# TEACHING GUIDE

**Module 6: Reasons Behind Reasoning**

## A. Learning Outcomes

**Content Standard:**
The learner demonstrates understanding of key concepts of axiomatic development of geometry.

**Performance Standard:**
The learner is able to communicate mathematical thinking with coherence and clarity in formulating, investigating, analyzing and solving real life problems.

## UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="2">SUBJECT:<br>Grade 8 Mathematics<br>QUARTER:<br>Third Quarter<br>STRAND:<br>Geometry<br>TOPIC:<br>Reasoning and Proofs<br>LESSONS:<br>1.If-then Statement<br>2.Inductive and Deductive Reasoning<br>3.Writing Proofs</td><td colspan="2">LEARNING COMPETENCIES<br>1. Identify the hypothesis and conclusions of if-then and other types of statements.<br>2. Formulate the inverse, converse, and contrapositive of an implication.<br>3. Distinguish between inductive and deductive reasoning.<br>4. Provide formal arguments that explain results of phenomenon or a situation.<br>5. Write formal arguments as a series of statements that make up a proof (both direct and indirect).<br>6. Explain the need for defined terms previously introduced.<br>7. Differentiate between postulate and theorem.</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:<br>Logic and reasoning are tools in geometry to facilitate mathematical thinking for making valid conclusions.<br><br>The use of inductive and/or deductive reasoning depends on the given situation.</td><td>ESSENTIAL QUESTION:<br>How do you establish valid conclusions?<br><br>When do we use inductive and/or deductive reasoning?</td></tr>
<tr><td colspan="3">TRANSFER GOAL:<br>Students will on their own make valid conclusion and communicate mathematical thinking with coherence and clarity on a real-life situation.</td></tr>
</table>

## B. Planning for Assessment

### Product/Performance

The following are products and performances that students are expected to accomplish with in this module.

### Assessment Map

| TYPE | KNOWLEDGE AND PROCESS/ SKILLS (ACQUISITION) | UNDERSTANDING (MEANING MAKING) | TRANSFER |
|---|---|---|---|
| **PRE-ASSESSMENT/ DIAGNOSTIC** | Pre–test | Pre–test<br>Inbox - Outbox Sheet<br>Interpretation, Explanation | Pre–test |
| **FORMATIVE ASSESSMENT** | Written Exercises | Interpretation, Explanation, Perspective | Picture Translation Sheet |
| | | Written Exercises<br>Interpretation, Explanation | Case Solved! Activity Sheet |
| | Quiz | Interpretation, Explanation, Perspective | |
| | | Investigating Cases<br>Interpretation, Explanation | |
| | | Prove it!<br>Interpretation, Explanation | |
| | | REAL Proving<br>Interpretation, Explanation | |
| | | One – minute essay<br>Explanation,<br>Self – knowledge | |
| | | Self – knowledge | Rubric on the Article in newspaper |

| | | | |
|---|---|---|---|
| **SUMMATIVE ASSESSMENT** | | Inbox – Outbox Sheet Interpretation, Explanation | Rubric on the Performance task |
| | Post Test | Post Test | Post Test |
| **SELF-ASSESSMENT** | | | Synthesis Journal |

**Assessment Matrix (Summative Test)**

| Levels of Assessment | What will I assess? | How will I assess? | How Will I Score? |
|---|---|---|---|
| Knowledge 15% | The learner demonstrates understanding of key concepts of axiomatic development of geometry. Identify the hypothesis and conclusions of if-then and other types of statements. Formulate the inverse, converse, and contrapositive of an implication. Distinguishes between inductive and deductive reasoning. Provides formal arguments that explain results of a phenomenon or a situation Write formal arguments as a series of statements that make up a proof (both direct and indirect). | Paper and Pencil Test | 1 point for every correct response |
| Process/Skills 25% | | Paper and Pencil Test | 1 point for every correct response |
| Understanding 30% | | Journal writting<br><br>Portfolio | Rubric<br><br>Criteria:<br>Relevance<br>Creativity<br>Insightful |
| Product 30% | The learner is able to formulate/ solve real-life problems involving reasoning. | **GRASPS Assessment**<br>Make a mathematical investigation that will enlighten the readers by providing valid conclusions.<br><br>Apply your understanding of the key concepts of reasoning. | Rubric on Mathematical Investigation<br><br>Criteria:<br>The written output of the investigation shall be evaluated based on to its coherence, clarity, judgment, and mathematical reasoning. |

## C. Planning for Teaching-Learning

**Introduction**:

The learner demonstrates understanding of the key concepts of axiomatic development of geometry. It consists of three lessons namely:

Lesson 1 – If- then Statements
Lesson 2 – Inductive and Deductive Reasoning
Lesson 3 – Writing Proof

As an introduction to the main lesson, ask them the following questions: Is it possible to make a valid conclusion without even going through the process of investigation? What will you do if you were asked to make a decision that will affect many people? Many aspects in our life involve decisions and proof.

In all lessons, students are given the opportunity to use their prior knowledge and skills in learning proofs in geometry.

They are also given varied activities to process the knowledge and skills learned and to deepen and transfer their understanding of the different lessons.

*Have you ever asked yourself how* do you establish valid conclusions?
When do we use inductive and/or deductive reasoning?

Entice the students to find out the answers to these questions and to determine the vast applications of proofs through this module.

**Objectives:**

**After the learners have gone through the lessons contained in this module, they are expected to:**

- Identify the hypothesis and conclusions of if-then and other types of statements.
- Formulate the inverse, converse and contrapositive of an implication.
- Distinguish between inductive and deductive reasoning.
- Provide formal arguments that explain results of a phenomenon or a situation.
- Use syllogism in writing formal arguments as a series of statements that make up a proof.
- Explain the need and importance for defined terms previously learned.
- Differentiate between postulate and theorem.

**Pre-Assessment:**

**Choose the letter of the correct answer**

1. Which of the following best describes deductive reasoning?
   a. using logic to draw conclusions based on accepted statements
   b. accepting the meaning of a term without definition
   c. defining mathematical terms in relation with physical objects
   d. inferring a general truth by examining a number of specific examples
   **Answer: A**

2. Theorem: A triangle has at most one obtuse angle.
   Francisco is proving the theorem above by contradiction. He began by assuming that in $\Delta ABC$, $\angle A$ and $\angle B$ are both obtuse. Which theorem will Francisco use to reach a contradiction?
   a. If two angles of a triangle are congruent, the sides opposite the angles are congruent.
   b. If two supplementary angles are congruent, each angle measures 90°.
   c. The largest angle in a triangle is opposite the longest side.
   d. The sum of the measures of the angles of a triangle is 180°.
   **Answer: D**

3. If $m\angle R + m\angle M = 90°$ then
   a. $\angle R \cong \angle M$.
   b. $\angle R$ and $\angle M$ are right angles.
   c. $\angle R$ and $\angle M$ are complementary.
   d. $\angle R$ and $\angle M$ are supplementary.
   **Answer: C**

4. The converse of the statement:" if you are in love then you are inspired", is,
   a. If you are not in love, then you are not inspired.
   b. If you are inspired, then you are in love.
   c. If you are not inspired, then you are not in love.
   d. if you are in love, you are not inspired.
   **Answer: B**

5. The if-then form of the statement: "Parallel lines never intersect", is:
   a. If two lines intersect, then they are parallel.
   b. If two lines are parallel, then they never intersect.
   c. if two lines are not parallel then they intersect.
   d. If two lines intersect, then they are not parallel.
   **Answer: B**

6. What is the inverse of the statement,:" If the number is divisible by 2 and 3, then it is divisible by 6".
   a. If the number is divisible by 6, then it is divisible by 2 and 3.
   b. If the number is not divisible by 2 and 3, then it is not divisible by 6.
   c. If the number is not divisible by 6, then it is not divisible by 2 and 3.
   d. If the number is divisible by 2 and 3, then it is not divisible by 6.
   **Answer: B**

7. What property is illustrated in : If $\angle A \cong \angle B$, $\angle B \cong \angle C$ then $\angle A \cong \angle C$.
   a. Reflexive Property
   b. Symmetric Property
   c. Transitive Property
   d. Addition Property
   **Answer: C**

8. Using the distributive property,
   $4\ (a + b) =$ _____.
   a. $4a + b$
   b. $B + 4a$
   c. $4a + 4b$
   d. $4 + a + b$
   **Answer: C**

9. Supply a valid conclusion for the given hypothesis: if $\overrightarrow{OM}$ bisects $\angle LON$ then
   a. $\angle LOM \cong \angle NOM$
   b. $\angle LOM \cong \angle LON$

c $\angle MON \cong \angle NOL$
d. $m\angle LON = m\angle LOM + m\angle MON$
**Answer: A**

10. The method of proof by contradiction is:
    a. direct proof
    b. formal proof
    c. indirect proof
    d. two column proof
    **Answer: c**

11. <u>If garbage are disposed properly</u> then dengue diseases will be prevented. What is the underlined portion called in the conditional statement?
    a. the conclusion
    b. the hypothesis
    c. the argument
    d. the converse
    **Answer: B**

12. How many dots are there in the three figure?

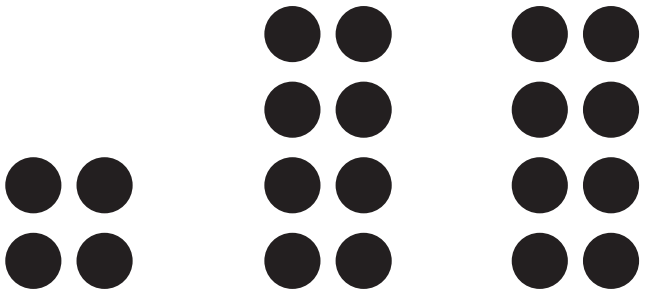


Figure 1 Figure 2 Figure 3

**Answer: 20**

13. Which of the following statements is true?
    a. If ∠1 has a measure of 90∠1,then ∠1 is obtuse.
    b. If ∠1 has a measure of 140°, then ∠1 is acute.
    c. If ∠1 has a measure of 35°, then ∠1 is acute.
    d. If ∠1 has a measure of 180°,then ∠1 is right.
    **Answer: C**

14. Which of the following statements is false?
    a. Any four non-collinear points lie in a distinct plane.
    b. A plane contains at least 3 non-collinear points.
    c. Any two lines intersect at a point.
    d. Through two given points we can draw three lines.
    **Answer: D**

15. Rewrite the statement in if-then form.
    a. A figure has four sides if and only if it is a quadrilateral.
    b. If a figure is a quadrilateral, then it has four sides.
    c. If a figure has four sides, then it is a quadrilateral.
    d. A figure is a quadrilateral if and only if it has four sides.
    **Answer: B**

16 Name the property which justifies the following conclusion.

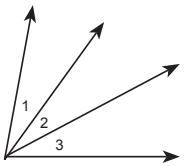
Given : JB = 28
Conclusion : JB + 4 = 32

    a. Addition property of equality
    b. Multiplication property of equality
    c. Substitution property of equality
    d. Transitive property of equality
    **Answer: A**

For 17-20 Give the answer.

17. $\angle 1$ and $\angle 2$ are complementary angles. $\angle 1$ and $\angle 3$ are vertical angles
If $m\angle 3 = 49°$, find $m\angle 2$.
**Answer: $m\angle 2 = 4$**

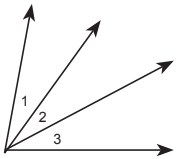
18. What is the missing reason in the following proof?
$m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 3$




**Answer: Transitive Property**

| Statement | Reason |
|---|---|
| 1. $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 3$ | 1. given |
| 2. $m\angle 2 = m\angle 3$ | 2. ___?____ |

19. Supply the missing statement in the following proof.




Given: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ **Answer: $m\angle 2 = m\angle 2$**
Prove; $m\angle 1 = m\angle 3$

| Statement | Reason |
|---|---|
| 1. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ | 1. given |
| 2. ----------?---------- | 2. Reflexive Property |
| 3. $m\angle 1 = m\angle 3$ | 3. Subtraction property |

20. What conclusion can be logically deduced based on the following statements?
If you are catholic, you are against the RH Bill
Mrs. Romano is a Catholic.
**Answer: Mrs. Romano is against RH Bill**

**What to Know**

Ask the students to accomplish the activity sheet called INBOX – OUTBOX sheet.

## *Teacher's Note and Reminders*


# Lesson 1 If- then Statements

**What to Know**

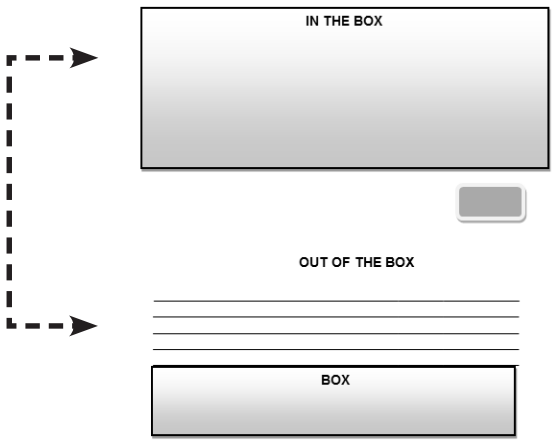
Let's begin this lesson by accomplishing the activity sheet below called INBOX – OUTBOX sheet.

### Activity 1 INBOX – OUTBOX SHEET

**Description:** This activity is intended to elicit your prior knowledge regarding the lesson.

**Direction:** Answer the question below and write your answer in the space provided IN THE BOX.

How do you make valid conclusions if faced with problems in life such as having failing grades, meeting deadlines and even in love- life troubles?



The students gave their initial ideas on how to make sound judgment and how useful it was. Now let them find out the other answers by doing the next part. What they will learn in the next sections will also enable them to do the final project which involves investigating mathematical concepts.

## *Teacher's Note and Reminders*


## Activity 2 ARTICLE ANALYSIS

**Description:** There are things in life which involve decision-making. Find out how valid decision making affects our life. The given article below deals with the effect of having or giving misguided conclusion.

**Direction:** Read the excerpts on the article from Bombo Radio Philippines entitled, "*Judge sinibak ng SC due to wrong decisions*" then answer the follow – up questions below.

**Judge sinibak ng SC due to wrong decisions**

Revoked of license by the Supreme Court (SC) is a judge in Cotabato City because of the issuance of a decision in the case of annulment of marriage without conducting a hearing.

In the per curiam decision of the Supreme Court en banc discharged on May 15, 2012, proven guilty of gross misconduct and dishonesty is Judge Cader Indar Al Haj, presiding judge of the Regional Trial Court (RTC), Branch 14, Cotabato City and also serve as acting presiding judge of RTC, Branch 15, Shariff Aguak, Maguindanao.

Along with its disbarment Indar was removed from the service and also the benefits that he should get from his retirement except leave credits.

Also stated in the decision was Indar can't work in any office of the government owned including any government owned and controlled corporations.

According to the court, Indar violated the Canons 1 and 7, as well as Rule 1.01 of the Code of Professional Responsibility, thus, also removing his name on the roll of attorneys.

The penalty of Indar stemmed from a report sent by the LCR of Manila and Quezon City to the Office of Court Administration (OCA) in relation to the decisions resolutions and orders in the marriage annulment issued by the same judge.

*http://www.bomboradyo.com/story-of-the-day/111056-judge-sinibak-ng-sc-dahil-sa-maling-desisyon*

This site provided the article above entitled "Judge sinibak ng SC due to wrong decisions"

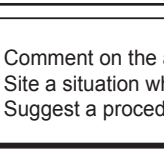
paki check po ito ung sa LM unti lang sa TG madami

## What to Process

The goal in this section is to learn and understand key concepts of reasoning and proving. The students will be dealing with If-then statement, deductive and inductive reasoning and writing proofs.
Let them do Activity 3.

Allow students to write something about the statements in their journal.
Tell them that they have just encountered conditional statements or the If-then statements. Let them read some notes about an if-then statement.

## *Teacher's Note and Reminders*


1. Comment on the article.
2. Site a situation where important decision making is needed.
3. Suggest a procedure on how to make a wise decision.

*You gave your initial ideas on how to make sound judgment and how useful it was. Let's now find out the other answers by doing the next part. What have you learned. In the next sections will also enable you to do the final project which involves investigating mathematical concepts.*

## What to Process

Your goal in this section is to learn and understand key concepts of reasoning and proving. You will be dealing with If-Then statement, Deductive and Inductive reasoning and writing proofs.

## Activity 3 JUDGE US!

**Description:** A lot in the statements that we encounter are logically constructed or written but NOT valid or acceptable. This activity deals with determining which statement is valid or not.

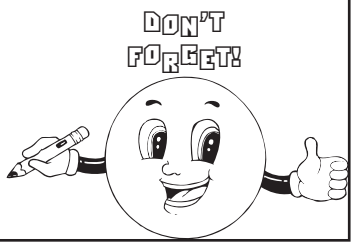
**Direction:** From the given statements tell whether the statement is valid or not.

1. Students who are good in mathematics are smart.
   Enchong is smart, then he is good in mathematics.
2. Young actresses are health conscious.
   Kim is a young actress then she is health conscious.
3. If it rains then the sports fest will be cancelled.
   It rains therefore the sports fest is cancelled.
4. If the lines are parallel they do not intersect
   Line $x$ and line $y$ do not intersect; therefore they, are parallel.
5. If two angles are right angles ,then they are congruent.
   $\angle A$ and $\angle B$ are congruent, then they are right angles.

For items 6 to 10 complete the statement and justify your answer

6 . Miss Earth candidates are environmentalists.
   Miss Jaybee is a candidate to the Miss Earth search, therefore
7. If you are at SM you got it all. Marie is at SM then

## Teacher's Note and Reminders




8. If you bank with BDO they find ways. Vincent has deposit at BDO then
9. If you drink coke you find happiness. Jay is drinking coke then…
10. Globe connects people. Dedeth is using globe simcard then..

QUESTIONS ?

a. What have you noticed about the statements given above?
b. Take one of the statements and tell something about it
c. .What is common to all of the statements?

Write your answers in your journal and have a small discussion with your group.

*You have just encountered conditional statements or the **If-then statements**.*

An *if-then* statement is composed of two clauses: the if- clause and the then- clause. We can denote a letter for each clause, $p$ for the if clause and $q$ for the then clause. The statement is in the form" If $p$ then $q$. Conditional statements are formed by joining two statements $p$ and $q$ using the words if and then. The $p$ statement is called the hypothesis and the $q$ statement is the conclusion.

A simple flow of reasoning from if-clause to the then-clause is called simple **implication**.

*There are some conditional statements not written in this form but you can rewrite them using the if-then form. How will you identify the hypothesis and the conclusion? You try this:*

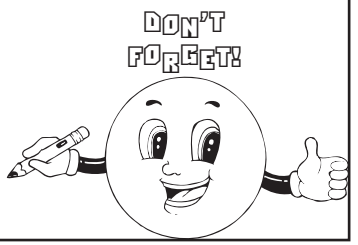
1. Cigarette smoking is dangerous to your health.
   If-then form _____
   Hypothesis_____
   Conclusion_____

2. It is more fun in the Philippines.
   If-then form _____
   Hypothesis_____
   Conclusion_____

3. A segment has exactly one midpoint.
   If-then form _____
   Hypothesis_____
   Conclusion_____

**Answer Key**
**Exercise 1**
1. If the polygon is a rectangle then the opposite sides are parallel
2. If you are Filipino then you are God-fearing people.
3. If two angles are complementary then the sum is 90°.
4. If Paolo is a good citizen then he obeys rules and regulations.
5. If the polygon is a triangle then it has three sides.

## *Teacher's Note and Reminders*




4. Angles in a linear pair are supplementary.
   If-then form _____
   Hypothesis _____
   Conclusion _____

5. Vertical angles are congruent.
   If-then form _____
   Hypothesis _____
   Conclusion _____

How do you distinguish the hypothesis from the conclusion when the statement is not in the if-then form? See the examples below.

1. National Disaster Risk Reduction Council volunteers are busy during calamities.
2. An eighteen year old Filipino can cast his/her vote during election.
3. All right angles are congruent.
4. Three non-collinear points **determine a plane**.
5. Perpendicular lines are **intersecting lines**.

Discuss with a partner the underlined part of the sentence and the one in bold letters. What part of the sentence are the underlined words ? What part of the sentence are in bold letters? Which is the hypothesis and which is the conclusion? Rewrite the statements to if-then form.

*Now that you know what conditional statements are, and you can identify the hypothesis and the conclusion, have more practice in answering the exercises below.*

**Exercise 1**

Convert each statement in if-then form, then identify the hypothesis and the conclusion.

1. Opposite sides of a rectangle are parallel.
2. Filipinos are God-fearing people.
3. The sum of the measures of complementary angles is 90°.
4. Good citizens obey rules and regulations.
5. A triangle is a polygon of three sides.
6. A quadrilateral has four sides.
7. Two points determine a line.
8. The intersection of two lines is a point.
9. Two intersecting lines line in one plane.
10. The sum of the angles forming a linear pair is 180°

*Now that you are well- versed in converting conditional statement to if-then form, you can easily identify the hypothesis and the conclusion. When do you say that the implication is true or false?*

## Teacher's Note and Reminders


The implication $p \rightarrow q$ is always true except in the case that $p$ is true and $q$ is false. See the truth table for implication below.

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

*This time let us make another statement from the given one. Let us do the activity.*

### Activity 4 JUMBLED WORDS

**Direction:** Make a sentence from the jumbled words.

**POLYGON TRIANGLE A IS A**

1. A TRIANGLE IS A POLYGON.
2. A POLYGON IS A TRIANGLE.

Let's take the #1 sentence as our first statement. First we can convert it to if-then form, then we can form its converse, inverse, and contrapositive.

Study the table below .

| Statement | If-then form | Converse | Inverse | Contrapositive |
|---|---|---|---|---|
| • A triangle is a polygon. | If an object is a triangle, then it is a polygon | If an object is a polygon , then it is a triangle. | If an object is not a triangle' then it is not a polygon. | If an object is not a polygon, then it is not a triangle. |

Discuss with your group how the converse is written? Inverse? and contrapositive of a given statement.

If $p$ is ; If the object is a triangle
$q$ is : then it is a polygon

What happen to $p$ and $q$ in the converse?
Compare the inverse and the original statement. What did you do with $p$? what did you do with $q$?

**Answer Key**
**Exercise 2**

A. Converse: If the angles have the same measure then they are congruent.
Inverse: If two angles are not congruent then they do not have the same measure
Contra positive: If two angles do not have the same measure then they are not congruent.

B. 1. If three points are not collinear, then they determine a plane.
2. If an object is a rectangle, then it has four right angles.
3. If two lines are perpendicular, then they intersect each other.

## *Teacher's Note and Reminders*


Observe the changes in the contrapositive.
Summarize your observation in terms of $p$ and $q$.

Let's take another statement: **An even number is divisible by two.**

If-then form _____
Converse _____
Inverse _____
Contrapositive _____

We can summarize how to convert the statement in terms of $p$ and $q$. See the table below.

| Statement | If $p$, then $q$ |
|---|---|
| Converse | If $q$, then $p$ |
| Inverse | If not $p$, then not $q$ |
| Contrapositive | If not $q$, then not $p$ |

**Exercise 2**

A. Fill up the table below.

| Statement | If two angles are congruent, then they have the same measure. |
|---|---|
| Converse | |
| Inverse | |
| Contrapositive. | |

B. State the converse of the following statements:
1. Three non-collinear points determine a plane.
2. A rectangle has four right angles.
3. Perpendicular lines intersect.

Go back to Activity 4 .

If $p$, then $q$ : If an object is a triangle then it is a polygon.

Converse
If $q$, then $p$: If an object is a polygon then it is a triangle.
Analyze the converse. Is it true? If not, give a counter example.

The converse is false because square is a a polygon. It is not a triangle.

Now that the students can identify the hypothesis and the conclusion in the if-then statement, and form its converse, inverse, and contrapositive, they are now ready to study the kinds of reasoning in the next section.

## *Teacher's Note and Reminders*


If $p$, then $q$: If a number is even then it is divisible by two.
If $q$, then $p$: If a number is divisible by two then it is even.

The converse is true.

Try to analyze the converse of the statements in B.
So what can you conclude about the converse of a statement?
Is the converse of a given statement always true?

### Activity 5 "PICTURE ME"

Observe the set of pictures. Translate the pictures into conditional statements. State the converse, inverse and contrapositive of the conditional statements.

Classify each as true or false and justify.

Go to other group, share each other's answers and come up with a common conclusion.

*Now that you can identify the hypothesis and the conclusion in the if-then statement, and form its converse, you are now ready to study the kinds of reasoning in the next section .*

Let them discuss with their group their answers to these questions.

- How did you arrive at your answer?
- Did you agree at once on your answer?
- Were there disagreements among the members?
- What you have shown is inductive reasoning . Can you give three examples?
- Based on the activity, what is inductive reasoning?

The teacher can ask the students to give more examples of inductive reasoning using the following situations:

- Classroom situation
- Supermarket situation
- Situation during family reunion
- JS Prom
- Election
- Summer vacation

This time tell the students that there is another kind of reasoning. Let them try to accomplish Activity 7 for them to discover what it is.

## *Teacher's Note and Reminders*


# Lesson 2 Inductive and Deductive Reasoning

## Activity 6 WHY OH WHY?

Each group will be given this activity sheet to accomplish.

1. Look carefully at the figures , what is next?

2. Study the pattern and draw the next figure.

3. My Math teacher is strict.
   My previous math teacher was strict.
   What can you say about all math teachers?

4. $1 \times 10 = 10$
   $2 \times 10 = 20$
   $3 \times 10 = 30$
   $5 \times 10 = 50$
   $24 \times 10 = 240$
   $2345 \times 10 =$ _____.

5. Every time Jackie visits her doctor she receives excellent services .With this she believes that.._____

Discuss the following with your group

- How did you arrive at your answer?
- Did you agree at once on your answer?
- Were there arguments among the members?
- What you have shown is inductive reasoning. Give 3 examples.
- Based on the activity, define inductive reasoning?

**Inductive reasoning** takes specific examples to make a general rule.

The students have just encountered deductive reasoning.
Ask them to give the difference between inductive and deductive reasoning, then let them answer exercise 3.

One of the tools used in proving is reasoning, specifically deductive **reasoning**. Deductive reasoning is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

A **proof** is a logical argument in which each statement you make is supported/ justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

Remember:
- **Postulate** is a statement that is accepted without proof.
- **Theorem** is a statement accepted after it is proved deductively.

**Answer Key**
**Exercise 3**
1. 25. Inductive reasoning
2. X, Y, Z are on the same plane. Deductive reasoning
3. BELEN is equilateral. Deductive reasoning
4. All teachers are ladies. Inductive reasoning
5. Julia is a peace-loving person. Deductive reasoning

## *Teacher's Note and Reminders*


## Activity 7 CUBRA CUBE

Complete the table below.

| Statement | Conclusion |
|---|---|
| 1. Filipinos are hospitable.<br>Bonifacio is a Filipino. | |
| 2. If points are collinear, then they lie on the same plane<br>Points R,M,andN are collinear . | |
| 3. A quadrilateral is a polygon of four sides | |
| 4. Smoking can cause cancer. Tomas is smoking | |
| 5. An angle is acute if its measure is between $0^0$ and $90^0$. Angle $B$ is acute. | |

You have just encountered Deductive reasoning. Can you give the difference between inductive and deductive reasoning?

Deductive reasoning is reasoning which begins using basic and general statements to prove more complicated statements.

Inductive reasoning is judging by experience while deductive reasoning is judging by logical progression.

**Exercise 3**

Draw conclusion from each given situation and identify the kind of reasoning used.

1. 5, 10,15, 20. The next number is ___.
2. Coplanar points are points on the same plane. $X, Y, Z$ are coplanar. Therefore_____
3. Regular polygon is equilateral. BELEN is a regular pentagon. Therefore_____
4. A child's teacher in pre school was a female, in his grades 1 and 2 his teachers were both female. The child may say_____
5. Filipinos are peace- loving people. Julia is a Filipino. Therefore_____-

The main focus in the study of geometry is to learn how to think logically. Do you still remember the If-then statement? Which one is the hypothesis? the conclusion?

In the next Activity the students are asked to give conclusion and reason. The teacher may recall definitions, axioms, postulates and proved theorems which are tools for deductive reasoning.

## *Teacher's Note and Reminders*


The parts of a deductive reasoning are:

- Hypothesis – the statement which is accepted or known at the beginning
- Conclusion – the statement drawn from the hypothesis.

### Activity 8 LET'S CONCLUDE

A . Supply the conclusion for the given hypothesis

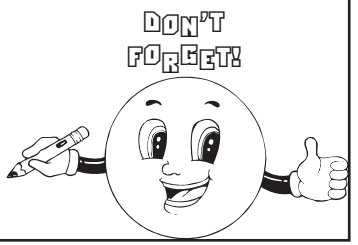
1. If $\angle 1 \cong \angle 2$, then _____
2. If $AB = CE$, then _____
3. If $\angle B$ and $\angle E$, are complementary then _____
4. $m\angle 3 + m\angle 5 = 180$, then _____
5. If $\angle A$ and $\angle X$ form a linear pair, then _____

B. Supply a valid conclusion for the given hypothesis on the first blank and the corresponding reason on the second blank

6. If $\angle B$ is a right angle,
   Then _____ _____
7. If $m\angle 3 + m\angle 4 = 180$
   Then _____ _____
8. If $\overrightarrow{PM}$ bisects $\angle APO$
   Then _____ _____
9. If $\overrightarrow{BP} \perp \overrightarrow{BC}$
   Then _____ _____
10. $\Delta$BOS is isosceles.
    Then _____ _____

*From the hypothesis we derive another statement that is the conclusion Where did you base your conclusion? Have you recalled your undefined terms, definitions and postulates? They will play a very important role in our next section.*

## Teacher's Note and Reminders




# Lesson 3 Proving Theorems

In proving theorems, the properties of equality and congruence are the bases for reasoning.

**Properties of Equality**

Addition Property of Equality (APE)

For all real numbers $a$, $b$, $c$ and $d$, if $a = b$ and $c = d$,
then $a + c = b + d$

Subtraction Property of Equality (SPE)

If $a = b$ and $c = d$, then $a - c = b - d$.

Multiplication Property of Equality (MPE)

If $a = b$, then $ac = bc$

Division Property of Equality (DPE)

If $a = b$ then $a/c = b/c$

Substitution Property of Equality

If $m\angle A = 60$, $m\angle B = 60$ then $m\angle A = m\angle B$

Distributive Property

$a(b + c) = ab + ac$

**Properties of Congruence**

Reflexive Property

$\overline{AB} \cong \overline{AB}$

Symmetric Property

If $\angle A \cong \angle B$ then $\angle B \cong \angle A$

Ask the students to answer exercise 4
In proving theorems you have to follow these steps:

- Read and understand the theorem
- Label the hypothesis as **Given** ,the conclusion as **Prove**
- Draw the **figure** and label the parts correctly.
- Write the proof which consists of **the statements** and **reasons**.

**Answer Key**
**Exercise 4**
1. Symmetric Prop.
2. Distributive Prop.
3. Substitution Prop. of Equality
4. Reflexive Prop.
5. Transitive Prop.

## *Teacher's Note and Reminders*


Transitive Property

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$ then $\angle A \cong \angle C$

Aside from the properties of equality and congruence, you should be equipped with the knowledge about undefined terms, definitions, and postulates in geometry. These are necessary to successfully support the statement of a proof.

**Exercise 4**

Justify each statement by giving the Property of Equality or Property of Congruence used.

If $TX = BK$, then $BK = TX$
$8(m + n) = 8m + 8n$
If $CT = 12$ and $PR + CT = 20$, then $PR + 12 = 20$.
$m\angle HIT = m\angle HIT$
If $\angle S \cong \angle P$, $\angle B \cong \angle S$, then $\angle P \cong \angle B$

One of the tools used in proving is reasoning, specifically deductive reasoning. **Deductive reasoning** is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

A **proof** is a logical argument in which each statement you make is supported/ justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

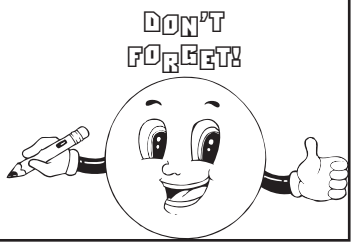
Remember:

- Postulate is a statement that is accepted without proof.
- Theorem is a statement accepted after it is proved deductively.

In proving theorems you have to follow these steps:

- Read and understand the theorem
- Label the hypothesis as given and the conclusion as Prove
- Draw the figure and label the parts correctly.
- Write the proof which consists of the statements and reasons.

## Teacher's Note and Reminders



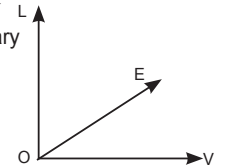

Proofs can be written in different ways

1. **Paragraph Form/ Informal Proof:**

The **paragraph** or **informal proof** is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

Given: $\angle LOE$ and $\angle EOV$ are complementary

Prove: $\overline{LO} \perp \overline{OV}$

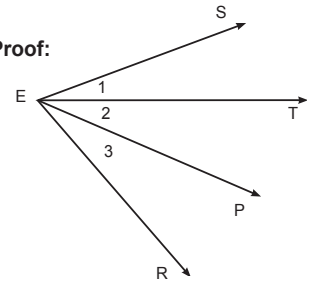



Proof: Since $\angle LOE$ and $\angle EOV$ are complementary, then $m\angle LOE + m\angle EOV = 90°$ by definition of complementary angles. Thus, $m\angle LOE + m\angle EOV = m\angle LOV$ by angle addition postulate and $m\angle LOV = 90°$ by transitive property of equality. So, $\angle LOV$ is a right angle by definition of right angles; and therefore, $\overline{LO} \perp \overline{OV}$ by definition of perpendicularity.

2. **Two-Column Form/ Formal Proof:**

Given: $m\angle SEP = m\angle TER$

Prove : $m\angle 1 = m\angle 3$




Write the missing reasons

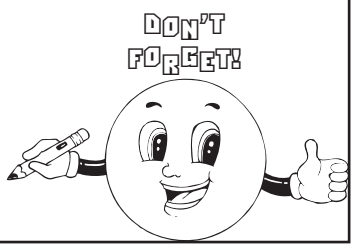
Proof:

| Statement | Reason |
|---|---|
| 1. $m\angle SEP = m\angle TER$ | 1. |
| 2. $m\angle SEP = m\angle 1 + m\angle 2$ | 2. Angle Addition Postulate |
| 3. $m\angle TER = m\angle 2 + m\angle 3$ | 3. |
| 4. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ | 4. Substitution Property |
| 5 $m\angle 2 = m\angle 2$ | 5 |
| 6. $m\angle 1 = m\angle 3$ | 6. Subtraction Property |

Study carefully the parts, especially the proof. How do we derive the statements and the reasons.

Try the flow chart form using the same example.

## Teacher's Note and Reminders

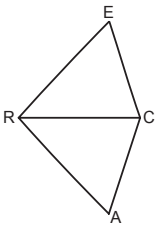


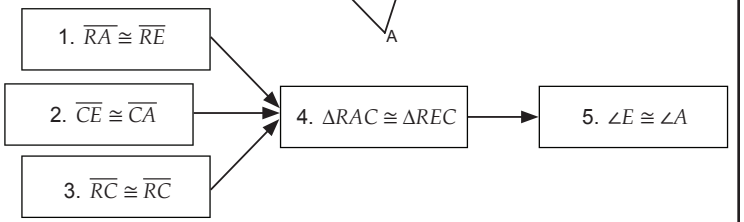

**3. Flowchart Form:**

A **flowchart-proof** organizes a series of statements in a logical order, starting with the given statements. Each statement together with its justification is written in a box. Arrows are used to show how each statement leads to another. It can make ones logic visible and help others follow the reasoning.

*Example 1*

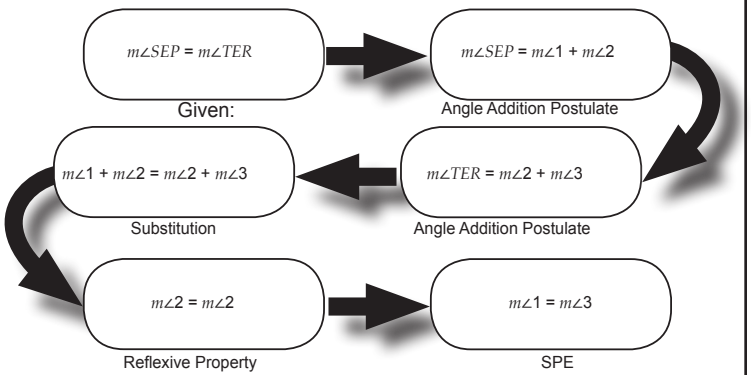
Flow Chart Proof
Given: $\overline{RA} \cong \overline{RE}$
$\overline{CE} \cong \overline{CA}$
Prove: $\angle E \cong \angle A$



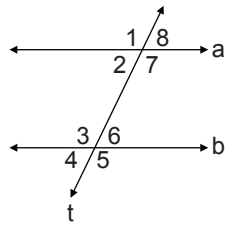

*Example 2*

CPCTC




You might want to watch a video lesson on this kind of proof, you may visit the following link:
*http://www.youtube.com/watch?feature=player_embedded&v=3Ti7-Ojr7Cg*

**Answer Key**
**Exercise 5**
Given: $\overleftrightarrow{a} \parallel \overleftrightarrow{b}$ with transversal $\overleftrightarrow{t}$ cutting $\overleftrightarrow{a}$ and $\overleftrightarrow{b}$.
Figure:




Prove: $\angle 2 \cong \angle 6$
Proof:

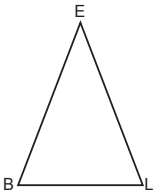
| Statement | Reason |
|---|---|
| $m\angle 2 + m\angle 7 = 180$<br>$m\angle 7 + m\angle 6 = 180$ | 1. Definition of Supplementary angles |
| $m\angle 2 + m\angle 7 = m\angle 7 + m\angle 6$ | 2. Transitive Prop. |
| $m\angle 7 = m\angle 7$ | 3. Reflexive Prop. |
| $m\angle 2 = m\angle 6$ | 4. Subtraction Prop. |
| $\therefore \angle 2 \cong \angle 6$ | 5. Def. of congruent angles. |

## *Teacher's Note and Reminders*


**4. Indirect Proof**

An indirect proof usually is paragraph form, the opposite of the statement to be proven is assumed true until the assumption leads to contradiction.
Example:

*Example :*




Given: $\Delta BEL$ is isosceles triangle with vertex $\angle B$
Prove: $\angle B \cong \angle L$
Proof: Assume that $\angle B \cong \angle L$
Given that $\Delta BEL$ is isosceles therefore
$\overline{BE} \cong \overline{BL}$ by the definition of isosceles triangle
$\angle B \cong \angle L$ because if two sides of a triangle are congruent then the angles opposite these sides are congruent; thus, the assumption is false and therefore $\angle B \cong \angle L$.

**Exercise 5**
*Work in group*
Prove that if two parallel lines are cut by a transversal, then the alternate interior angles are congruent. Discuss and show the proof.

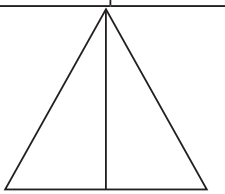
1. Given:_____

Prove: _____ Figure:

Proof:

| Statement | Reason |
|---|---|
| | |
| | |
| | |
| | |

Show the proof of the following.
2. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\Delta COD \cong \Delta BOD$



3. Given: $\overline{OP} \perp \overline{PE}$; $\overline{EB} \perp \overline{PE}$
***T* is the midpoint of $\overline{PE}$**
Prove: $\overline{OP} \cong \overline{BE}$

## What to Understand

This is a picture analysis activity, where students can form conditional statements out of the picture.

## Teacher's Note and Reminders


*In this section, the discussion was about proofs.*

*Go back to the previous section and compare your initial ideas with the iedeas discussed. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?*

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

## What to Understand

What will you do before jumping into a conclusion?

*This is a picture analysis activity, where you can form conditional statements out of the picture. Proceed by forming another statement drawn from the original one.*

## Activity 9 "CASE SOLVED"

From the sets of geometric representations on real-life situations problems, write your reasons inductively or deductively on the "Reason Out Activity Sheet".

Be ready to present your arguments to persuade your classmates.

Your arguments will be rated in terms of coherence, mathematical thinking and conclusions made.

In this section, the discussion was about proofs.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

## *Teacher's Note and Reminders*


### Activity 10 SHALL WE MEET OR NOT?

The city's newspaper will release its 2nd volume this year. It contains a new column that will explain phenomena that deals with the concepts of mathematics. As a contributor writer, you were tasked to write an article that will explain the "phenomenon of perspective" it shows that when two parallel lines are seen from a far, the lines intersect. Your article will be evaluated by the head writer and editor-in-chief based on its coherence, mathematical thinking, and conclusions made.

*In this section, the discussion was about proofs.*

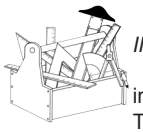
*What new realizations do you have about the topic? What new connections have you made for yourself?*

*Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.*

### What to Transfer

In this section, you will be applying your understanding of inductive and deductive reasoning through the following activity that reflect meaningful and relevant situations.

### Activity 11 "IMBESTIGADOR"



Math Magazine will release its November issue themed "*MATH INVESTIGATES*".

As one of the investigators you were tasked to make a mathematical investigation that will enlighten the readers by providing valid conclusions. The written output of your investigation will be presented to the Head writer, writers, and Editor-In-Chief and shall be evaluated based on to its coherence, clarity, judgment, and mathematical reasoning.

In this section, your task was on math investigations.

How did you find the performance task? How did the task help you see the application use of the topic?

## *Teacher's Note and Reminders*