line?
5. What have you observed?

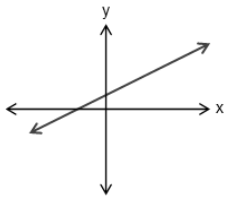
The Vertical Line Test

If every vertical line intersects the graph no more than once, the graph represents a function.

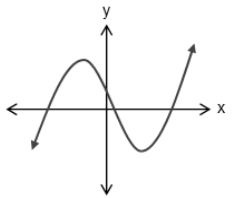
Exercise 12

Identify which graph represents a function. Describe each graph.

1.



3.



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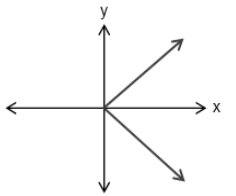
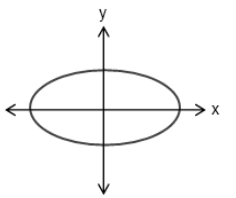


5,



Questions:

1. Which are functions? Why?
2. Can you give graphs which are that of functions? If yes, give three graphs.
3. Can you give graphs which are not that of functions? If yes, give another three graphs which do not represent functions.
4. How do you know that a graph represents a function?
5. How is function represented using graphs?

Web Links

Watch the video by clicking the websites below.

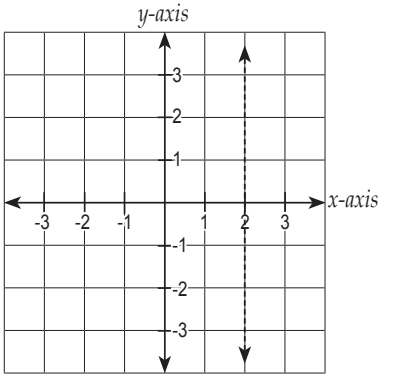
1. http://www.youtube.com/watch?NR=1&v=uJyx8eAHazo&feature=endscreen
2. http://www.youtube.com/watch?v=-xvD-n4FOJQ&feature=endscreen&NR=1

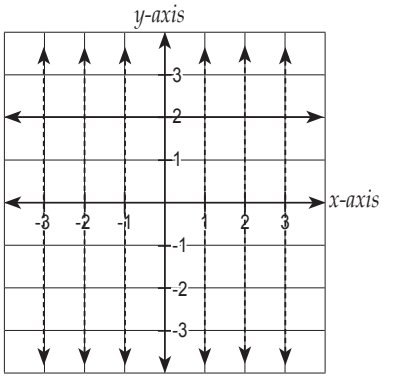
Tell the students that both vertical and horizontal lines represent a relation but only one, that is, vertical line represents a function.

Teacher's Note and Reminders

Introduce rule or equation as a representation of relation. Tell the students that a rule may either be a function or not. Let them observe Illustrative Examples 1 and Illustrative Examples 2. Use the process questions through oral questioning to enable students to draw a generalization that a rule is a function if and only if it can be written in the form $y = f(x)$.

Consider the following graphs:





Questions:

Which graph is a function? Which line fails the Vertical Line Test? Explain.

Horizontal and Vertical Lines

The horizontal line represents a function. It can be described by the equation $y = c$, where c is any constant. It is called a Constant Function. However, a vertical line which can be described by the equation $x = c$ is not a function.

A relation may also be represented by an equation in two variables or the so-called rule. Consider the next example.

Illustrative Example 1

The rule $3x + y = 4$ represents a relation. If we substitute the value of $x = -2$ in the equation, then the value of y would be:

$3x + y = 4$	
$3(-2) + y = 4$	Subsituting x by -2.
$-6 + y = 4$	Simplification
$-6 + y + 6 = 4 + 6$	Addition Property of Equality
$y = 10$	Simplification

Similarly, if $x = -1$, then $y = 7$, and so on. Thus, we can have a set of ordered pairs {…, (-2, 10), (-1, 7), (0, 4), (1, 1), (2, -2),...}. Besides, a rule is a function if it can be written in $y = f(x)$.

Teacher's Note and Reminders

Let the students identify points on the graph of the given equation, look into their x-coordinates, and identify whether the equation represents a function or not. Let them realize that an equation represents a function if no exponent of y is an even number. Links are provided for further reference. Ask them to perform Activity 6.

Illustrative Example 2

Tell whether the rule $3x + y = 4$ a function or not.

Solutions

$3x + y = 4$

$3x + y + (-3x) = 4 + (-3x)$ Why?

$y = -3x + 4$ Why?

The rule above is a function since it can be written in $y = f(x)$; that is, $y = -3x + 4$.

Illustrative Example 3

Tell whether the rule $x^2 + y^2 = 4$ a function or not.

$x^2 + y^2 = 4$

$x^2 + y^2 + (-x^2) = 4 + (-x^2)$ Why?

$y^2 = 4 - x^2$ Why?

$y = \pm\sqrt{4 - x^2}$ Getting the square root of both sides.

Notice that for every value of x, there are two values of y. Let's find the values of y if $x = 0$.

$y = \pm\sqrt{4 - x^2}$

$y = \pm\sqrt{4 - 0^2}$

$y = \pm\sqrt{4}$

$y = \pm 2$

As shown above, if $x = 0$, then the values of y are 2 and -2. Thus, the ordered pairs are (0, 2) and (0, -2) and therefore, it is not a function.

Activity 6 IDENTIFY ME!

Description: An equation in two variables can also represent a relation. With this activity, you are able to determine whether a rule is a function or not.

Direction: Given the rule, determine whether the rule represents a function or not. Answer the questions that follow. Examples are done for you.

Equation	Solutions	Coordinates
a. $y = 2x + 1$	$x = -2$ $y = 2x + 1 = 2(-2) + 1 = -4 + 1 = -3$	(-2, -3)
	$x = -1$ $y = 2x + 1 = 2(-1) + 1 = -2 + 1 = -1$	(-1, -1)
	$x = 0$ $y = 2x + 1 = 2(0) + 1 = 0 + 1 = 1$	(0, 1)
	$x = 1$ $y = 2x + 1 = 2(1) + 1 = 2 + 1 = 3$	(1, 3)
	$x = 2$ $y = 2x + 1 = 2(2) + 1 = 4 + 1 = 5$	(2, 5)

Teacher's Note and Reminders

Answer to Exercise 13

1. Function	6. Not Function
2. Function	7. Not Function
3. Function	8. Function
4. Function	9. Function
5. Not Function	10. Not Function

b. $x = y^2$	$x = 0$ $x = y^2 = 0$; hence, $y = 0$. Why?	(0, 0)
	$x = 1$ $x = y^2 = 1$; hence, $y = 1$ or -1. Why?	(1, 1), (1, -1)
	$x = 4$ $x = y^2 = 4$; hence, $y = 2$ or -2. Why?	(4, 2), (4, -2)

Write the set of ordered pairs of each rule.

a. $y = 2x + 1$: _____

b. $x = y^2$: _____

QUESTIONS ?

1. Are there any two ordered pairs whose abscissas are equal? If yes, which ones? Which rule does this set of ordered pairs belong?
2. Does the equation $y = 2x + 1$ define a function? Why or why not?
3. Does the equation $x = y^2$ define a function? Why or why not?
4. What is the exponent of y in the equation $y = 2x + 1$? What about the exponent of y in the equation $x = y^2$?
5. What can you deduce? How do we know that an equation illustrates a function? How do we know that an equation illustrates a mere relation?
6. Which among the equations below define functions? Explain.
   a. $y = 5x - 4$
   b. $3x - 2y = 2$
   c. $y = x^2$
   d. $x^2 + y^2 = 9$
   e. $y^3 = x$
7. Can you give some equations which represent a function? How about those which do not represent a function? Give three examples each.

Exercise 13

Determine whether each rule below represents a function or not.

1. $y = 3x + 9$
2. $y = -2x - 7$
3. $x + y = 10$
4. $x^2 + y = 2$
5. $2x^2 + y^2 = 8$
6. $x + y^2 = 10$
7. $x = y^4$
8. $y = x^2$
9. $y = \sqrt{4 + 1}$
10. $x^2 - y^2 = 16$

Web Links

For your reference, you can visit the websites:

1. http://www.webgraphing.com/

2. http://www.youtube.com/watch?v=hdwH24ToqZI

Let the students classify the variables as independent and dependent. Ask them to perform Activity 7.

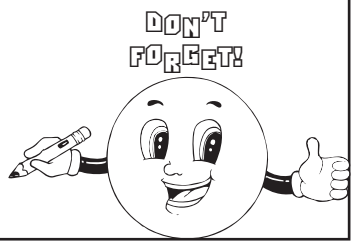
Answers to Activity 7:

1. the number of hours of work and salary in a certain private company
   Independent variable: **the number of hours of work**
   Dependent variable: **salary**
2. the number of hours boiling and the number of ounces of water left in pot
   Independent variable: **the number of hours boiling**
   Dependent variable: **the number of ounces of water left in pot**
3. the distance covered and the volume of the gasoline
   Independent variable: **the volume of the gasoline**
   Dependent variable: **the distance covered**
4. the number of hours studied to grade on test
   Independent variable: **the number of hours studied**
   Dependent variable: **grade on test**
5. height of a plant to the number of months grown
   Independent variable: **the number of months grown**
   Dependent variable: **height of a plant**

Answers to Questions of Activity 7:

a.	independent, controls	f.	independent, controls
b.	dependent, depends	g.	independent, controls
c.	independent, controls	h.	dependent, depends
d.	dependent, depends	i.	dependent, depends
e.	dependent, depends	j.	independent, controls

Teacher's Note and Reminders



Note that a rule represents a function if and only if it can be written in the form $y = f(x)$.

Activity 7 MINDS-ON

Description: Variables may be dependent and independent. Dependent variable depends on the independent variable while the independent variable controls the dependent variable.

Direction: Classify the variables as independent or dependent.

1. time and salary
   Independent variable: _____
   Dependent variable: _____
2. the number of hours boiling and the number of ounces of water in pot
   Independent variable: _____
   Dependent variable: _____
3. the distance covered and the volume of the gasoline
   Independent variable: _____
   Dependent variable: _____
4. the number of hours studied to grade on test
   Independent variable: _____
   Dependent variable: _____
5. height of a plant to the number of months grown
   Independent variable: _____
   Dependent variable: _____

QUESTIONS ?

1. Fill in the blanks.
   a. I consider time as a/an _____ variable because it _____ the salary.
   b. I consider salary as a/an _____ variable because it _____ on the number of hours worked.
   c. I consider the number of hours boiling as a/an _____ variable because it _____ the number of ounces of water in pot.
   d. I consider the number of ounces of water in pot as a/an _____ variable because it _____ on the number of hours boiling.

Teacher's Note and Reminders

Let the students give counterexamples of variables which involve relations. Instruct them to identify which variable is independent and is dependent. Ask them to perform Activities 8 and 9. Give these as a group assignment. Allow them to conduct interview.

e. I consider the distance covered as a/an _____ variable because it _____ on the volume of the gasoline.
f. I consider the volume of the gasoline as a/an _____ variable because it _____ the distance covered.
g. I consider the number of hours studied as a/an _____ variable because it _____ grade on test.
h. I consider grade on test as a/an _____ variable because it _____ on the number of hours studied.
i. I consider height of the plant as a/an _____ variable because it _____ on the number of months grown.
j. I consider the number of months grown as a/an _____ variable because it _____ the number of months grown.

2. How do you differentiate the dependent from the independent variable?

Dependent and Independent Variables

The variable x is considered the independent variable because any value could be assigned to it. However, the variable y is the dependent variable because its value depends on the value of x.

Activity 8 AM I RELATED (PART I)?

Description: This task provides counterexamples to the previous activity. This can be done by group of 5 members.

Direction: Think of two quantities related to each other. Identify the independent and dependent variables. Give as many three examples.

QUESTIONS ?

1. What three pairs of quantities did you choose? Why?
2. Can we see/experience them in real life?

Historical Note:

Function Notation

(1707 – 1783) was a Swiss mathematician who taught and wrote about mathematics in both St. Petersburg, Russia, and Berlin, Germany. He made contributions to many branches of mathematics and was particularly successful in devising useful notations. Among his notations was the $f(x)$ notation to represent the value of a function.

Discuss Function Notation as well as evaluation of function at a given value of x. Give examples. Ask the students to give their counterexamples. Emphasize to them that a function is usually represented by f, g or h. $f(x)$ is not a function but rather it is the output for every input x.

Teacher's Note and Reminders

Activity 9 AM I RELATED (PART II)?

Description: Among the variables mentioned in the previous activity, make a table of values and set of ordered pairs and identify whether or not each illustrates a function.

Direction: Among the three pairs you have identified in Activity 9, choose only one for your group. You may conduct an interview with experts. Then, make a table of values and a set of ordered pairs. Identify whether it illustrates a function or not.

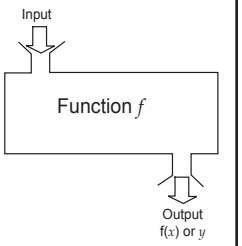
QUESTIONS ?

1. What difficulty did you encounter in collecting the data?
2. How were you able to prepare the table of values?
3. Is the relation a function? Why?

In the previous section, you have learned how a function is defined. This time, you will enrich your knowledge about functions starting with function notation.

Function Notation

The $f(x)$ notation can also be used to define a function. If f is a function, the symbol $f(x)$, read as "f of x," is used to denote the value of the function f at a given value of x. In simpler way, $f(x)$ denotes the y-value (element of the range) that the function f associates with x-value (element of the domain). Thus, $f(1)$ denotes the value of y at x = 1. Note that f(1) does not mean f times 1. The letters such as g, h and the like can also denote functions.



Furthermore, every element x in the domain of the function is called the **pre-image**. However, evey element y or $f(x)$ in the range is called the **image**. The figure at the right illustrates concretely the input (the value of x) and the output (the value of y or $f(x)$) in the rule or function. It shows that for every value of x there corresponds one and only one value of y.

Example:

Consider the rule or the function f defined by $f(x) = 3x - 1$.
If x = 2, then the value of the function would be 5.

Solution:

$f(x) = 3x - 1$	Rule/Function
$f(2) = 3(2) - 1$	Substituting x by 2
$f(2) = 6 - 1$	Simplification
$f(2) = 5$	Simplification

Discuss the Domain and Range of a Function. Let the students recall the domain and range of a relation if a table, mapping diagram, or a set of ordered pairs is known. Stress the ideas of the arrow heads and of the asymptote. Present the illustrative example provided and explain to them. You may also give another graph with a vertical or horizontal asymptote as an example and explain.

Teacher's Note and Reminders

The input is 2 (the value of x) and the output is 5 (the value of y or $f(x)$).
How about if $x = 3$?

Solution:

$f(x) = 3x - 1$	Rule/Function
$f(3) = 3(3) - 1$	Substituting $x = 3$
$f(3) = 9 - 1$	Simplification
$f(3) = 8$	Simplification

The input is 3 (the value of x) while the output is 8 (the value of function).

Domain and Range of a Function

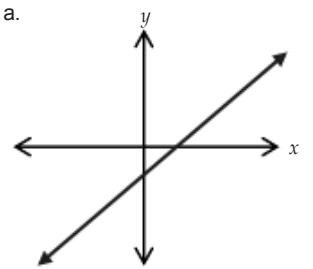
In the previous section, you have learned how the domain and the range of a relation are defined. The domain of the function is the set of all permissible values of x that give real values for y. Similarly, the range of the function is the set of permissible values for y or $f(x)$ that give the values of x real numbers.

You have taken the domain and the range of the relation given in the table of values in the previous lesson, the set of ordered pairs and the graph. Can you give the domain and the range if the graph of the function is known? Try this one!

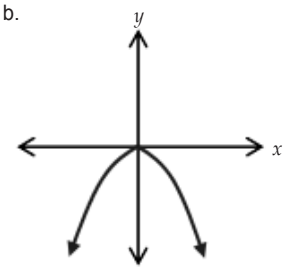
Illustrative Example

Find the domain and the range of each graph below.

a.



b.



Solutions:

In (a), arrow heads indicate that the graph of the function extends in both directions. It extends to the left and right without bound; thus, the domain D of the function is the set of real numbers. Similarly, it extends upward and downward without bound; thus, the range R of function is the set of all real numbers. In symbols,

$$D = \{x | x \in \Re\}, \quad R = \{y | y \in \Re\}$$

Answers to Exercise 14:

1. Domain: $\{x|x \in \Re\}$ Range: $\{y|y \geq 0\}$
2. Domain: $\{x|x \in \Re\}$ Range: $\{y|y = 5\}$ or $\{5\}$
3. Domain: $\{x|x \in \Re\}$ Range: $\{y|y \in \Re\}$
4. Domain: $\{x|x \in \Re\}$ Range: $\{y|y > 0\}$
5. Domain: $\{x|x \geq 0\}$ Range: $\{y|y \geq 0\}$
6. Domain: $\{x|x \in \Re\}$ Range: $\{y|y \geq -2\}$

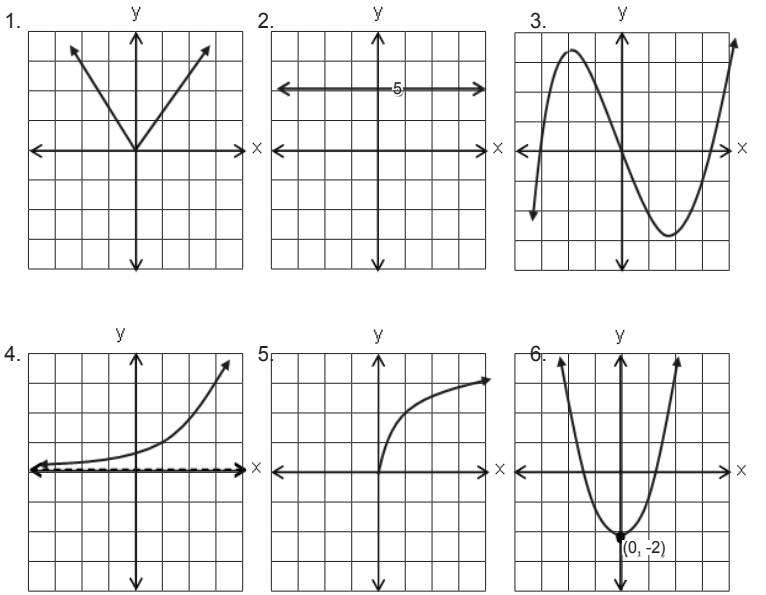
Teacher's Note and Reminders

In (b), arrow heads indicate that the graph of the function is extended to the left and right without bound, and downward, but not upward, without bound. Thus, the domain of the function is the set of real numbers, while the range is any real number less than or equal to 0. That is,

$$D = \{x|x \in \Re\},\ R = \{y|y \leq 0\}$$

Exercise 14

Determine the domain and the range of the functions below.



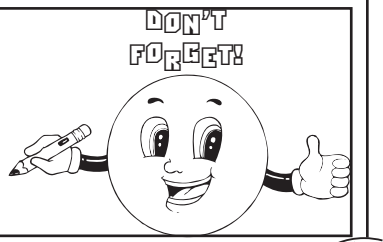
Note: The broken line in item number 4 is an *asymptote*. This is a line that the graph of a function approaches, but never intersects. (Hint: The value of x = 0 is not part of the domain of the function.)

Let the students identify the domain of the function illustrated below. Note that the graph of $f(x) = \frac{1}{x}$ is asymptotic to the x-axis and to the y-axis. That is, the graph of a function approaches but never intersects to the x-axis and the y-axis. For further investigation, allow the students to use calculator. Explain to them the meaning of Error or Math Error in the calculator. Ask them to perform Activity 10. Varied answers of students in Question 7 of the activity are expected. Give examples and discuss them.

Answers to the Questions of Activity 10:

1. By Vertical Line Test, every graph above represents a function.
2. The domains of the graphs are as follows:
   First graph: $\{x|x \in \Re, x \neq 0\}$
   Second graph: $\{x|x \geq 0\}$
   Third graph: $\{x|x \in \Re\}$
3. The first graph does not touch the y-axis because the value of the function f defined by $f(x) = \frac{1}{x}$, when $x = 0$, is undefined, which appears Error or Math Error in the calculator. This means that the graph of the function does not intersect the line $x = 0$ or the y-axis. Thus, the domain of the function is $\{x|x \in \Re, x \neq 0\}$.
4. In $f(x) = \sqrt{x}$, the value of the function is a real number for every real number x which is greater than or equal to zero. When x is negative, the value of the function is imaginary in which calculators yield an Error or Math Error. This also means that the graph of the function does not lies on the left side of the line $x = 0$ or the y-axis. Thus, the domain of the function is $\{x|x \geq 0\}$.
5. In $f(x) = x^2$, there is no value of x that makes the function f undefined. Thus, the domain of the function is $\{x|x \in \Re\}$.
6. The value of the function is not a real number when it is undefined or is imaginary.

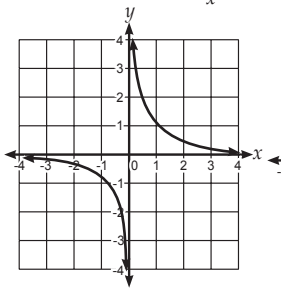
Teacher's Note and Reminders



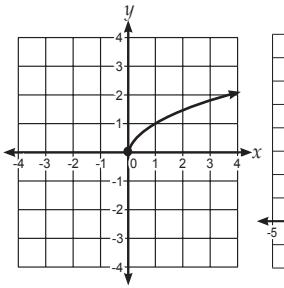
Activity 10 GRAPH ANALYSIS

Description: This activity will enable you to determine the domain of the function.
Direction: Consider the graphs below. Answer the questions that follow.

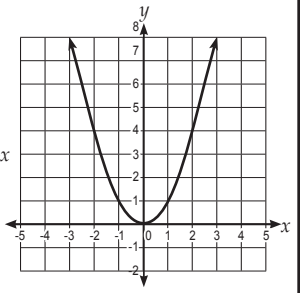
The graph of $f(x) = \frac{1}{x}$



The graph of $f(x) = \sqrt{x}$



The graph of $f(x) = x^2$



QUESTIONS ?

1. Does each graph represent a function? Why?
2. What is the domain of the first graph? Second graph? Third graph? Explain each.
3. Does the first graph touch the y-axis? Why or why not?
4. In $f(x) = \frac{1}{x}$, what happens to the value of the function if $x = 0$? Does this value affect the domain of the function?
5. In $f(x) = \sqrt{x}$, what happens to the value of the function if $x < 0$, or negative? Does this value help in determining the domain of the function?
6. In $f(x) = x^2$, is there a value of x that will make the function undefined? If yes, specify: _____.
7. Make a reflection about the activity.

You have tried identifying the domain and the range of the graph of the function. What about if you are asked to find the domain of the function itself without its graph. Try this one!

Illustrative Example

Determine the domain of each function below. Check the solution using calculator.

1. $f(x) = 3x$
2. $f(x) = x^2$
3. $f(x) = \sqrt{x - 2}$
4. $f(x) = \frac{x + 1}{x}$

Teacher's Note and Reminders

Answers to Exercise 15:

1. $\{x|x \in \Re\}$
2. $\{x|x \in \Re\}$
3. $\{x|x \geq 0\}$
4. $\{x|x \geq -1\}$
5. $\{x|x \in \Re, x \neq 2\}$
6. $\{x|x \in \Re, x \neq 1\}$
7. $\{x|x \geq 8\}$
8. $\{x|x \in \Re, x \neq -6\}$
9. $\{x|x \geq 2\}$
10. $\{x|x \in \Re, x \neq {}^{5}/_{3}\}$

Solutions:

1. In $f(x) = 3x$, there is no value of x that makes the function f undefined. Thus, the domain of $f(x) = 3x$ is the set of real numbers or $\{x|x \in \Re\}$.

2. In $f(x) = x^2$, there is no value of x that makes the function f undefined. Thus, the domain of $f(x) = x^2$ is the set of real numbers or $\{x|x \in \Re\}$.

3. In $f(x) = \sqrt{x - 2}$, the domain of the function is the set of values of x that will not make $\sqrt{x - 2}$ an imaginary number. Examples of these values of x are 2, 2.1, 3, 3.74, 4, 5, and so on. However, $x = 1$ cannot be because it can give the value of the function

$$\sqrt{1 - 2} = \sqrt{-1}$$

which is imaginary where the calculator yields an Error or a Math Error. The numbers between 1 and 2 neither work. Thus, the domain of the function is x is greater than or equal to 2, or $\{x|x \geq 2\}$. For you to find easily the domain of the function, we say the radicand ≥ 0. That is, $x - 2 \geq 0$ which gives $x \geq 2$ if simplified.

4. In $f(x) = \frac{x + 1}{x}$, the domain of the function is the set of values of x that will not make $\frac{x + 1}{x}$ undefined. The value $x = 0$ will make the expression $\frac{x + 1}{x}$ undefined. When the answer is undefined, the calculator yields an Error or a Math Error. Thus, $x = 0$ is not part of the domain. The domain, therefore, of the function is the set of real numbers except 0, or $\{x|x \in \Re, x \neq 0\}$. To find easily the domain of the function, we say denominator is not equal to zero, or $x \neq 0$.

Note that the value of the function will not be a real number if it is an imaginary number or undefined.

Exercise 15

Find the domain of each function.

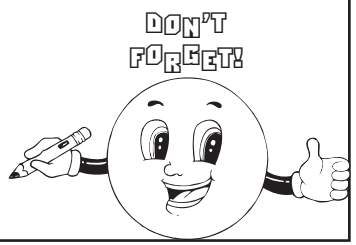
1. $g(x) = 5x + 1$
2. $g(x) = x - 7$
3. $g(x) = \sqrt{x}$
4. $g(x) = \sqrt{x + 1}$
5. $g(x) = \frac{x + 4}{x - 2}$
6. $g(x) = \frac{3x + 4}{x - 1}$
7. $g(x) = \sqrt{x - 8}$
8. $g(x) = \frac{3x}{x + 6}$
9. $g(x) = \sqrt{2x - 4}$
10. $g(x) = \frac{x + 4}{3x - 5}$

Let the students do Activity 11 by revisiting the IRF Worksheet. Consider this activity as part of a formative assessment. Compare their revised answers to their initial answers. Pose again the topical Essential Question: **How are the quantities related to each other?**

What to Understand

Have students take a closer look at the next activity. The questions in this activity are quite difficult. Tell them to analyze the questions well and write their answers accurately. Allow them to discuss the activity by pair.

Teacher's Note and Reminders



Answers to A of Activity 12:

1. $\{x|x \leq 1\}$
2. $\{x|x \in \Re\}$
3. $\{x|-4 \leq x \leq 4\}$
4. $\{x|x \geq 4\}$
5. $\{x|x \in \Re, x \neq -2\}$

Activity 11 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will write your present knowledge about the concept.

Direction: Give your revised answers of the questions provided in the first column and write them in the third column. Compare your revised answers from your initial answers.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is a relation?			
2. What is a function?			
3. How are relations and functions represented?			
4. How are the quantities related to each other?			

Go back to the previous section and find out if your initial ideas are correct or not. How much of your initial ideas are discussed. Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand

Your goal in this section is to take a closer look at some aspects of the topic.

Activity 12 QUIZ

Description: This activity will evaluate your knowledge about the domain of the given relation.

Direction: Do as directed.

A. State the domain of the relation.

1. $h(x) = \sqrt{1 - x}$
2. $x + y = 4$
3. $x^2 + y^2 = 16$
4. $t(x) = 2\sqrt{x - 4}$
5. $r(x) = \dfrac{2x^2 + 3x - 2}{x + 2}$

Answers to B of Activity 12:

1. $\{x|x \leq 1\}$
2. $\{x|x \in \Re\}$
3. $\{x|-4 \leq x \leq 4\}$
4. $\{x|x \geq 1\}$
5. $\{x|x \in \Re, x \neq -2\}$

Answers to B of Activity 12:

1. The domain of the function f is $\{x|x \in \Re, x \neq 4\}$ while that of g is $\{x|x \in \Re\}$. At $x = 4$, function f is undefined while function g is not.
2. The domain of the function W is $\{x|x \in \Re\}$.
3. Answers may vary.
4. The statement is true. The domain of the function f is $\{x|x > 1\}$ because $\sqrt{x-1} \neq 0$ and $x - 1 \geq 0$ which gives $x \neq 1$ and $x \geq 1$, respectively.

Solutions:

$\sqrt{x-1} \neq 0$	Given
$x - 1 \neq 0$	Squaring both sides
$x - 1 \neq 0$	Addition Property of Equality
$x \neq 1$	Simplification
$x - 1 \geq 0$	Given
$x - 1 + 1 \neq 0 + 1$	Addition Property of Equality
$x \geq 1$	Simplification

Answers to C of Activity 12:

1. Yes, the graph represents a relation because any graph is a representation of a relation.
2. By Vertical Line Test, the graph represents a function.
3. The domain of the graph is $\{x|x \in \Re, x \neq 0\}$.
4. The range of the graph is $\{y|y \in \Re, y \neq -2\}$.
5. The value of y increases as the value of x increases.

B. Answer the following questions.

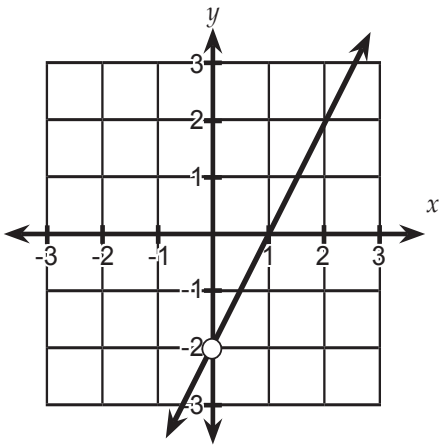
1. Is the domain of $f(x) = \frac{(x+4)(x-4)}{x-4}$ equal to the domain of $g(x) = x + 4$? Justify your answer.

2. (Biology) The weight of the muscles of a man is a function of his body weight x and can be expressed as $W(x) = 0.4x$. Determine the domain of this function. Explain your answer.

3. Give a function whose domain is described below:
   a. $\{x|x \in \Re\}$
   b. $\{x|x \in \Re, x \neq 1\}$
   c. $\{x|x \geq 4\}$
   d. $\{x|x \leq -1\}$

4. Accept or reject the following statement and justify your response: "The domain of the function $f(x) = \frac{x+5}{\sqrt{x-1}}$ is $\{x|x > 1\}$."

C. Study the graph given and use it to answer the questions that follow.



1. Does the graph represent a relation? Explain.
2. Does the graph represent a function? Explain.
3. Determine the domain of the graph.
4. Determine the range of the graph.
5. **How are the quantities related to each other? Does the value of y increase as x increases?**

Before the students move to the next section of this lesson, give a short test (formative test) to find out how well they understood the lesson. Let them give their present knowledge about the concept. This is one way of assessing the student's self-knowledge on the topic.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

Teacher's Note and Reminders

Activity 13 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will give your present knowledge about the concept.

Direction: Give your final answers of the questions provided in the first column and write them in the third column. Compare your revised answers from your initial and revised answers.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is a relation?			
2. What is a function?			
3. How are relations and functions represented?			
4. How are the quantities related to each other?			

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

Activity 14 GALLERY WALK

Description: Your output of this activity is one of your projects for the second quarter. It summarizes the representations of relations and functions. This could be done by groups of 5 to 8 members each. Before doing this project, you are required to have a research on making a leaflet.

Direction: You make an informative leaflet providing the information about the representations of relations and functions. Each member in the group will give a relation and write its representations. Arrange these in a creative manner. Your group output will be assessed using the rubric on the next page.

Teacher's Note and Reminders

RUBRIC: INFORMATIVE LEAFLET

CRITERIA	Exemplary 4	Satisfactory 3	Developing 2	Beginning 1
Required Elements	The leaflet includes all required elements as well as additional information.	All required elements are included on the leaflet.	All but 1 or 2 of the required elements are not included on the leaflet.	Several required elements were missing.
Graphics - Relevance / Color	All graphics are related to the topic and make it easier to understand. All borrowed graphics have a source citation.	All graphics are related to the topic. All borrowed graphics have a source citation.	All graphics relate to the topic. One or two borrowed graphics were not cited.	Graphics do not relate to the topic or several borrowed graphics were not cited.
Attractiveness/ Formatting	The leaflet is exceptionally attractive in terms of design, layout, and neatness.	The leaflet is attractive in terms of design, layout and neatness.	The leaflet is acceptably attractive though it may be a bit messy.	The leaflet is distractingly messy or very poorly designed. It is not attractive.

In this section, your task was to make an informative leaflet. How did you find the performance task?

Continue studying the next lesson for further understanding about functions.

What to Know

Provide students the opportunity to recall translating verbal phrases to mathematical phrases and vice-versa. Ask them to answer Activities 1 and 2. Answers in Activity 2 may vary.

Answers to Activity 1:

1. B	6. C	11. A
2. H	7. P	12. K
3. G	8. Q	13. L
4. F	9. O	14. J
5. E	10. M	15. N

Teacher's Note and Reminders

Lesson 3 Linear Function and Its Applications

What to Know

Let's start this lesson by recalling translation of English phrases to mathematical expressions and vice versa. As you go through this module, keep in mind this question: **How can you predict the value of a quantity given the rate of change?**

Activity 1 FIND MY PAIR!

Description: This activity will enable you to recall on translations of verbal phrases to mathematical phrases.

Direction: Match the verbal phrase in Column A to the mathematical phrase in Column B. Write the letter that corresponds to your answer on the space provided before each item.

Column A	Column B
___ 1. The sum of the numbers x and y	A. $7xy$
___ 2. The square of the sum of x and y	B. $x + y$
___ 3. The sum of the squares of x and y	C. $2(x + y)$
___ 4. Nine less than the sum of x and y	D. $9 - x + y$
___ 5. Nine less the sum of x and y	E. $9 - (x + y)$
___ 6. Twice the sum of x and y	F. $(x + y) - 9$
___ 7. Thrice the product of x and y	G. $x^2 + y^2$
___ 8. Thrice the quotient of x and y	H. $(x + y)^2$
___ 9. The difference between x and y divided by four	I. $4x^3 + y^3$
___10. Eight more than the product of x and y	J. $4(x^3 + y^3)$
___11. The product of 7, x and y	K. $4(x + y)$
___12. The product of four and the sum of x and y	L. $x + y^2 - 10$
___13. The sum of x and the square of y diminished by ten	M. $8 + xy$
___14. Four times the sum of the cubes of x and y	N. $2\lvert x - y\rvert$
___15. Two multiplied by the absolute value of the difference of x and y	O. $\frac{x - y}{4}$
	P. $3xy$
	Q. $3\left(\frac{x}{y}\right)$

Possible Answers to Activity 2:

1. the sum of a and b
2. twice the difference of a and b
3. the sum of thrice the number a and four times the number b
4. b less 5
5. b less than 5
6. the square of the number a added to the square of the number b
7. the number a added to twice the number b
8. the product of the numbers a and b divided by 2
9. twice the square of the number a diminished by thrice of the number b
10. the quotient of a and b added to 7

Give some examples of sentences which can be translated to mathematical equations.

Teacher's Note and Reminders

QUESTIONS ?

1. What is the difference between:
   a. "x less than y" and "x less y?"
   b. "the sum of the squares of x and y" and "the square of the sum of x and y?"
2. Have you encountered any difficulty in translating English phrases to mathematical expressions? Explain your answer.

In **Activity 1**, *you translated verbal phrases to mathematical phrases. However, in the next activity, you will write the verbal phrases for a given set of mathematical phrases.*

Activity 2 WRITE YOUR OWN VERBAL PHRASE!

Description: This activity will enable you to translate mathematical phrases to verbal phrases.

Direction: Write the verbal phrase for each mathematical phrase below.

1. $a + b$
2. $2(a - b)$
3. $3a + 4b$
4. $b - 5$
5. $5 - b$
6. $a^2 + b^2$
7. $a + 2b$
8. $\frac{ab}{2}$
9. $2a^2 - 3b$
10. $\frac{a}{b} + 7$

It is also necessary to recall translating verbal sentences to equations. Try the next activity.

Illustrative Example

Represent the sentences below algebraically:

1. Four times a number increased by 5 is 21.

Four	times	a number	increased by	5	is	21.
↓	↓	↓	↓	↓	↓	↓
4	•	x	+	5	=	21

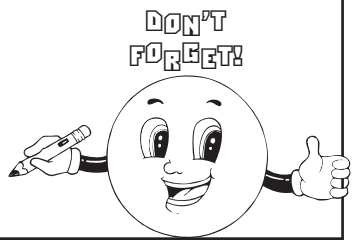
The mathematical equation for the verbal sentence is $4x + 5 = 21$.

Answers to Activity 3:

1. $2x = 6$
2. $4 + x = 10$
3. $25 - 2x = 12$
4. $3x + 7 = 98$
5. $x^2 + 3^2 = 16$
6. $3x - 9 = 100$
7. $x + (x + 1) = 25$
8. $x(x + 1) = 182$
9. $(x + 4)(x - 3) = 30$
10. $x + y = 47$ or $M + S = 47$

Let the students recall also evaluating algebraic expressions. Tell them to answer Activity 4. This is pre-requisite to evaluating linear functions which will be discussed later.

Teacher's Note and Reminders



2. The difference of two numbers is 8.

   The term "difference" means the answer of the subtraction. The two numbers can be represented by two variables, say x and y. Thus, the correct mathematical equation is $x - y = 8$.

3. The perimeter of the triangle whose sides are x, $x + 4$ and $2x + 5$ is 57.

   The perimeter of any triangle is the sum of the lengths of its three sides. The perimeter P of the triangle is $x + (x + 4) + (2x + 5)$ and is equal to 57. Thus, the correct mathematical equation is $x + (x + 4) + (2x + 5) = 57$.

Activity 3 WRITE THE CORRECT EQUATION

Description: This activity will enable you to translate each verbal sentence into mathematical equation and vice versa.

Direction: Represent each of the following algebraically.

1. Twice a number is 6.
2. Four added to a number gives ten.
3. Twenty-five decreased by twice a number is twelve.
4. If thrice a number is added to seven, the sum is ninety-eight.
5. The sum of the squares of a number x and 3 yields 25.
6. The difference between thrice a number and nine is 100.
7. The sum of two consecutive integers is equal to 25.
8. The product of two consecutive integers is 182.
9. The area of the rectangle whose length is $(x + 4)$ and width is $(x - 3)$ is 30.
10. The sum of the ages of Mark and Sheila equals 47.

QUESTIONS ?

1. What are the common terms used to represent the "=" sign?
2. Use the phrase "is equal to" on your own sentence.
3. Translate the formulae below to verbal phrases.
   a. $P = a + b + c$ (Perimeter of a triangle)
   b. $A = lw$ (Area of a rectangle)
   c. $A = s^2$ (Area of a square)
   d. $C =$ (Circumference of a circle)
   e. $SA = 2lw + 2lh + 2wh$ (Surface area of a rectangular prism)
4. Write five pairs of mathematical phrases and their verbal translations.

Recalling evaluation of algebraic expressions is also important. Try the next activity.

Answers to Activity 4:

1. 4	6. 34
2. -1	7. -3
3. 32	8. 10
4. -5	9. 4
5. -1	10. $^7/_2$

Teacher's Note and Reminders

Activity 4 EVALUATE ME!

Description: This activity will enable you to evaluate algebraic expressions.
Direction: Evaluate the following algebraic expressions.

1. $2xy$ when $x = 2$ and $y = 1$
2. $x - 4y$ when $x = -1$ and $y = 0$
3. $x^2 + y$ when $x = -5$ and $y = 7$
4. $\sqrt{3x} + 2y$ when $x = 3$ and $y = -4$
5. $\frac{x+4}{x^2-30}(8y)$ when $x = 2$ and $y = \frac{1}{2}$
6. $3(x + y) - 2(x - 8y)$ when $x = 8$ and $y = -2$
7. $(3x)(\frac{y-8}{y-2})$ when $x = 4$ and $y = 0$
8. $\frac{x^2+4x-5}{y^2-y-2}$ when $x = 5$ and $y = 3$
9. $\sqrt{2x-4} + 7y$ when $x = 4$ and $y = \frac{2}{7}$
10. $(x + 3) \div 4 - 15 \div 2xy$ when $x = 5$ and $y = -1$

QUESTIONS ?

1. How do you evaluate an algebraic expression?
2. What rule did you use to evaluate algebraic expressions?
3. If exponent and parenthesis appear simultaneously, which one will you perform first?
4. If an expression allows you to multiply and divide in any order, is it correct to always perform multiplication first before division?
5. In the expression 6 ÷ (3)(4), which operation will you perform first, multiplication or division? Explain your answer.
6. If an expression allows you to add and subtract, is it correct to always perform addition first before subtraction? Why?
7. In the expression 2 – 1 + 8, which operation will you perform first, addition or subtraction? Explain your answer.
8. State the GEMDAS Rule.

Elicit students' present knowledge of Linear Functions by answering the "Initial Answer" column in the IRF Worksheet.

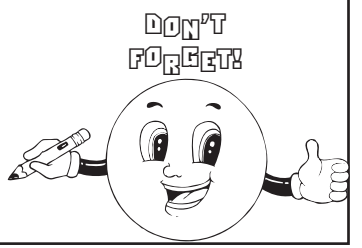
After letting the students answer the IRF Worksheet, tell them that at the end of the lesson, they are expected to formulate and solve real-life problem, and make an informative leaflet about electric bill and power consumption, and orally present this to the other *barangay* officials as a demonstration of your understanding.

What to Process

These are enabling activities/experiences that the learner will have to go through to understand linear function and its applications. Interactive activities are provided for the students to check their understanding on the lesson.

Let the students identify whether the function is linear or not based on the definition. Give examples and discuss them. After giving many examples, allow the students to give their own examples of linear function in $f(x)$ notation.

Teacher's Note and Reminders



Activity 5 IRF WORKSHEET

Description: Below is the IRF Worksheet in which you will write your present knowledge about the concept.

Direction: Give your initial answers of the questions provided in the first column and write them in the second column.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is linear function?			
2. How do you describe a linear function?			
3. How do you graph a linear function?			
4. How do you find an equation of the line?			
5. How can the value of a quantity given the rate of change be predicted?			

You have just reviewed translations of English phrases and sentences to mathematical expressions and equations and vice versa. The next section will enable you to understand linear functions and its applications, to formulate and solve real-life problems, and to make a leaflet about electric bill and power consumption to be presented to the different members of the community.

What to Process

Your goal in this section is to learn and understand the key concepts of Linear Function and Its Application.

Linear Function

A linear function is defined by $f(x) = mx + b$, where m is the slope and b is the y-intercept, m and $b \in \Re$ and $m \neq 0$. The degree of the function is one and its graph is a line.

Teacher's Note and Reminders

Answers to Exercise 1:

Function	Degree	Yes	No	m	b
1. $f(x) = 5x + 1$	1	/		5	1
2. $f(x) = -6x - 7$	1	/		-6	-7
3. $f(x) = 3x$	1	/		3	0
4. $f(x) = x - 4$	1	/		1	-4
5. $f(x) = 5x - 3$	1	/		5	-3
6. $f(x) = 2(x - 3)$	1	/		2	-6
7. $f(x) = -(x + 5)$	1	/		-1	-5
8. $f(x) = -4x^2$	2		/		
9. $f(x) = 10x^2 + 7x$	2		/		
10. $f(x) = 3x^2 - 5x + 1$	2		/		

Illustrative Example 1

Is the function f defined by $f(x) = 2x + 3$ a linear function? If yes, determine the slope m and the y-intercept b.

Solution:

Yes, the function f defined by $f(x) = 2x + 3$ is a linear function since the highest exponent (degree) of x is one and it is written in the form $f(x) = mx + b$. The slope m is 2 while the y-intercept b is 3.

Illustrative Example 2

Is the function g defined by $g(x) = -x$ a linear function? If yes, determine its slope and y-intercept.

Solution:

Yes, the function g is a linear function because it has a degree one. Since $g(x) = -x$ can be written as $g(x) = \mathbf{-1x + 0}$, its slope is -1 and y-intercept is 0.

Illustrative Example 3

Is the function h defined by $h(x) = x^2 + 5x + 4$ a linear function?

Solution:

The function h is not a linear function because its degree (the highest exponent of x) is 2, not 1.

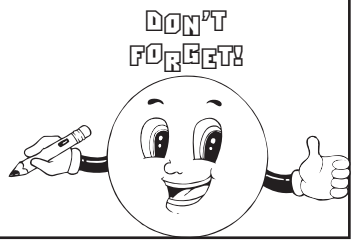
Exercise 1

Determine whether each is a linear function or not. Check Yes if it is a linear function and No if it is not. Write the degree of the function. For linear functions, identify its slope m and y-intercept b.

Function	Degree	Yes	No	m	b
1. $f(x) = 5x + 1$					
2. $f(x) = -6x - 7$					
3. $f(x) = 3x$					
4. $f(x) = x - 4$					
5. $f(x) = 5x - 3$					
6. $f(x) = 2(x - 3)$					
7. $f(x) = -(x + 5)$					
8. $f(x) = -4x^2$					
9. $f(x) = 10x^2 + 7x$					
10. $f(x) = 3x^2 - 5x + 1$					

Let the students evaluate a linear function given some values of x and let them give the coordinates.

Teacher's Note and Reminders



A linear function can be described using its graph.

Illustrative Example

Determine the values of the function f if $f(x) = 2x - 1$ at $x = -3$, 0, and 2. Give their meanings and ordered pairs.

Solution:

If $x = -3$, then $f(x) = 2x - 1$ becomes

$f(-3) = 2(-3) - 1$

$f(-3) = -6 - 1$

$f(-3) = -7$, which means the value of f at $x = -3$ is -7.

Or, if $x = -3$, then $y = -7$. This gives the ordered pair (-3, -7). Recall that an ordered pair can be written (x, y).

If $x = 0$, then $f(x) = 2x - 1$ becomes

$f(0) = 2(0) - 1$

$f(0) = 0 - 1$

$f(0) = -1$, which means the value of f at $x = 0$ is -1.

Or, if $x = 0$, then $y = -1$. This gives another ordered pair (0, -1).

If $x = 2$, then $f(x) = 2x - 1$ becomes

$f(2) = 2(2) - 1$

$f(2) = 4 - 1$

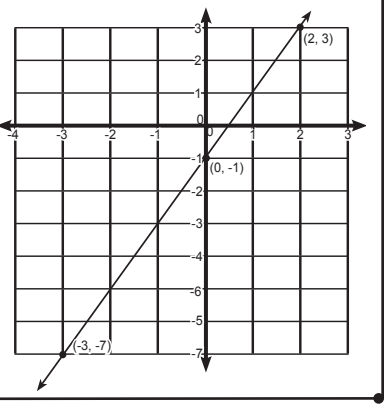
$f(2) = 3$, which means the value of f at $x = 2$ is 3.

Or, if $x = 2$, then $y = 3$. This gives the ordered pair (2, 3).

This implies that the graph of the function f will pass through the points (-3, -7), (0, -1) and (2, 3). Out of the values, we can have the table below:

x	-3	0	2
$f(x)$	-7	-1	3

With the use of table of values of x and y, the function can be graphed as shown at the right.



Web Links

For your enrichment, visit this link: http://www.youtube.com/watch?v=UgtMbCI4G_I&feature=related

Answers in Activity 6:

A. 1. $f(x) = 2x$ f(-3) = -6 f(1) = 2 f(4) = 8
2. $f(x) = 2x + 1$ $f(-3) = -5$ $f(1) = 3$ $f(4) = 9$
3. $f(x) = -3x$ $f(-3) = 9$ $f(1) = -3$ $f(4) = -12$
4. $f(x) = -3x - 4$ $f(-3) = 5$ $f(1) = -7$ $f(4) = -16$
5. $f(x) = 2 - 3x$ $f(-3) = 11$ $f(1) = -1$ $f(4) = -10$

B.

Function	The values of			Ordered Pairs	Table
	$f(-3)$	$f(1)$	$f(4)$		
1. $f(x) = 2x$	**-6**	**2**	**8**	**(-3, -6), (1, 2), (4, 9)**	x: -3, 1, 4; $f(x)$: -6, 2, 8
2. $f(x) = 2x + 1$	**-5**	**3**	**9**	**(-3, -5), (1, 3), (4, 8)**	x: -3, 1, 4; $f(x)$: -5, 3, 9
3. $f(x) = -3x$	**9**	**-3**	**-12**	**(-3, 9), (1, -3), (4, -12)**	x: -3, 1, 4; $f(x)$: 9, -3, -12
4. $f(x) = -3x - 4$	**5**	**-7**	**-16**	**(-3, 5), (1, -7), (4, -16)**	x: -3, 1, 4; $f(x)$: 5, -7, -16
5. $f(x) = 2 - 3x$	**11**	**-1**	**-10**	**(-3, 11), (1, -1), (4, -10)**	x: -3, 1, 4; $f(x)$: 11, -1, -10

Teacher's Note and Reminders

Note that an ordered pair (x, y) can be written as $(x, f(x))$ for any function in $f(x)$ notation.

Activity 6 DESCRIBE ME (PART I)!

Description: This activity will enable you to describe a linear function using the set of ordered pairs and table by finding the value of the function at x.

Direction: Do as directed the given tasks.

A. Determine the values (*a*) $f(-3)$, (*b*) $f(1)$, and (c) $f(4)$ in each of the following functions.

1. $f(x) = 2x$
2. $f(x) = 2x + 1$
3. $f(x) = -3x$
4. $f(x) = -3x - 4$
5. $f(x) = 2 - 3x$

B. Complete the table below.

Function	The values of			Ordered Pairs	Table
	$f(-3)$	$f(1)$	$f(4)$		
1. $f(x) = 2x$					x: ; $f(x)$:
2. $f(x) = 2x + 1$					x: ; $f(x)$:
3. $f(x) = -3x$					x: ; $f(x)$:
4. $f(x) = -3x - 4$					x: ; $f(x)$:
5. $f(x) = 2 - 3x$					x: ; $f(x)$:

C

Function	The values of...	Meaning
1. $f(x) = 2x$	$f(-3) = -6$	The value of f at $x = -3$ is -6.
	$f(1) = 2$	The value of f at $x = 1$ is 2.
	$f(4) = 8$	The value of f at $x = 4$ is 8.
2. $f(x) = 2x + 1$	$f(-3) = -5$	The value of f at $x = -3$ is -5.
	$f(1) = 3$	The value of f at $x = 1$ is 3.
	$f(4) = 9$	The value of f at $x = 4$ is 9.
3. $f(x) = -3x$	$f(-3) = 9$	The value of f at $x = -3$ is 9.
	$f(1) = -3$	The value of f at $x = 1$ is -3.
	$f(4) = -12$	The value of f at $x = 4$ is -12.
4. $f(x) = -3x - 4$	$f(-3) = 5$	The value of f at $x = -3$ is 5.
	$f(1) = -7$	The value of f at $x = 1$ is -7.
	$f(4) = -16$	The value of f at $x = 4$ is -16.
5. $f(x) = 2 - 3x$	$f(-3) = 11$	The value of f at $x = -3$ is 11.
	$f(1) = -1$	The value of f at $x = 1$ is -1.
	$f(4) = -10$	The value of f at $x = 4$ is -10.

Process the guide questions and let them realize that as x increases, the value of the function may either increase or decrease.

Teacher's Note and Reminders

C. Complete the table below. An example is done for you.

Function	The values of...	Meaning
1. $f(x) = 2x$	$f(-3) = -6$	The value of f at $x = -3$ is -6.
	$f(1) = 2$	The value of f at $x = 1$ is 2.
	$f(4) = 8$	The value of f at $x = 4$ is 8.
2. $f(x) = 2x + 1$	$f(-3) =$ ___	
	$f(1) =$ ___	
	$f(4) =$ ___	
3. $f(x) = -3x$	$f(-3) =$ ___	
	$f(1) =$ ___	
	$f(4) =$ ___	
4. $f(x) = -3x - 4$	$f(-3) =$ ___	
	$f(1) =$ ___	
	$f(4) =$ ___	
5. $f(x) = 2 - 3x$	$f(-3) =$ ___	
	$f(1) =$ ___	
	$f(4) =$ ___	

QUESTIONS ?

1. How did you determine the values of $f(-3)$, $f(1)$ and $f(4)$ of each function?
2. In each of the functions below, what have you observed about the values of f as x increases?
   a. $f(x) = 2x$
   b. $f(x) = 2x + 1$
   c. $f(x) = -3x$
   d. $f(x) = -3x - 4$
   e. $f(x) = 2 - 3x$
3. Does the value of the function increase as x increases?
4. What affects the change of values of the function?
5. Have you observed a pattern? If yes, state so.
6. **How can the value of a quantity given the rate of change be predicted?**

Let the students describe a linear function using mapping diagram and graph. To do this, let the students evaluate $f(-2)$, $f(-1)$, $f(0)$, $f(1)$ and $f(2)$ of each function and complete each table.

Answers to Activity 7:

a. $f(x) = x + 5$

x	-2	-1	0	1	2
$f(x)$	3	4	5	6	7

c. $f(x) = -x + 5$

x	-2	-1	0	1	2
$f(x)$	7	6	5	4	3

b. $f(x) = 3x$

x	-2	-1	0	1	2
$f(x)$	-6	-3	0	3	6

d. $f(x) = -3x$

x	-2	-1	0	1	2
$f(x)$	6	3	0	-3	-6

The mapping diagram of each function is one-to-one correspondence. Each element in the domain corresponds to one and only one element in the range. For function defined by $f(x) = x + 5$ and $f(x) = 3x$, the value of the function increases as x increases. However, for functions defined by $f(x) = -x + 5$ and $f(x) = -3x$, the value of the function decreases as x increases. The value of m affects the trend of the function.

Let the students tell whether a function represented by a table is linear or not. Allow them to observe the first differences on the x-values and the first differences on the y-values in answering Activity 8.

Teacher's Note and Reminders

Activity 7 DESCRIBE ME (PART II)!

Description: This activity will enable you to describe a linear function using mapping diagram and graph.

Direction: Given the functions below, evaluate the following: $f(-2)$, $f(-1)$, $f(0)$, $f(1)$ and $f(2)$. Complete the table of values of each function below. Illustrate with a mapping diagram and draw the graph in a graphing paper.

a. $f(x) = x + 5$

x			
$f(x)$			

c. $f(x) = -x + 5$

x			
$f(x)$			

b. $f(x) = 3x$

x			
$f(x)$			

d. $f(x) = -3x$

x			
$f(x)$			

QUESTIONS ?

1. How did you determine the values of f(-2), f(-1), f(0), f(1) and f(2) of each function?
2. What type of correspondence are the mapping diagrams? Does each element in the domain correspond to one and only one element in the range?
3. Have you observed any pattern from the domain and range of each function? Based from the values obtained, is the function increasing or decreasing?
4. Which function has an increasing value of y as x increases?
5. Which function has a decreasing value of y as x increases?
6. How can you predict the value of a quantity given the rate of change?

Activity 8 WHAT ARE THE FIRST DIFFERENCES ON Y-VALUES?

Description: This activity will enable you to determine whether a function is linear given the table.

Direction: Do as directed.

A. Consider the function f defined by $f(x) = 3x - 1$.

1. Find the values of the functions and complete the table below:

x	0	1	2	3	4
$f(x)$ or y					

Answer to Activity 8

A.

	1	1	1	1	
x	0	1	2	3	4
$g(x)$ or y	-1	2	5	8	11
	3	3	3	3	

B.

	2	2	2	2	
x	0	3	5	7	9
$h(x)$ or y	6	10	14	18	22
	4	4	4	4	

C.

	1	1	1	1	
x	-2	-1	0	1	2
$f(x)$ or y	5	2	1	2	5
	-3	-1	1	3	

2. Find the first differences on x-coordinates. Write your answers on the boxes above the table:

	1				
x	0	1	2	3	4
$f(x)$ or y					

3. Find the first differences on y-coordinates and write your answers on the boxes below the table:

x	0	1	2	3	4
$f(x)$ or y					

QUESTIONS ?

1. How did you find the values of the function?
2. What are the first differences on x-coordinates? How did you find them? Are they equal?
3. What are the first differences on y-coordinates? How did you find them? Are they equal?
4. Is the given function linear? Explain.
5. How is the slope m of the function related to the first differences on y-coordinates?

B. Consider the function g defined by $g(x) = 2x + 4$.

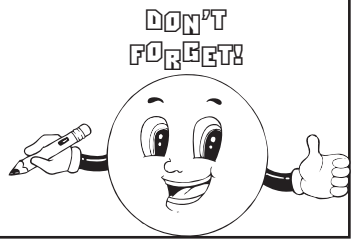
1. Find the values of the functions and complete the following table:

x	1	3	5	7	9
$g(x)$ or y					

2. Find the first differences on x-coordinates and write your answers on the boxes above the table:

x	1	3	5	7	9
$g(x)$ or y					

Teacher's Note and Reminders



Use process questions to enable students' generalize the following:

The function is linear if first differences on x-coordinates are **equal** and the first differences on y-coordinates are **equal**. However, the function is not linear if the first differences on x-coordinates are equal and the first differences on y-coordinates are **not equal**.

3. Find the first differences on y-coordinates and write your answers on the boxes below the table:

x	1	3	5	7	9
$g(x)$ or y					

QUESTIONS ?

1. How did you find the values of the function?
2. What are the first differences on x-coordinates? How did you find them? Are they equal?
3. What are the first differences on y-coordinates? How did you find them? Are they equal?
4. Is the given function linear? Explain.
5. How is the slope m of the function related to the first differences on y-coordinates?

C. Consider the function h defined by $h(x) = x^2 + 1$.

1. Find the values of the functions and complete the following table:

x	-2	-1	0	1	2
$h(x)$ or y					

2. Find the first differences on x-coordinates and write your answers on the boxes above the table:

x	-2	-1	0	1	2
$h(x)$ or y					

3. Find the first differences on y-coordinates and write your answers on the boxes below the table:

x	-2	-1	0	1	2
$h(x)$ or y					

Answers in Activity 8:

1. **L**

x	-2	-1	0	1	2
$f(x)$ or y	1	2	3	4	5

2. **L**

x	-2	-1	0	1	2
$f(x)$ or y	-3	-1	1	3	5

3. **L**

x	-2	-1	0	1	2
$f(x)$ or y	5	2	-1	-4	-7

4. **NL**

x	1	2	3	4	5
$f(x)$ or y	4	1	0	1	4

5. **NL**

x	-2	0	2	4	6
$f(x)$ or y	4	-2	-4	-2	4

6. **L**

x	-2	-1	0	1	2
$f(x)$ or y	-1	2	5	8	11

7. **L**

x	5	4	3	2	1
$f(x)$ or y	-1	2	5	8	11

8. **L**

x	-5	-4	-3	-2	-1
$f(x)$ or y	15	11	7	3	-1

9. **NL**

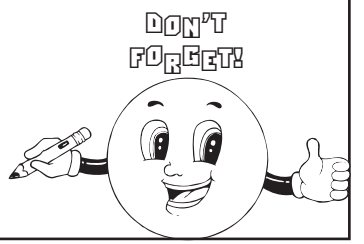
x	-2	-1	0	1	2
$f(x)$ or y	1	0	1	4	9

10. **L**

x	-4	-2	1	3	4
$f(x)$ or y	-21	-11	4	14	19

Let the students recall the domain and the range of a relation if a set of ordered pairs, a mapping diagram, a table, or a graph is known. Process the questions provided so that the students are able to generalize that the domain and the range of any linear function is a set of real numbers.

Teacher's Note and Reminders



QUESTIONS ?

1. How did you find the values of the function?
2. What are the first differences on x-coordinates? How did you find them? Are they equal?
3. What are the first differences on y-coordinates? How did you find them? Are they equal?
4. Is the given function linear? Explain.
5. What have you realized? State your realization by completing the statement below.
   The function is linear if first differences on x-coordinates are _____ and the first differences on y-coordinates are _____. However, the function is not linear if the first differences on x-coordinates are equal and the first differences on y-coordinates are _____.

Exercise 2

Determine whether the function below is linear given the table.

1.

x	-2	-1	0	1	2
$f(x)$ or y	1	2	3	4	5

2.

x	-2	-1	0	1	2
$f(x)$ or y	-3	-1	1	3	5

3.

x	-2	-1	0	1	2
$f(x)$ or y	5	2	-1	-4	-7

4.

x	1	2	3	4	5
$f(x)$ or y	4	1	0	1	4

5.

x	-2	0	2	4	6
$f(x)$ or y	4	-2	-4	-2	4

6.

x	-2	-1	0	1	2
$f(x)$ or y	-1	2	5	8	11

7.

x	5	4	3	2	1
$f(x)$ or y	-1	2	5	8	11

8.

x	-5	-4	-3	-2	-1
$f(x)$ or y	15	11	7	3	-1

9.

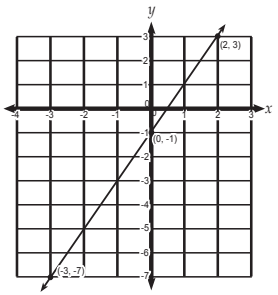
x	-2	-1	0	1	2
$f(x)$ or y	1	0	1	4	9

10.

x	-4	-2	1	3	4
$f(x)$ or y	-21	-11	4	14	19

Domain and Range of a Linear Function

Again, consider the function f defined by $f(x) = 2x - 1$. Study the graph carefully. What have you noticed about the arrow heads of the graph? What can you say about it?



Teacher's Note and Reminders

Answers to Exercise 3:

Function	Domain	Range
1. $f(x) = 2x$	$\{x \mid x \in \Re\}$	$\{y \mid y \in \Re\}$
2. $f(x) = 4x + 1$	$\{x \mid x \in \Re\}$	$\{y \mid y \in \Re\}$
3. $f(x) = -7x - 4$	$\{x \mid x \in \Re\}$	$\{y \mid y \in \Re\}$
4. $f(x) = 8x - 5$	$\{x \mid x \in \Re\}$	$\{y \mid y \in \Re\}$
5. $f(x) = x - 9$	$\{x \mid x \in \Re\}$	$\{y \mid y \in \Re\}$

Discuss rewriting linear equations from the form $Ax + By = C$ into $y = mx + b$ and vice-versa. Give some examples.

QUESTIONS ?

1. What do the arrow heads indicate?
2. Does the graph extend to the left and right without bound?
3. What is its domain?
4. Does the graph extend upward and downward without bound?
5. What is its range?
6. What is the domain of the linear function? Justify your answer.
7. What is the range of the linear function? Justify your answer.

If function f is defined by $f(x) = mx + b$, then the domain of the function D_f is $\Re$ and its range of the function R_f is $\Re$. In symbols,

$D_f = \{x \mid x \in \Re\}$, read as: "the domain of the function f is the set of all x such that x is an element of the set of real numbers," and

$R_f = \{y \mid y \in \Re\}$, read as: "the range of the function f is the set of all y such that y is an element of the set of real numbers."

Exercise 3

Complete the following table.

Function	Domain	Range
1. $f(x) = 2x$		
2. $f(x) = 4x + 1$		
3. $f(x) = -7x - 4$		
4. $f(x) = 8x - 5$		
5. $f(x) = x - 9$		

Linear Equations

Aside from the sets of ordered pairs and the graph, a linear function f defined by $f(x) = mx + b$ can also be represented by its equation.

Question:

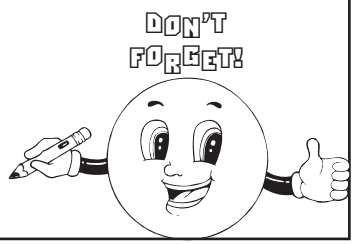
Does the equation $3x + 2y = 6$ describe a linear function? If yes, determine the slope and the y-intercept.

Solution:

The equation $3x + 2y = 6$ can be solved for y:

$3x + 2y = 6$	Given
$3x + 2y + (-3x) = 6 + (-3x)$	Addition Property of Equality

Teacher's Note and Reminders



$2y = -3x + 6$	Simplification
$\frac{1}{2}(2y) = \frac{1}{2}(-3x + 6)$	Multiplication Property of Equality
$y = -\frac{3}{2}x + 3$	Simplification

The function $f(x) = -\frac{3}{2}x + 3$ or $y = -\frac{3}{2}x + 3$ can be expressed in the form $3x + 2y = 6$ with slope $m = -\frac{3}{2}$ while the y-intercept $b = 3$.

A linear equation is an equation in two variables which can be written in two forms:

Standard Form: $Ax + By = C$, where A, B and $C \in \Re$, $A \neq 0$ and $B \neq 0$; and
Slope-Intercept Form: $y = mx + b$, where m is the slope and b is the y-intercept, m and $b \in \Re$, and $m \neq 0$.

Illustrative Example 1

How do we rewrite the equation $3x - 5y = 10$ in the form $y = mx + b$? Determine its slope and y-intercept.

Solution:

$3x - 5y = 10$	Given
$3x - 5y + (-3x) = 10 + (-3x)$	Addition Property of Equality
$-5y = -3x + 10$	Simplification
$-\frac{1}{5}(-5y) = -\frac{1}{5}(-3x + 10)$	Multiplication Property of Equality
$y = \frac{3}{5}x - 2$	Simplification

The slope is $\frac{3}{5}$ and the y-intercept is -2.

Illustrative Example 2

How do we rewrite the equation $y = \frac{1}{2}x + 3$ in the form $Ax + By = C$?

Solution:

$y = \frac{1}{2}x + 3$	Given
$2(y) = 2(\frac{1}{2}x + 3)$	Multiplication Property of Equality
$2y = x + 6$	Simplification
$2y + (-x) = x + 6 + (-x)$	Addition Property of Equality
$-x + 2y = 6$	Simplification
$(-1)(-x + 2y) = (-1)(6)$	Multiplication Property of Equality
$x - 2y = -6$	Simplification

Answers to Exercise 4:

1. $x + y = 4$
2. $2x + y = 6$
3. $5x - y = -7$
4. $3x - y = 8$
5. $x - 2y = 0$
6. $x - 2y = -6$
7. $2x - 3y = 9$
8. $8x - 4y = 1$
9. $5x - 2y = -3$
10. $10x - 8y = -3$

Answers to Exercise 5:

1. $y = -2x + 9$
2. $y = -\frac{1}{2}x + 2$
3. $y = 3x - 2$
4. $y = -\frac{5}{2}x + \frac{7}{2}$
5. $y = x + \frac{1}{3}$
6. $y = \frac{5}{7}x - \frac{2}{7}$
7. $y = -6x + 8$
8. $y = 2x - 3$
9. $y = -\frac{5}{4}x + \frac{15}{2}$
10. $y = \frac{10}{3}x - 3$

Discuss slope of a line. Then, give examples. Start with formula $m = \frac{rise}{run}$ and let them derive the formula for two points with the use of process questions through oral questioning.

Teacher's Note and Reminders

Exercise 4

Rewrite the following equations in the form $Ax + By = C$.

1. $y = -x + 4$
2. $y = -2x + 6$
3. $y = 5x + 7$
4. $y = 3x - 8$
5. $y = \frac{1}{2}x$
6. $y = \frac{1}{2}x + 3$
7. $y = \frac{2}{3}x - 3$
8. $y = 2x + \frac{1}{4}$
9. $y = \frac{5}{2}x + \frac{3}{2}$
10. $y = \frac{5}{4}x + \frac{3}{8}$

Exercise 5

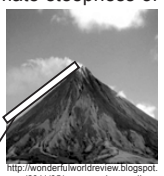
Rewrite the following equations in the form $y = mx + b$ and identify the values of m and b.

1. $2x + y = 9$
2. $x + 2y = 4$
3. $3x - y = 2$
4. $5x + 2y = 7$
5. $-3x + 3y - 1 = 0$
6. $5x - 7y = 2$
7. $3x + \frac{1}{2}y = 4$
8. $\frac{2}{3}x - \frac{1}{3}y = 1$
9. $\frac{5}{2}x + \frac{2}{3}y - 5 = 0$
10. $\frac{2}{3}x - \frac{1}{5}y = \frac{3}{5}$

Slope of a Line

Shown at the right is the Mount Mayon. It is one of the fascinating volcanoes in the Philippines because of its almost symmetrical conical shape. The approximate steepness of the volcano is labelled by the line.

The slope of the line can be used to describe how steep Mount Mayon is.



http://wonderfulworldreview.blogspot.com/2011/05/mayon-volcano-albay-philippines.html

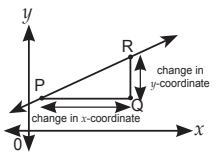
slope

A line can be described by its steepness or slope. The slope m of a line can be computed by finding the quotient of rise and run. That is,

$$m = \frac{rise}{run}$$

The rise refers to the vertical change or change in y-coordinate while the run is the horizontal change or change in x-coordinate. That is,

$$m = \frac{rise}{run} = \frac{vertical\ change}{horizontal\ change} = \frac{change\ in\ y\text{-coordinate}}{change\ in\ x\text{-coordinate}}$$



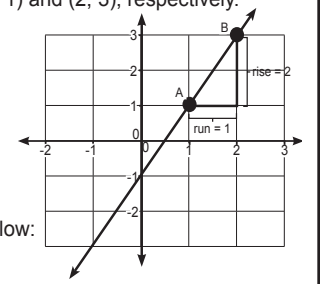
Teacher's Note and Reminders

Answers to Exercise 6:

1. $^7/_2$
2. -1
3. 0
4. ½
5. undefined

How do you solve the change in y-coordinate? What about the change in x-coordinate?

Suppose two points A and B have coordinates (1, 1) and (2, 3), respectively.



How is rise = 2 arrived at? Explain.
How is run = 1 arrived at? Explain.
What is the slope? How did you find the slope?
How did you find the change in y-coordinate?
How did you find the change in x-coordinate?
What have you realized?

Express your realization by completing the box below:

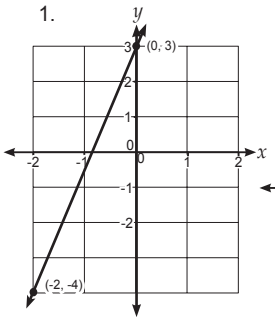
> If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then the slope m of the line can be computed by the formula:
>
> m = ----------

> *The slope m of the line passing through two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by*
>
> $m = \frac{y_2 - y_1}{x_2 - x_1}$ *or* $m = \frac{y_1 - y_2}{x_1 - x_2}$, *where* $x_1 \neq x_2$.

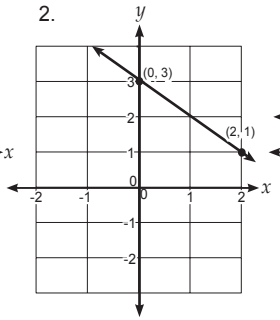
Exercise 6

Find the slope of each line below.

1.



2.



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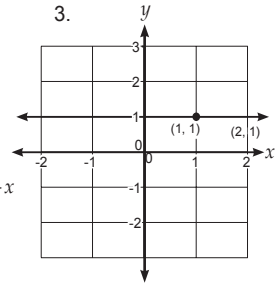


Teacher's Note and Reminders

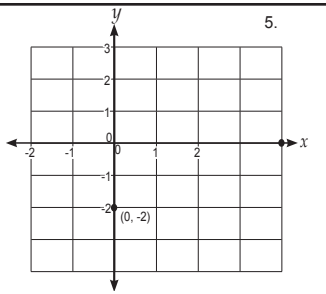
Discuss the trend of the graph and differentiate the lines whose slopes are positive (m > 0) and negative (m < 0) as well as the lines whose slopes are zero and undefined.

Answers to Challenge Questions:

1. a. $a = -\,{}^{7}/_{2}$
   b. $a = -\,{}^{1}/_{5}$

2. The slope is $-\,A/B$

4.



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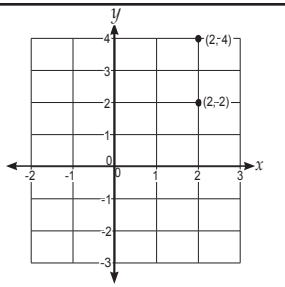


QUESTIONS ?

1. How did you find the slope of the line?
2. What is the trend of the graph? Is it increasing? Or decreasing?
3. What is the slope of each increasing graph? What are the signs of the slopes?
4. What is the slope of the decreasing graph? What is the sign of the slope?
5. Do the graphs represent linear functions? Why or why not?
6. What is the slope of the horizontal line? How about the vertical line?

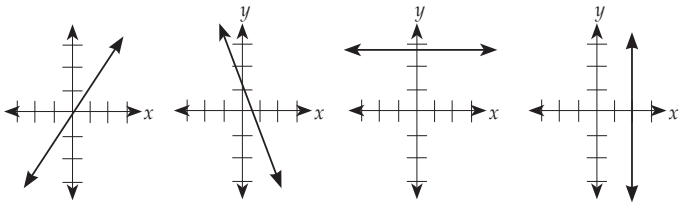
Note that:

★ *A basic property of a line, other than vertical line, is that its slope is constant.*

★ *The slope of the horizontal line is zero while that of the vertical line is undefined. Both lines do not represent linear functions.*

★ *The value of the slope m tells the trend of the graph.*

- *If m is positive, then the graph is increasing from left to right.*
- *If m is negative, then the graph is decreasing from left to right.*
- *If m is zero, then the graph is a horizontal line.*
- *If m is undefined, then the graph is a vertical line.*



Challenge Questions

1. Determine the value of a that will make the slope of the line through the two given points equal to the given value of m.
   a. (4, -3) and (2, a); $m = \frac{1}{4}$
   b. (a + 3, 5) and (1, a – 2); m = 4
2. If A, B, and $C \in \Re$ and the line is described by $Ax + By = C$, find its slope.

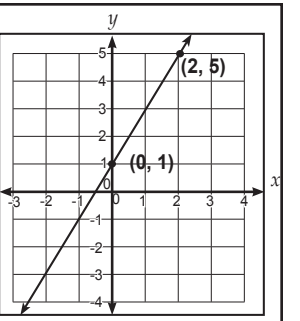
Teacher's Note and Reminders

Answers to Exercise 7:

1. $m = 2$	6. $m = 2$
2. $m = -3$	7. $m = {}^{7}/_{3}$
3. $m = 1$	8. $m = -2$
4. $m = ¼$	9. m is undefined; vertical line
5. $m = {}^{2}/_{3}$	10. $m = 0$; horizontal line

Ask the students to perform Activity 9. Allow them to go out of the classroom and look for any inclined object. Let them find its slope. This will enable them to see the connection of the concept to real life.

Consider the graph of the function f defined $f(x) = 2x + 1$ at the right.



Question to Ponder:

1. What is the slope of the line using any of the formulae?
2. Compare the slope you have computed to the numerical coefficient of x in the given function

The slope of the function f defined by $f(x) = mx + b$ is the value of m.

Exercise 7

Determine the slope of each line, if any. Identify which of the lines is vertical or horizontal.

1. $f(x) = 2x - 5$
2. $f(x) = -3x + 7$
3. $f(x) = x + 6$
4. $f(x) = \frac{1}{4}x - 8$
5. $f(x) = \frac{2}{3}x - \frac{1}{2}$
6. $2x - y = 5$
7. $7x - 3y - 10 = 0$
8. $\frac{1}{2}x + \frac{1}{4}y - 8 = 0$
9. $x = 8$
10. $2y + 1 = 0$

Activity 9 STEEP UP!

Description: This activity will enable you to use the concept of slope in real life. This can be done by group of 5 members.

Direction: Find any inclined object or location that you could see in your school and then determine its steepness.

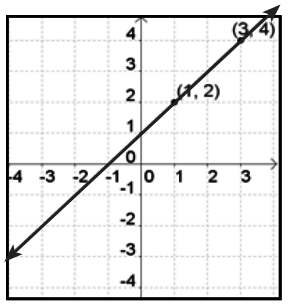
QUESTIONS ?

1. How did you find the steepness of the inclined object?
2. Have you encountered any difficulty in determining the steepness of the object? Explain your answer.

Discuss graphing linear equations. Start with any two points. Let them recall that a linear equation is an equation whose graph is a line and in Geometry, two points determine a line. That's why, two points are sufficient to draw the graph of a linear equation.

Answers to Exercise 8:

1.



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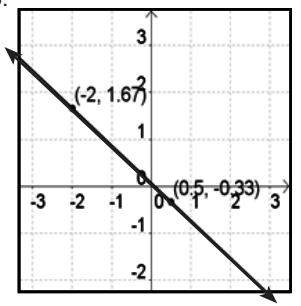

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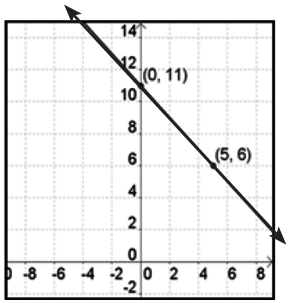

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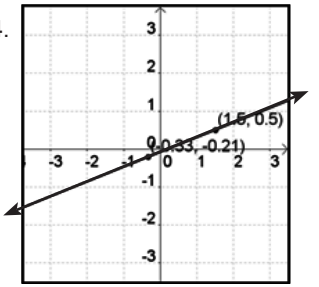
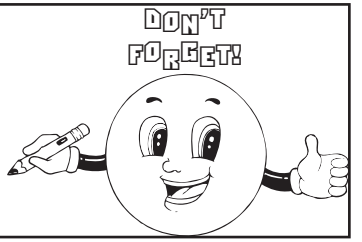


Teacher's Note and Reminders



Graphs of Linear Equations

You have learned earlier that a linear function can be described by its equation, either in the form $y = mx + b$ or $Ax + By = C$. A linear equation can also be described by its graph. Graphing linear equations can be done using any of the four methods:

1. Using two points
2. Using x- and y-intercepts
3. Using the slope and the y-intercept
4. Using the slope and a point

Using Two Points

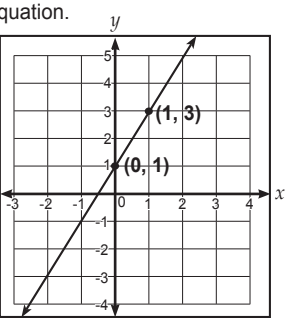
One method of graphing a linear equation is using two points. In Geometry, you learned that two points determine a line. Since the graph of the linear equation is a line, thus two points are enough to draw a graph of a linear equation.

Illustrative Example

Graph the function $y = 2x + 1$.

You may assign any two values for x, say 0 and 1. By substitution,

$y = 2x + 1$	$y = 2x + 1$
$y = 2(0) + 1$	$y = 2(1) + 1$
$y = 0 + 1$	$y = 2 + 1$
$y = 1$	$y = 3$



If $x = 0$, then $y = 1$. Furthermore, if $x = 1$, then $y = 3$. So, the ordered pairs are (0, 1) and (1, 3). This means that the line passes through these points.

After finding the ordered pairs of the two points, plot and connect them. Your output is the graph of the linear equation.

Exercise 8

Graph each linear equation that passes through the given pair of points.

1. (1, 2) and (3, 4)
2. (5, 6) and (0, 11)
3. $(-2, \frac{5}{3})$ and $(\frac{1}{2}, -\frac{1}{3})$
4. $(-\frac{1}{3}, -\frac{1}{5})$ and $(\frac{3}{2}, \frac{1}{2})$

Using x-Intercept and y-Intercept

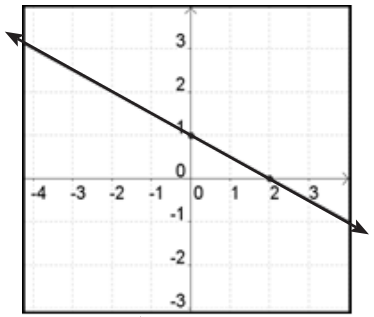
Secondly, the linear equation can be graphed by using x-intercept a and y-intercept b. The x- and y-intercepts of the line could represent two points, which are $(a, 0)$ and $(0, b)$. Thus, the intercepts are enough to graph the linear equation.

Discuss graphing linear equations using x- and y-intercepts. Emphasize that x-intercept a is the abcissa of the coordinates of the point (a, 0) that intersects the x-axis while y-intercept b is the ordinate of the coordinates of the point (0, b) that intersects the y-axis. This means that two points exist to represent x- and y-intercepts. Thus, x-intercept and y-intercept are sufficient to draw the graph of the linear equation.

Processes on how to solve for x- and y-intercepts are provided. Links are also provided for further references.

Answers to Exercise 9:

1.



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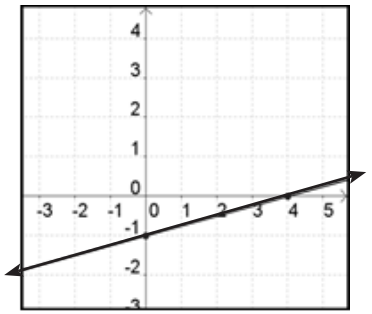

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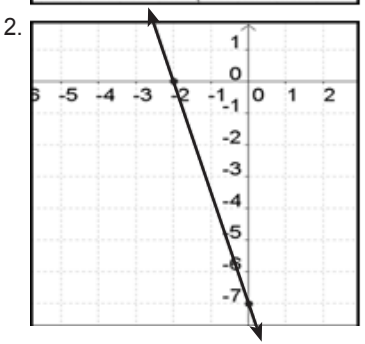

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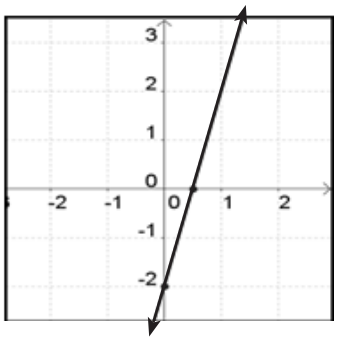


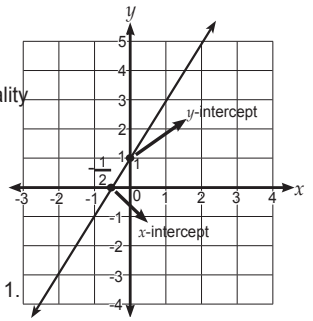
To graph the equation $y = 2x + 1$ using this method, you need to solve the x-intercept by letting $y = 0$ and the y-intercept by letting $x = 0$.

Letting $y = 0$, the equation $y = 2x + 1$ becomes

$0 = 2x + 1$	Substitution
$-2x = 1$	Addition Property of Equality
$x = -\frac{1}{2}$	Multiplication Property of Equality

Letting $x = 0$, $y = 2x + 1$ becomes

$y = 2(0) + 1$	Substitution
$y = 0 + 1$	Simplification
$y = 1$	Simplification



The x-intercept a is $-\frac{1}{2}$ while the y-intercept b is 1.

Now, plot the x- and y-intercepts, then connect them.

The **x-intercept** is the abscissa of the coordinates of the point in which the graph intersects the x-axis. However, the **y-intercept** is the ordinate of the coordinates of the point in which the graph intersects the y-axis.

Web Links
Click these links for further references:

1. http://www.youtube.com/watch?v=mvsUD3tDnHk&feature=related.
2. http://www.youtube.com/watch?v=mxBoni8N70Y

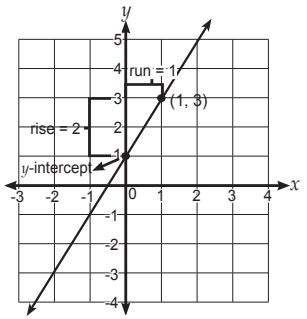
Exercise 9

Graph each linear equation whose x-intercept a and y-intercept b are given below.

1. $a = 2$ and $b = 1$
2. $a = 4$ and $b = -1$
3. $a = -2$ and $b = -7$
4. $a = \frac{1}{2}$ and $b = -2$

Using Slope and y-Intercept

The third method is by using the slope and the y-intercept. This can be done by identifying the slope and the y-intercept of the linear equation.

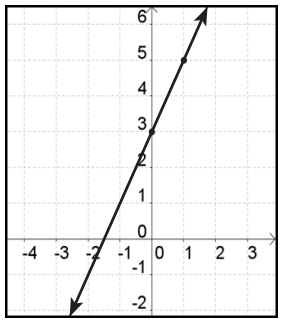


In the same equation $y = 2x + 1$, the slope m is 2 and y-intercept b is 1. Plot first the y-intercept, then use the slope to find the other point. Note that 2 means $\frac{2}{1}$, which means rise = 2 and run = 1. Using the y-intercept as the starting point, we move 2 units upward since rise = 2, and 1 unit to the right since run = 1.

Discuss graphing linear equations using slope and y-intercept. A y-intercept represents a point. Thus, it is necessary to find for another point. That could be done by using the slope and y-intercept.
Processes on how to find another point using slope and y-intercept are provided. Links are also provided for further references.

Answers to Exercise 10:

1.



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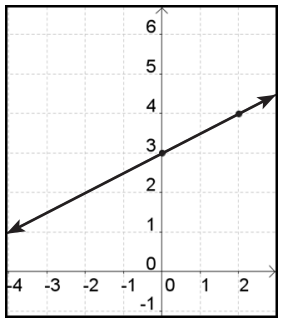

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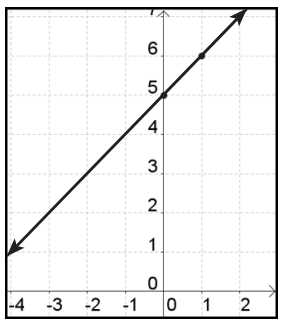
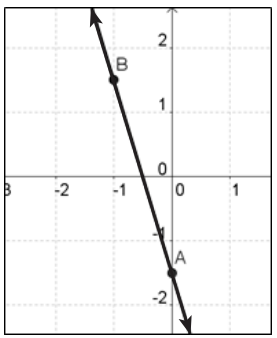


4.



Web Links
Click these links for more examples:

1. http://www.youtube.com/watch?v=Qlp3zMTTACE
2. http://www.youtube.com/watch?v=jd-ZRCsYaec
3. http://www.youtube.com/watch?v=EbuRufY41pc&feature=related

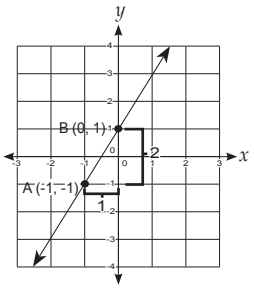
Note that if rise is less than zero (or negative), we move downward from the first point to look for the second point. Similarly, if run is less than zero (or negative), we move to the left from the first point to look for the second point. Moreover, a negative rational number $-\frac{1}{2}$ can be written as either $\frac{-1}{2}$ or $\frac{1}{-2}$ but not $\frac{-1}{-2}$.

Exercise 10

Graph each linear equation given slope m and y-intercept b.

1. $m = 2$ and $b = 3$
2. $m = 1$ and $b = 5$
3. $m = \frac{1}{2}$ and $b = 3$
4. $m = -3$ and $b = -\frac{3}{2}$

Using Slope and One Point



The fourth method in graphing linear equation is by using the slope and one point. This can be done by plotting first the given point, then finding the other point using the slope.

The linear equation $y = 2x + 1$ has a slope of 2 and a point (-1, -1). To find a point from this equation, we may assign any value for x in the given equation. Let's say, $x = -1$. The value of y could be computed in the following manner:

$y = 2x + 1$	Given
$y = 2(-1) + 1$	Substitution
$y = -2 + 1$	Simplification
$y = -1$	Simplification

Complete the statement below:

The line passes through the point _____.

The point found above is named A whose coordinates are (-1, -1). Since the slope of the line is 2 which is equal to $\frac{2}{1}$, use the rise of 2 and run of 1 to determine the coordinates of B (refer to the graph). This can also be done this way.

B = (-1 + 1, -1 + 2) = (0, 1)

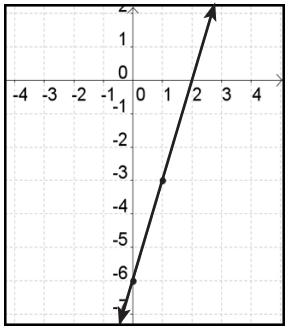
Web Links
Use this link http://www.youtube.com/watch?v=f58Jkjypr_I which is a video lesson for another example.

Note that 2 (the rise) must be added to the y-coordinate while 1 (the run) must be added to x-coordinate.

Discuss graphing linear equations using slope and one point. It is necessary to find for another point. That could be done by using the slope and one point. Processes on how to find another point using slope and a point are provided. Links are also provided for further references.

Answers to Exercise 11:

1.



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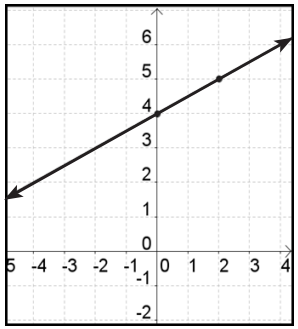

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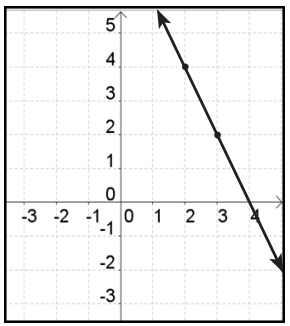

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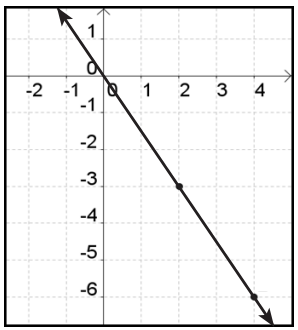


Exercise 11

Graph the following equations given slope m and a point.

1. $m = 3$ and (0, -6)
2. $m = -2$ and (2, 4)
3. $m = \frac{1}{2}$ and (0, 4)
4. $m = \frac{3}{2}$ and (2, -3)

Activity 10 WRITE THE STEPS

Description: This activity will enable you to summarize the methods of graphing a linear equation.

Direction: Fill in the diagram below by writing the steps in graphing a linear equation using 4 different methods.

Using Two Points
-
-

Using x- and y-Intercepts
-
-

Using Slope and y-Intercept
-
-
-

Using Slope and One Point
-
-
-

QUESTIONS ?

1. Among the four methods of graphing a linear equation, which one is easiest for you? Justify your answer.
2. Have you encountered any difficulty in doing any of the four methods? Explain your answer.

Assess students' knowledge about the steps on drawing the graph of the linear equation using the four methods. Allow them to go back to how these methods are done.

Allow the students to create their own story about the given graph in performing Activity 11. Varied answers to this activity are expected.

Let the students describe the graph of the linear function using its x-intercept, y-intercept, slope, trend and equation. You may give additional graph for further practice. Ask them to answer Activity 12.

Answers to Activity 12:

1. x-intercept: -3
2. y-intercept: 2
3. rise: 3
4. run: 2
5. slope: $^3/_2$
6. trend: increasing

   Table:

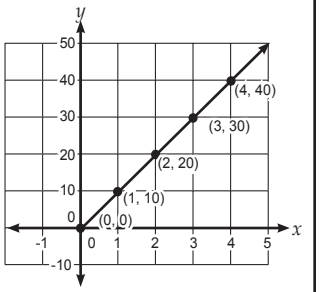
x	0	-3
y	2	0

Teacher's Note and Reminders

Activity 11 MY STORY

Description: This activity will enable you to analyze the graph and connect this to real life.

Direction: Create a story out of the graph of the linear equation at the right. Share this to your classmate.



QUESTIONS ?

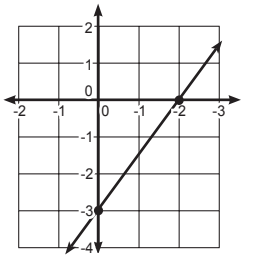
1. Do you have the same story with your classmates?
2. Is your story realistic? Why?

Activity 12 DESCRIBE ME (PART III)!

Description: This activity will enable you to describe the graph of a linear equation in terms of its intercepts, slope and points.

Direction: Given the graph at the right, find the following:

1. x-intercept
2. y-intercept
3. rise
4. run
5. slope
6. trend



Complete the table below:

x		
y		

QUESTIONS ?

1. How did you identify the x-intercept and y-intercept?
2. In your own words, define x-intercept and y-intercept.
3. How did you find the rise and run?
4. How did you find the slope?
5. Is it increasing or decreasing from left to right? Justify your answer.
6. Have you observed a pattern?
7. What happen to the value of y as the value of x increases?
8. How can the value of a quantity given the rate of change be predicted?

Discuss finding the equation of the line. Start it by using the slope –intercept form $y = mx + b$.

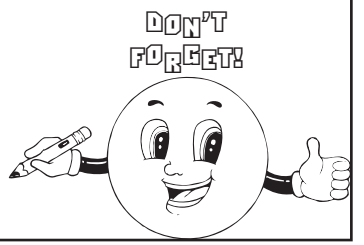
Answers to Questions of Activity 13:

1. The value of m in each equation
2. The value of b in each equation
3.

Equation of the Line	Slope	y-Intercept
a. $y = 2x$	2	0
b. $y = 2x + 4$	2	4
c. $y = 2x – 5$	2	-5
d. $y = x + 5$	1	5
e. $y = -2x + 4$	-2	4

4. The value of m is the slope of the line $y = mx + b$ and the value of b is its y-intercept.
5. The slope of $y = 7x + 1$ is 7 while its y-intercept is 1.

Teacher's Note and Reminders



Finding the Equation of the Line

The equation of the line can be determined using the following formulae:

a. slope-intercept form: $y = mx + b$;
b. point-slope form: $y – y_1 = m(x – x_1)$; and
c. two-point form: $y – y_1 = \frac{y_2 – y_1}{x_2 – x_1}(x – x_1)$.

Activity 13 SLOPE AND Y-INTERCEPT

Description: This activity will enable you to find the equation of the line using slope-intercept form.

Materials: graphing paper
pencil or ballpen

Direction: Graph these equations in one Cartesian plane.

a. $y = 2x$ c. $y = 2x – 5$ e. $y = -2x + 4$
b. $y = 2x + 4$ d. $y = x + 5$

QUESTIONS ?

1. What is the slope of each line? Use the formula $m = \frac{rise}{run}$ to answer this question.
2. What is the y-intercept of each line?
3. Complete the table below using your answers in 1 and 2.

Equation of the Line	Slope	y-Intercept
a. $y = 2x$		
b. $y = 2x + 4$		
c. $y = 2x – 5$		
d. $y = x + 5$		
e. $y = -2x + 4$		

4. What can you say about the values of m and b in the equation $y = mx + b$ and the slope and the y-intercept of each line? Write a short description below.

   _____

5. Consider the equation $y = 7x + 1$. Without plotting points and computing for m, what would you expect the slope to be? How about the y-intercept? Check your answer by graphing. Are your expectations about the slope and the y-intercept of the line correct?

Example:
Find the equation of the line whose slope is 3 and y-intercept is -5.

Solution:
The equation of the line is $y = 3x – 5$.

Assess the students' knowledge about the slope and the y-intercept of the line whose equation is in the form $Ax + By = C$. Do not give this as an assignment.

Answers to Activity 14:

1. $2x + \boxed{5}\,y = \boxed{10}$

   $m = -\dfrac{\boxed{2}}{5}$ $\quad b = 2$

2. $\boxed{3}x - 6y = 7$

   $m = \dfrac{\boxed{1}}{2}$ $\quad b = -\dfrac{\boxed{7}}{6}$

3. $3\boxed{6}\,x + \boxed{9}\,y = 1\boxed{8}$

   $m = -\boxed{4}$ $\quad b = 2$

Answers to Activity 15:

1. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

2. Since the two points of the line are represented by $(x_1 - y_1)$ and (x, y), its slope is $m = \dfrac{y - y_1}{x - x_1}$

3. Two-Point Form of the linear equation

4. $m = \dfrac{y - \boxed{y_1}}{x - \boxed{x_1}}$

   $y - \boxed{y_1} = m(x - \boxed{x_1})$

Slope-Intercept Form of the Equation of a Line

The linear equation $y = mx + b$ is in slope-intercept form. The slope of the line is m and the y-intercept is b.

Activity 14 FILL IN THE BOX

Description: This activity will assess what you have learned in identifying the slope and y-intercept of the line whose equation is in the form $Ax + By = C$.

Direction: Complete the boxes below in such a way that m and b are slope and y-intercept of the equation, respectively. You are allowed to write the numbers 1 to 10 once only.

1. $2x + \square\,y = \square$

   $m = -\dfrac{\square}{5}$ $\quad b = 2$

2. $\square x - 6y = 7$

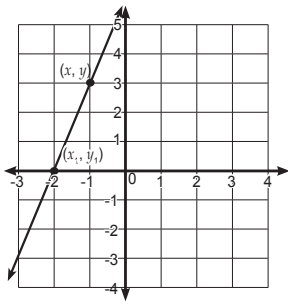
   $m = \dfrac{\square}{2}$ $\quad b = -\dfrac{\square}{6}$

3. $3\square\,x + \square\,y = 1\square$

   $m = -\square$ $\quad b = 2$

Activity 15 THINK-PAIR-SHARE

Description: This activity will enable you to generate Point-Slope Form of the equation of a line. Shown at the right is a line that contains the points (x_1, y_1) and (x, y). Note that the (x_1, y_1) is a fixed point on the line while (x, y) is any point contained on the line.



Direction: Give what are asked.

1. Recall the formula for slope given two points.
2. How do you compute the slope of this line?
3. What formula did you use?
4. Solve for the Point-Slope Form of a line by completing the following:

   $m = \dfrac{y - \square}{x - \square}$

   $y - \square = m(x - \square)$ Why?

Let the students derive the formula of Point-Slope and Two-Point Forms of the linear equation. Let each student find a partner and discuss Activities 15 and 16. You may also assist each pair of students in the derivation of the formulae. Discuss their answers and give examples.

Answers to Exercise 12:

1. $y = 2x + 4$
2. $y = x - 7$
3. $y = -5x - 6$
4. $y = -7x + 27$
5. $y = -x + 9$
6. $y = \frac{1}{2}x - 3$
7. $y = \frac{2}{3}x + 8$
8. $y = -\frac{7}{2}x - 11$
9. $y = -\frac{7}{4}x + \frac{9}{2}$
10. $y = \frac{1}{2}x + \frac{35}{12}$

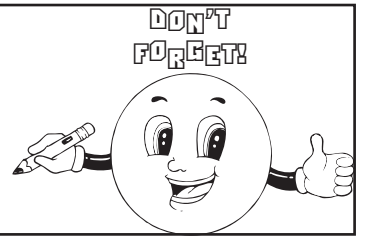
Answers to Activity 16:

1. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2. $y - y_1 = m(x - x_1)$
3. Subsitute the formula of m to the **Point-Slope Form**.

Answers to Exercise 13:

1. $y = 3x - 5$
2. $y = -3x + 28$
3. $y = -x + 2$
4. $y = -6x - 43$
5. $y = 5x + 15$
6. $y = -\frac{1}{4}x + \frac{1}{2}$
7. $y = -\frac{1}{4}x + \frac{15}{8}$
8. $y = -4x - \frac{9}{2}$
9. $y = \frac{1}{3}$
10. $y = -\frac{7}{12}x + \frac{1}{24}$

Teacher's Note and Reminders



Point-Slope Form of the Equation of a Line

The linear equation $y - y_1 = m(x - x_1)$ is the point-slope form. The value of m is the slope of the line which contains a fixed point $P_1(x_1, y_1)$.

Exercise 12

Find the equation of the line of the form y = mx + b given the slope and a point.

1. $m = 2$; (0, 4)
2. $m = 1$; (5, -2)
3. $m = -5$; (-3, 9)
4. $m = -7$; (4, -1)
5. $m = -1$; (7, 2)
6. $m = \frac{1}{2}$; (-6, 0)
7. $m = \frac{2}{3}$; (0, 8)
8. $m = -\frac{7}{2}$;(-4, 3)
9. $m = -\frac{7}{4}$;(-2, 8)
10. $m = \frac{1}{2}$, $(-\frac{1}{2}, \frac{8}{3})$

Activity 16 THINK-PAIR-SHARE

Description: This activity will enable you to derive the Two-Point Form of the equation of the line. Again, recall the formula for the slope and the Point-Slope Form of the equation of the line.

Direction: Answer the following guide questions

1. Write the formula of slope m of the line given two points in the box.

2. Write the Point-Slope Form of the equation of the line in the box.

3. State the justification in the second statement below.

$y - y_1 = m(x - x_1)$	Point-Slope Form
$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$	Why?

Two-Point Form of the Equation of a Line

The linear equation $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the Two-Point Form, where (x_1, y_1) and (x_2, y_2) are the coordinates of P_1 and P_2, respectively.

Teacher's Note and Reminders

Elicit present knowledge about the linear functions by answering the "Revised Answer" column in the IRF Worksheet. Compare their revised answers of the questions to their initial answers.

Exercise 13

Find the equation of the line of the form $y = mx + b$ that passes through the following pairs of points.

1. (3, 4) and (4, 7)
2. (8, 4) and (6, 10)
3. (3, -1) and (7, -5)
4. (-8, 5) and (-9, 11)
5. (-1, 10) and (0, 15)
6. $(0, \frac{1}{2})$ and $(1, -\frac{1}{2})$
7. $(\frac{7}{2}, 1)$ and $(-\frac{1}{2}, 2)$
8. $(-\frac{1}{2}, -\frac{5}{2})$ and $(-\frac{3}{2}, \frac{3}{2})$
9. $(-\frac{15}{2}, \frac{1}{3})$ and $(-\frac{1}{2}, \frac{1}{3})$
10. $(-\frac{5}{2}, \frac{3}{2})$ and $(\frac{1}{2}, -\frac{1}{4})$

To enrich your skills in finding the equation of the line, which is horizontal, vertical or slanting, go to this link http://www.mathplayground.com/SaveTheZogs/SaveTheZogs_IWB.html.

You can also visit the link in finding the equation of the line, where two points can be moved from one place to another http://www.mathwarehouse.com/algebra/linear_equation/linear-equation-interactive-activity.php

Activity 17 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will give your present knowledge about the concept.

Direction: Give your revised answers of the questions provided in the first column and write them in the third column. Compare your revised answers from your initial answers.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is linear function?			
2. How do you describe a linear function?			
3. How do you graph a linear function?			
4. How do you find an equation of the line?			
5. How can the value of a quantity given the rate of change be predicted?			

Provide students the opportunities to think deeper and apply their knowledge and skills in solving word problems involving linear functions. Flow chart and guide questions are provided. Even though your little assistance is encouraged, allow the students to solve the problems on their own. Ask the students to do Activities 18, 19, and 20.

Answers to Activity 18:

Distance (in meters) x	0	200	400	600	800	1000
Amount (in Php) y	**40**	**43.50**	**47**	**50.50**	**54**	**57.50**

1. The dependent variable is the amount because it depends on the distance.
2. The independent variable is the distance because it controls the amount.
3. It represents a line.
4. The y-intercept of the line is 40.
5. The slope is $^{7}/_{400}$.
6. The linear function f is $f(x) = {}^{7}/_{400}x + 40$.
   (a) Emman will have to pay Php 50.50.
   (b) He will have to pay Php 145.
   (c) If he pays Php 68, then he traveled 1600 meters or less than 1600 meters but greater than 1400 meters. If he pays Php 75, then he traveled 2000 meters or less than 2000 meters but greater than 1800 meters. If he pays Php 89, then he traveled 2800 meters or less than 2800 meters but greater than 2600 meters. Finally, if he pays Php 92.50, then he traveled 3000 meters or less than 3000 meters but greater than 2800 meters.
7. The linear equation is $7x - 400y = -1600$, instead of $7x - 800y = -1600$.

In this section, the discussions were about linear functions. Go back to the previous section and compare your initial ideas with the discussions. How much of your initial ideas are found in the discussions? Which ideas are different and need revision? Now you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand

Your goal in this section is to take a closer look ate the real-life problems involving linear equations and relations.

Activity 18 RIDING IN A TAXI

Description: This activity will enable you to solve real-life problems involving linear functions.

Direction: Consider the situation below and answer the questions that follow.

Emman often rides a taxi from one place to another. The standard fare in riding a taxi is Php 40 as a flag down rate plus Php 3.50 for every 200 meters or a fraction of it.

Complete the table below:

Distance (in meters) x	0	200	400	600	800	1000
Amount (in Php) y						

QUESTIONS ?

1. What is the dependent variable? Explain your answer.
2. What is the independent variable? Explain your answer.
3. Based on the completed table, would the relation represent a line?
4. What is the y-intercept? Explain your answer.
5. What is the slope? Explain your answer.
6. Write the linear function and answer the following questions.
   (a) If Emman rides a taxi from his workplace to the post office with an approximate distance of 600 meters, how much will he pay?

Answers to Activity 19:

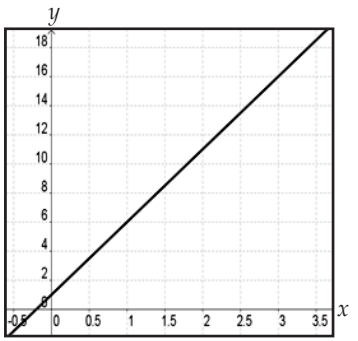
Step 1: The dog's weight is 1 kg at birth. Its weight is 6 kg after a month.

Step 2: The dependent variable is the dog's weight while the independent variable is the time.

Step 3:

x	0	1	2	3
y	1	6	11	16

Step 4:



Step 5: The slope m = 5, y-intercept b = 1. The equation is $y = 5x + 1$.

Teacher's Note and Reminders

(a) If he rides a taxi from his residence to an airport with an approximate distance of 6 kilometers, how much will he pay?
(b) If Emman pays Php 68, how many kilometers did he travel? How about Php 75? Php 89? Php 92.50?

7. Write the equation of the line in the form $Ax + By = C$ using your answer in number 6.
8. Draw the graph of the equation you have formulated in item 7.

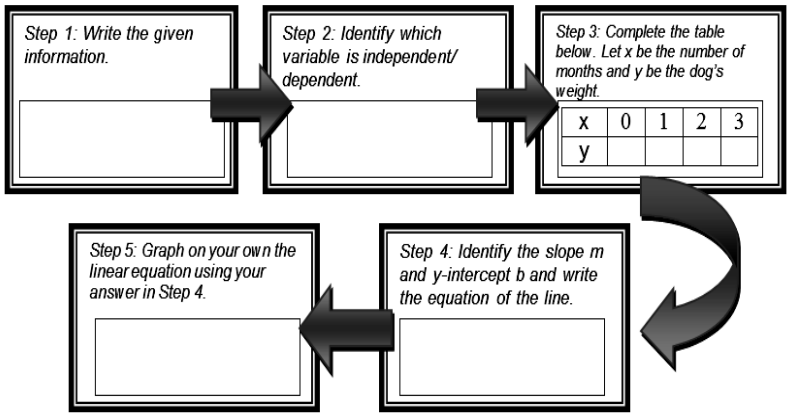
Activity 19 GERMAN SHEPHERD

Description: This activity will enable you to solve problems involving linear functions by following the steps provided.

Direction: Do the activity as directed.

You own a newly-born German shepherd. Suppose the dog weighs 1 kg at birth. You've known from your friend that the monthly average weight gained by the dog is 5 kg. If the rate of increase of dog's weight every month is constant, determine an equation that will describe the dog's weight. Predict the dog's weight after five months using mathematical equation and graphical representation.

Complete the flow chart below then use it to answer the questions that follow.



Ask the students to answer Activity 20. Allow the them to use the flow chart given in Activity 19. In answering item 1 of this activity, emphasize that x must be the time that exceeds after 3 minutes, or consider the domain is $\{x|x \geq 3\}$. This is important because if we fail to do it, the graph of the function is not a line anymore. Give more real-life problems involving linear functions.

Answers to Activity 20:

1. A caller will have to pay Php 10. Let x be the time that exceeds after 3 minutes and let y be the charge. The rule is $y = x + 5$.

2. The formula to be used in solving this problem is $t = \mathrm{d}/\mathrm{r}$ or $t = {}^{1}/\mathrm{r}\ (\mathrm{d})$, where t is time, r is rate and d is distance. Given in this problem are r = 60 kph, which is constant, and d = 240 kilometers. So, the rule in this problem is $t = {}^{1}/_{60}(d)$. If d = 240 kilometers, then t = 4 hours.

3. Let x be the number of donuts sold and let y be the total price. The rule that best describes the function is $y = 18x + 5$. It is assumed that there are 1 to 24 donuts sold; thus, the domain of the relation is the $\{x|1 \leq x \leq 24\}$. There would be 12 donuts in the box whose price is Php 221.

Teacher's Note and Reminders

QUESTIONS ?

1. What equation describes the dog's weight?
2. What method did you use in graphing the linear equation?
3. How will you predict the dog's weight given the rate of change in his weight?

Activity 20 WORD PROBLEMS

Description: This activity will enable you to solve more word problems involving linear functions. In this activity, you are allowed to use the flow chart given in Activity 19.

Direction: Solve the following. Show your solutions and graphs.

1. A pay phone service charges Php 5 for the first three minutes and Php 1 for every minute additional or a fraction thereof. How much will a caller have to pay if his call lasts for 8 minutes? Write a rule that best describes the problem and draw its graph using any method.

2. A motorist drives at a constant rate of 60 kph. If his destination is 240 kilometers away from his starting point, how many hours will it take him to reach the destination? Write a rule that best describes the problem and draw its graph using any method.

3. Jolli Donuts charges Php 18 each for a special doughnut plus a fixed charge of Php 5 for the box which can hold as many as 24 donuts. How many doughnuts would be in a box priced at Php 221? Write a rule that best describes the problem and draw its graph. In your graph, assume that only 1 to 24 doughnuts are sold.

Activity 21 FORMULATE YOUR OWN WORD PROBLEM!

Description: This activity will enable you to formulate your own word problem involving linear functions and to answer it with or without using the 5-step procedure.

Direction: Formulate a word problem involving linear functions then solve. You may or may not use the flow chart to solve the problem. Be guided by the given rubric found in the next page.

Let the students formulate real-life problems involving linear function in Activity 21. This activity can be done by groups of five members each. Allow students to use the 5-step procedure of the flow chart provided in the previous activity.

Teacher's Note and Reminders

This activity is a Scaffold Level 3 of the transfer task. A little of your guidance is important in order for students' to be ready to perform the final task in Activity 24.

QUESTIONS ?

Did you encounter any difficulty in formulating real-life problem involving linear functions? Explain your answer.

RUBRIC: PROBLEMS FORMULATED AND SOLVED

Score	Descriptors
6	Poses a more complex problem with 2 or more correct possible solutions and communicates ideas clearly; shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate.
5	Poses a more complex problem and finishes all significant parts of the solution and communicates ideas clearly; shows in-depth comprehension of the pertinent concepts and/or processes.
4	Poses a complex problem and finishes all significant parts of the solution and communicates ideas clearly; shows in-depth comprehension of the pertinent concepts and/or processes.
3	Poses a complex problem and finishes most significant parts of the solution and communicates ideas clearly; shows comprehension of major concepts although neglects or misinterprets less significant ideas or details.
2	Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension.
1	Poses a problem but demonstrates minor comprehension, not being able to develop an approach.

Activity 22 YOU ARE THE SCHOOL PRINCIPAL

This is a preparatory activity which will lead you to perform well the transfer task in the next activity. This can be a group work.

Situation:

You are the school principal of a certain school. Every week you conduct an information drive on the different issues or concerns in your school through announcements during flag ceremony or flag retreat or during meetings with the department heads and teachers. For this week, you noticed that water consumption is high. You will make and present an informative leaflet with design to the members of the academic community. In your leaflet design, you must clearly show water bill and water consumption and how these two quantities relate each other. The leaflet must also reflect data on the quantity of water bill for the previous five months, and a detailed mathematical computation and a graphical presentation that will aid in predicting the amount of water bill that the school will pay.

Elicit students' present knowledge of Linear Functions by answering the "Final Answer' column in the IRF Worksheet. Compare their final answers to their initial and revised answers.
Before giving the transfer task, ask first the students if they have realizations about the topic. Also, ask them the question: What new connections have you made for yourself? Then say, now that you have a deeper understanding of the topic, you are ready to do the task in the next section.

Teacher's Note and Reminders

What to Transfer

Allow the students to perform Activity 24 without your assistance, if possible. Rate their performance based on the criteria of the rubric provided.

recommendations to save water. Your leaflet as a whole will be assessed using the following criteria: use of appropriate mathematical concepts and accuracy, organization, quality of presentation, and practicality of recommendations.

Activity 23 IRF WORKSHEET REVISITED

Description: Below is the IRF Worksheet in which you will write your present knowledge about the concept.

Direction: Complete IRF sheet below.

Questions	Initial Answer	Revised Answer	Final Answer
1. What is linear function?			
2. How do you describe a linear function?			
3. How do you graph a linear function?			
4. How do you find an equation of the line?			
5. How can the value of a quantity given the rate of change be predicted?			

What new realizations do you have about the topic? What new connections have you made for yourself? Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

Teacher's Note and Reminders

Activity 24 YOU ARE A BARANGAY COUNCILOR

This activity is the transfer task. You have to perform this in your own community.

Situation:

You are a *barangay* councilor in San Sebastian. Every month, you conduct information drive on the different issues that concern every member in the community. For the next month, your focus is on electricity consumption of every household. You are tasked to prepare a leaflet design which will clearly explain about electricity bill and consumption. You are to include recommendations to save water. You are expected to orally present your design to the other officials in your *barangay*. Your output will be assessed according to the rubric below.

RUBRIC: LEAFLET DESIGN

CRITERIA	Exemplary 4	Satisfactory 3	Developing 2	Beginning 1
Use of mathematical concepts and accuracy	The mathematical concepts used are correct and the computations are accurate. Brief explanation is provided.	The mathematical concepts used are correct and the computations are accurate.	The mathematical concepts used are correct but the computations are inaccurate.	The mathematical concepts used are wrong and the computations are inaccurate.
Organization	The ideas and facts are complete, orderly presented, and well prepared.	The ideas and facts are completely orderly presented.	The ideas and facts are mostly orderly presented.	The ideas and facts are not well presented.
Quality of presentation	The presentation uses appropriate and creative visual designs.	The presentation uses appropriate visual designs.	The presentation uses some visual designs which are inappropriate.	The presentation does not include any visual design/s.
Practicality of recommendations	The recommendations are sensible, doable and new to the community.	The recommendations are sensible and doable.	Some recommendations are sensible and doable.	The recommendations are insensible and undoable.

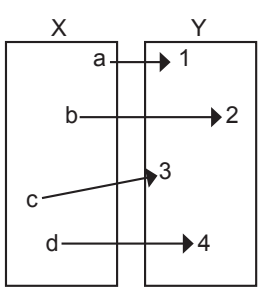
You have just completed this lesson. Before you go to the next lesson, you have to answer the post-assessment.

POST-ASSESSMENT

1. What is abscissa?
   a. It is a y-coordinate.
   b. It is a x-coordinate.
   c. It is a point on the xy-plane.
   d. It divides the plane into four regions called quadrant.

2. Which best describes the point (3, -4)?
   a. It is 4 units above the x-axis and 3 units to the left of the y-axis.
   b. It is 4 units below the x-axis and 3 units to the left of the y-axis.
   c. It is 4 units above the x-axis and 3 units to the right of the y-axis.
   d. It is 4 units below the x-axis and 3 units to the right of the y-axis.

3. Which relation below does NOT define a function?

a.


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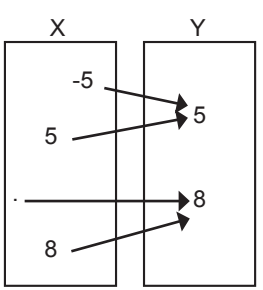

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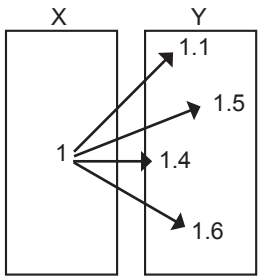

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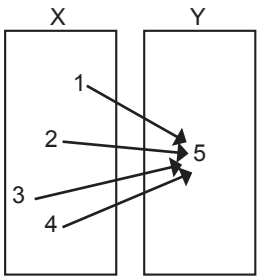
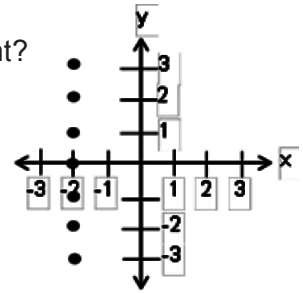


4. What is the range of the relation at the right?

a. $\{x|-3 \le x \le 3, x \in \Re\}$
b. $\{x|-3 < x < 3, x \in \Re\}$
c. $\{x|-3 \le x \le 3, x \in Z\}$
d. $\{x|-3 < x < 3, x \in Z\}$



5. The correct table of the function f defined by f(x) = 3x + 1 is

a.

x	y
-2	-5
0	1
2	7
4	13
6	19

b.

x	y
-2	-5
-1	1
0	7
1	13
2	19

c.

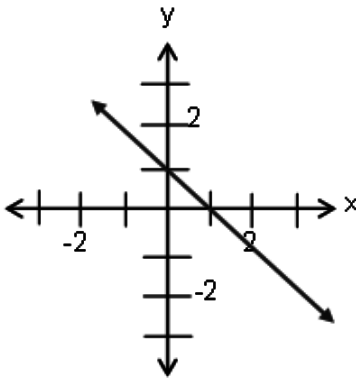
x	y
-2	-6
0	-3
2	0
4	3
6	6

d.

x	y
-2	-5
-1	-3
0	0
1	3
2	6

6. What is the equation of the line at the right?

a. $x + y = 1$
b. $x - y = 1$
c. $2x + y = 1$
d. $2x - y = 1$



7. Find the equation of the line passing through the point (-3, 5) and whose slope is 2?

a. $y = 2x - 1$
b. $y = 2x + 2$
c. $y = 2x + 8$
d. $y = 2x + 11$

8. Three steps to rewrite $3x - 4y = 7$ into $y = mx + b$ are shown below.

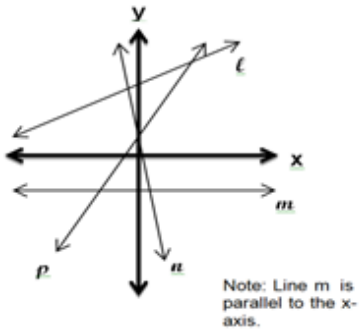
I.	$y = \frac{3}{4}x - \frac{7}{4}$
II.	$-\frac{1}{4}(-4y) = -\frac{1}{4}(-3x + 7)$
III.	$-4y = -3x + 7$

What is the correct order of these steps?

a. II, III, I
b. I, II, III
c. III, II, I
d. II, I, III

9. Which line in the figure at the right has a slope of zero?

a. line l
b. line m
c. line n
d. line p



10. What will happen to the value of y in the equation $2x + 3y = 12$ when the value of x decreases?

a. The value of y will increase.
b. The value of y will decrease.
c. The value of y will not change.
d. The value of y cannot be determined.

11. John rode a taxi from a bus terminal to JB Mall whose distance is approximately four kilometers. After riding, he paid an amount of ₱110. Which variable is dependent?

a. taxi riding
b. the amount paid
c. the distance travelled
d. the person riding the taxi

For item numbers 12 and 13, refer to the situation below.

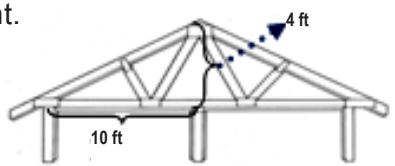
The height h of the candle in centimeters is a function of time t in hours it has been burning. It is described by the table below:

t	0	1	2	3	4	5
$h(t)$	10	8	6	4	2	0

12. Write the linear function h described by the table above?
    a. $h(t) = 2t - 10$ c. $h(t) = 10 - 2t$
    v. $h(t) = 2t + 10$ d. $h(t) = 10 - t$

13. How long will it take the candle be completely melted?
    a. 3
    b. 4
    c. 5
    d. 6

14. Find the slope of the roof indicated at the right.
    a. $^4/_5$
    b. $^5/_4$
    c. $^2/_5$
    d. $^5/_2$



For items 15 to 18, refer to the situation below:

Jose, who is the SSG Business Manager, was given the task by the SSG President to canvass for a tarpaulin printing. He knew that in printing ad, the charge of tarpaulin printing is Php 12 per square foot and Php 100 for the layouting.

15. Which of the following equations best represents the total cost y with x number of square feet including layouting fee?

a. $y = 12x - 100$
b. $y = 12x + 100$
c. $y = 100x - 12$
d. $y = 100x + 12$

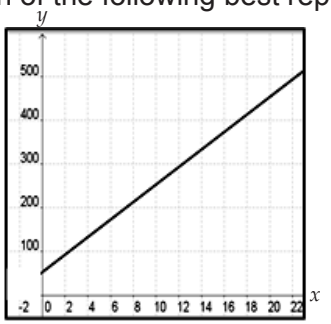
16. What qualities you must look into in tarpaulin printing?

I. The printing and layouting cost
II. The quality of the printing output
III. The brand of the PC used in layouting
IV. The quality of the layout artist's output

a. I and II only
b. I, II and III only
c. I, II and IV only
d. I, II, III and IV

17. Which of the following best represents the relationship of the total cost y and the x number of square feet?

a.



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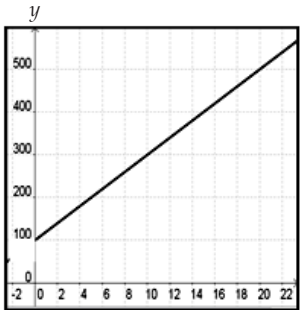
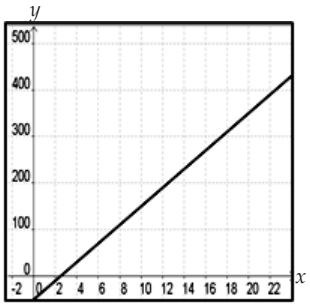
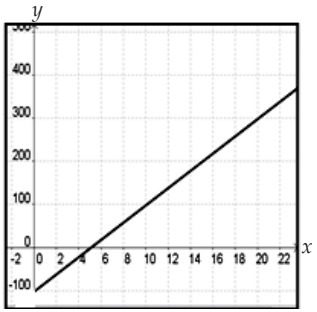


b.



d.



18. The SSG President told Jose that the dimensions of the tarpaulin are 5 feet by 4 feet. How many square feet is the tarpaulin? How much should Jose pay for the printing ad?
    a. 20 square feet; Php 420
    b. 20 square feet; Php 340
    c. 9 square feet; Php 320
    d. 9 square feet; Php 208

For items 19 to 20, refer to the situation below.

> In a certain barangay, you are elected as the "*Punong Barangay*." Hon. Bacus, who is a councilor, was assigned as the chairman of Committee on Energy. You gave him a task to make a Powerpoint presentation illustrating the relationship between electric bill and power consumption and to provide recommendations and friendly reminders to help minimize energy consumption.

19. As a "*Punong Barangay*," what criteria should you consider to assess Hon. Bacus' PowerPoint presentation to ensure good quality of the delivery of presentation?
    I. colors and attractiveness
    II. content and delivery
    III. layout and design
    IV. font and font size used in the texts

    a. I only
    b. II and III only
    c. III and IV only
    d. II, III and IV

20. If Hon. Lapuz has to choose one best representation of the relationship between electric bill and power consumption in his powerpoint presentation, what do you expect John should use to present his ideas in the clearest way?
    a. graph
    b. table
    c. mapping diagram
    d. rule or equation

Answer Key:

1.	**B**	**6.**	**A**	**11.**	**B**	**16.**	**C**
2.	**B**	**7.**	**D**	**12.**	**C**	**17.**	**C**
3.	**B**	**8.**	**C**	**13.**	**C**	**18.**	**B**
4.	**C**	**9.**	**B**	**14.**	**C**	**19.**	**D**
5.	**A**	**10.**	**A**	**15.**	**B**	**20.**	**A**

SUMMARY/SYNTHESIS/GENERALIZATION

This module was about relations and functions. It involved three lessons, namely: Rectangular Coordinate System, Representations of Relations and Functions, and Linear Functions.

In the first lesson, students were expected to properly plot points in Cartesian plane and apply this to real life. In the second lesson, students were exposed to the different types of representing relations and functions. They were able to differentiate a function from a relation.

Finally, in the last lesson, students were expected to solve, graph, and write in different ways a linear function. More importantly, they were given the chance to formulate real-life problems, solve these using a variety of strategies, and demonstrate their understanding of the lesson by doing some practical tasks.

GLOSSARY

Cartesian plane Also known as the Rectangular Coordinate System which is composed of two perpendicular number lines (vertical and horizontal) that meet at the point of origin (0, 0).

degree of a function f The highest exponent of x that occurs in the function f.

dependent variable The variable (usually) y that depends on the value of the independent variable (usually) x.

domain of the relation The set of first coordinates of the ordered pairs.

function A relation in which each element in the domain is mapped to exactly one element in the range.

function notation A notation in which a function is written in the form $f(x)$ in terms of x.

horizontal line A line parallel to the x-axis.

independent variable The variable (usually) x that controls the value of the dependent variable (usually) y.

line A straight line in Euclidean Geometry.

Linear Function A function of first degree in the form $f(x) = mx + b$, where m is the slope and b is the y-intercept.

mapping diagram A representation of a relation in which every element in the domain corresponds to one or more elements in the range.

mathematical phrase An algebraic expression that combines numbers and/or variables using mathematical operators.

ordered pair A representation of point in the form (x, y).

point-slope form The linear equation $y - y_1 = m(x - x_1)$ is the point-slope form, where m is the slope and x_1 and y_1 are coordinates of the fixed point.

quadrants The four regions of the xy-plane separated by the x- and y-axes.

range of the relation The set of second coordinates of the ordered pairs.

rate of change The slope m of the line and is the quotient of change in y-coordinate and the change in x-coordinate.

Rectangular Coordinate System Also known as Cartesian plane or xy-plane

relation Any set of ordered pairs.

slope of a line Refers to the steepness of a line which can be solved using the formulae:
$m = \frac{rise}{run}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope-intercept form The linear equation $y = mx + b$ is in slope-intercept form, where m is the slope and b is the y-intercept.

standard form The linear equation in the form $Ax + By = C$, where A, B and C are real numbers.

trend Tells whether the line is increasing or decreasing and can be determined using the value of m (or slope).

two-point form The linear equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the two-point form, where x_1 and y_1 are coordinates of the first point while x_2 and y_2 are coordinates of the second point.

vertical line A line parallel to the y-axis.

Vertical Line Test If every vertical line intersects the graph no more than once, the graph represents a function.

x-axis The horizontal axis of the Cartesian plane.

x-intercept The x-coordinate of the point at which the graph intersects the x-axis.

y-axis The vertical axis of the Cartesian plane.

y-intercept The y-coordinate of the point at which the graph intersects the y-axis.

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http://www.purplemath.com/modules/fcns.htm
http://www.teachbuzz.com/lessons/graphing-functions
http://www.webgraphing.com/
http://www.youtube.com/watch?NR=1&v=uJyx8eAHazo&feature=endscreen
http://www.youtube.com/watch?v=EbuRufY41pc&feature=related
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http://www.youtube.com/watch?v=QIp3zMTTACE
http://www.youtube.com/watch?v=-xvD-n4FOJQ&feature=endscreen&NR=1
http://www.youtube.com/watch?v=UgtMbCI4G_I&feature=related
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