

TEACHING GUIDE

Module 7: Triangle Congruence

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of the key concepts of triangle congruence.

Performance Standard:

The learner is able to communicate mathematical thinking with coherence and clarity, in formulating, investigating, analyzing and solving real life problems involving proving triangle congruence.

UNPACKING THE STANDARDS FOR UNDERSTANDING

SUBJECT:	LEARNING COMPETENCIES	
Grade 8 Mathematics QUARTER: Third Quarter STRAND: Geometry TOPIC: Triangle Congruence LESSONS: 1. Definition of Congruent Triangles 2. Congruence Postulates 3. Proving Congruence of Triangles 4. Application of Triangle Congruence	1. (K) Illustrates triangle congruence 2. (K,S) States and illustrates the SAS, ASA, and SSS congruence postulates 3. (S) Applies the postulates and theorems on triangle congruence to prove statements on congruence including right triangles 4. (S) Applies triangle congruence to geometric constructions of perpendicular and angle bisector.	
	ESSENTIAL UNDERSTANDING: Students will understand that: Triangle congruence can be proven deductively. Real life problems requiring stability involve the use of triangle congruence.	ESSENTIAL QUESTION: How do you know that the given triangles are congruent? What ensures the stability of any structure?
	TRANSFER GOAL: Students will, on their own, solve real life problems pertaining to stability and structure, using triangle congruence.	

B. Planning for Assessment

Product/Performance

The following are products and performances that students are expected to accomplish with in this module.

Assessment Map

TYPE	KNOWLEDGE AND PROCESS/ SKILLS (ACQUISITION)	UNDERSTANDING (MEANING MAKING)	TRANSFER
PRE-ASSESSMENT/ DIAGNOSTIC	Pre – test	Pre Test Interpretation Explanation	
FORMATIVE ASSESSMENT	Written Exercises	Testing for congruence understanding Interpretation Explanation	
	Quiz Congruence Postulates	Worksheet Interpretation Application	Journal
SUMMATIVE ASSESSMENT		Proving Interpretation Explanation Application	Rubric
		Long Examination Interpretation Explanation Application	
SELF ASSESSMENT	Post Test		Journal Writing

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	The learner demonstrates understanding of key concepts of triangle congruence.	Paper and Pencil Test Part I items 1, 2, 3, 12, 13	1 point for every correct response
Process/Skills 25%	State and illustrate the SAS, ASA, and SSS congruence.		1 point for every correct response
Understanding 30%	<p>Prove triangle congruence by SAS, ASA, and SSS.</p> <p>Apply triangle congruence to geometric construction of perpendicular and angle bisector.</p> <p>Solve problems involving triangle congruence.</p>	Part II items 1 d-e, 3, 4, and 5	<p>1 point for every correct response</p> <p>Rubric for explanation Criteria: Clear Coherent Justified</p> <p>Rubric for drawing Criteria: Neat and Clear Accurate Justified Appropriate Relevant</p>

Product 30%	The learner is able to formulate real-life problems involving triangle congruence and solve these with utmost accuracy using a variety of strategies.	<p>Portfolio</p> <p>Cite two situations in real-life where the concept is illustrated.</p> <p>Formulate problems out of these situations then solve them in as many ways as you can.</p> <p>GRASPS Assessment</p> <p>Make a design or a sketch plan of a suspension bridge. Apply your understanding of the key concepts of triangle congruence.</p>	<p>Rubric on Problem Posing/ Formulation and Problem Solving</p> <p>Criteria: Relevant Creative Insightful Authentic Clear</p> <p>Rubric on Design/Sketch Plan</p> <p>Criteria:</p>
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C. Planning for Teaching-Learning

Introduction:

This module covers key concepts of triangle congruence. It is divided into four lessons,

Lesson 1: Definition of congruent triangles

Lesson 2: Congruence postulates

Lesson 3: Proving congruence of triangles

Lesson 4: Application of triangle congruence

In all lessons, students are given the opportunity to use their prior knowledge and skills in learning triangle congruence. They are also given varied activities to process the knowledge and skills learned and reflect and further understand and transfer their understanding of the different lessons.

As an introduction to the module, the teacher ask the following questions:

- o Have you ever wondered how bridges and buildings are designed?
- o What factors are being considered in making bridges and buildings?
- o How do problems on structure stability be solved?
- o When do you say triangles are congruent?
- o What ensures the stability of any structure?



Entice the students to find out the answers to these questions and to determine the vast applications of triangle congruence in this module.

Objectives:

After the learners have gone through the lessons contained in this module, they are expected to:

- Define and illustrate congruent triangles
- State and illustrate the SAS, ASA and SSS Congruence Postulates;
- Apply the postulates and theorems on triangle congruence to prove statements on congruence, including right triangles; and
- Apply triangle congruence to geometric construction of perpendicular bisector and angle bisector.

Learning Goals and Targets:

In this lesson, students are expected to do the following:

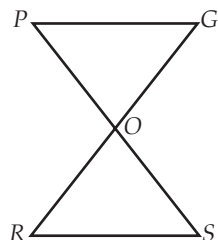
1. Cite real-life situations where congruent triangles are illustrated and describe how these mathematics concepts are applied.
2. Define unfamiliar terms.
3. Explore the websites for better understanding of the lesson.
4. Write a journal on their experience in using the module.
5. Make a portfolio of their output.
6. Collaborate with their teacher and peers.

Pre-Assessment:

1. In the figure $\triangle POG \cong \triangle SOR$, what is the side corresponding to \overline{PO} ?

- a. \overline{OS}
- b. \overline{RD}
- c. \overline{RS}
- d. \overline{SO}

Answer: D



2. Listed below are the six pairs of corresponding parts of congruent triangles. Name the congruent triangles.

$$\begin{array}{ll} \overline{SA} \cong \overline{JO} & \angle D \cong \angle Y \\ \overline{AD} \cong \overline{OY} & \angle A \cong \angle O \\ \overline{SD} \cong \overline{JY} & \angle S \cong \angle J \end{array}$$

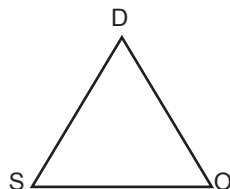
- a. $\triangle ASD \cong \triangle JOY$
- b. $\triangle ADS \cong \triangle YJO$
- c. $\triangle SAD \cong \triangle JOY$
- d. $\triangle SAD \cong \triangle JYO$

Answer: A

3. In $\triangle DOS$, what side is included between $\angle D$ and $\angle O$?

- a. \overline{DO}
- b. \overline{DS}
- c. \overline{SD}
- d. \overline{SO}

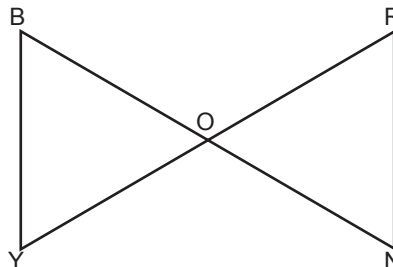
Answer: A



4. Name the corresponding congruent parts as marked that will make each pair of triangles congruent by SAS.

- a. $\overline{BY} \cong \overline{NR}$, $\angle BOY \cong \angle NOR$, $\overline{BO} \cong \overline{NO}$
 b. $\overline{BO} \cong \overline{NO}$, $\angle BOY \cong \angle NOR$, $\overline{RO} \cong \overline{YO}$
 c. $\overline{YO} \cong \overline{OR}$, $\overline{BO} \cong \overline{ON}$, $\angle BOY \cong \angle NOR$
 d. $\angle B \cong \angle N$, $\overline{BO} \cong \overline{NO}$, $\overline{OY} \cong \overline{OR}$

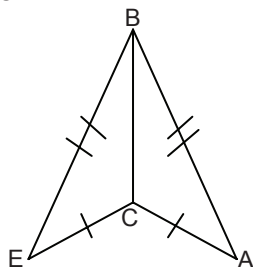
Answer: B



5. If corresponding congruent parts are marked, how can you prove $\triangle BEC \cong \triangle BAC$?

- a. ASA
 b. LL
 c. SAS
 d. SSS

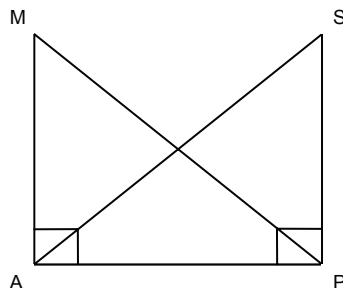
Answer: D



6. Identify the pairs of congruent right triangles and tell the congruence theorem used.

- a. $\triangle PMA \cong \triangle APS$
 b. $\triangle MAP \cong \triangle SPA$
 c. $\triangle MPA \cong \triangle SPA$
 d. $\triangle AMP \cong \triangle PAS$

Answer: A

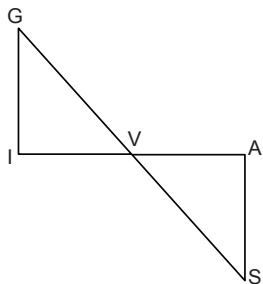


7. What property of congruence is illustrated in the statement? If $\overline{AB} \cong \overline{DE}$, $\overline{EF} \cong \overline{DE}$ then $\overline{AB} \cong \overline{EF}$.

- A. Symmetric
B. Transitive
C. Reflexive
D. Multiplication

Answer: B

8. $\triangle GIV \cong \triangle SAV$ deduce a statement about point V .



- a. V is in the interior of $\triangle GIV$.
b. V is in the exterior of $\triangle SAV$.
c. V is in the midpoint of \overline{GS} .
d. V is collinear with G and I .

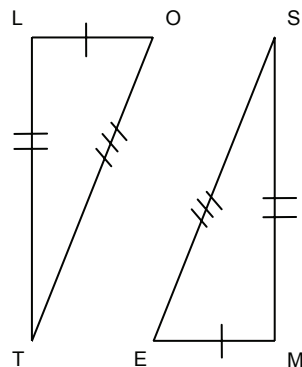
Answer: C

9. Is the statement “corresponding parts of congruent triangles are congruent” based on

- a. Definition
b. Postulate
c. Theorem
d. Axiom

Answer: B

10. Use the marked triangles to write proper congruence statement.



- A. $\overline{LT} \cong \overline{MS}$
 $\overline{LO} \cong \overline{ME}$
 $\overline{OT} \cong \overline{ES}$
 $\triangle LOT \cong \triangle MES$
- B. $\overline{LT} \cong \overline{SM}$
 $\overline{LO} \cong \overline{ME}$
 $\overline{OT} \cong \overline{ES}$
- C. $\overline{LT} \cong \overline{MS}$
 $\overline{OL} \cong \overline{ME}$
 $\overline{OT} \cong \overline{SE}$
 $\triangle LOT \cong \triangle MSE$
- D. $\overline{TL} \cong \overline{MS}$
 $\overline{LO} \cong \overline{ME}$
 $\overline{OT} \cong \overline{ME}$
 $\triangle TOL \cong \triangle SME$

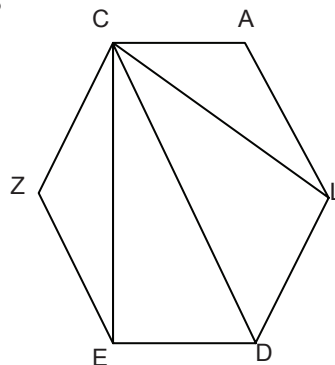
Answer: A

11. Hexagon $CALDEZ$ has six congruent sides. \overline{CE} , \overline{CD} , \overline{CL} are drawn on the hexagon forming 4 triangles.

Which triangles can you prove congruent?

- a. $\triangle CEZ \cong \triangle CDE$
 $\triangle CDE \cong \triangle CAL$
- b. $\triangle CEZ \cong \triangle CAL$
 $\triangle CED \cong \triangle CLD$
- c. $\triangle CED \cong \triangle CEZ$
 $\triangle CLA \cong \triangle CLD$
- d. $\triangle CZE \cong \triangle CED$
 $\triangle DEC \cong \triangle LCD$

Answer: B



12. $\triangle ABC \cong \triangle DEF$, which segment is congruent to \overline{AB} :

- a. BC
- b. AC
- c. DE
- d. EB

Answer: C

13. $\triangle SUM \cong \triangle PRO$, which angle is congruent to $\angle M$?

- a. S
- b. P
- c. R
- d. O

Answer: D

14. $\triangle TIN \cong \triangle CAN$, then $\triangle NAC$ is congruent to ____.

- a. $\triangle ITN$
- b. $\triangle NIT$
- c. $\triangle TNI$
- d. $\triangle INT$

Answer: B

15. Jancent knows that $AB = XY$ and $AC = XZ$. What other information must he know to prove $\triangle ABC \cong \triangle XYZ$ by SAS postulate?

- a. $\angle B \cong \angle Y$
- b. $\angle C \cong \angle Z$
- c. $\angle A \cong \angle X$

Answer: C

16. Miguel knows that in $\triangle MIG$ and $\triangle JAN$, $MI = JA$, $IG = AN$, and $MG = JN$. Which postulate or theorem can he use to prove the triangles congruent?

- a. ASA
- b. AAS
- c. ASA
- d. SSS

Answer: D

17. In $\triangle ABC$, $AB = AC$. If $m\angle B = 80$, find the measure of $\angle A$.

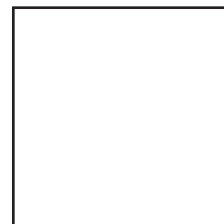
- a. 20
- b. 80
- c. 100
- d. 180

Answer: A

18. You are tasked to make a design of the flooring of a chapel using triangles. The available materials are square tiles. How are you going to make the design?

- Applying triangle congruence by ASA
- Applying triangle congruence by SAS.
- Applying triangle congruence by SSS
- Applying triangle congruence by AAS

Answer: C



For items 19 to 20

Complete the proof. Choose the letter of the correct answer to fill the blank

- $\overline{CO} \cong \overline{CO}$
- ASA
- SAS
- $\angle BCO \cong \angle ACO$

In $\triangle ABC$, let O be a point in AB such that CO bisects $\angle ACB$, if $\overline{AC} \cong \overline{BC}$.
Prove that $\triangle ACO \cong \triangle BCO$.

Statements

Reasons

- $\overline{AC} \cong \overline{BC}$
- \overline{CO} bisects $\angle ACB$
- ____ (19) ____
- $\overline{CO} \cong \overline{CO}$
- $\triangle ACO \cong \triangle BCO$

- Given
- Given
- Definition of angle bisector
- Reflexive Property of Congruence
- ____ (20) ____

Answer: D

Answer: C

What to Know



Before doing Activity 1, Let the students answer the following questions on their journal.

Activating Prior Knowledge

1. What is the symbol for congruence?
2. If $\triangle ABC \cong \triangle XYZ$, what are the six pairs of corresponding congruent parts?
3. How do we measure an angle?
4. How can you draw an angle of specified measure?
5. What is the sum of the measures of the angles of a triangle?

For numbers six to ten define or illustrate each of the following:

6. Midpoint
7. Vertical angles
8. Right Triangle
9. Hypotenuse
10. Isosceles Triangle

My idea of Congruent Triangles is _____

Provide the students opportunities to recall congruent figures. Define congruent triangles by doing activities 1 and 2.

Lesson

1

Definition of Congruent Triangles

What to Know



Let's begin this lesson by finding out what congruent triangles are. As you go over the activities, keep on thinking "**When are two triangles congruent?**"

Activating Prior Knowledge

1. What is the symbol for congruence?
2. If $\triangle ABC \cong \triangle XYZ$, what are the six pairs of corresponding congruent parts?
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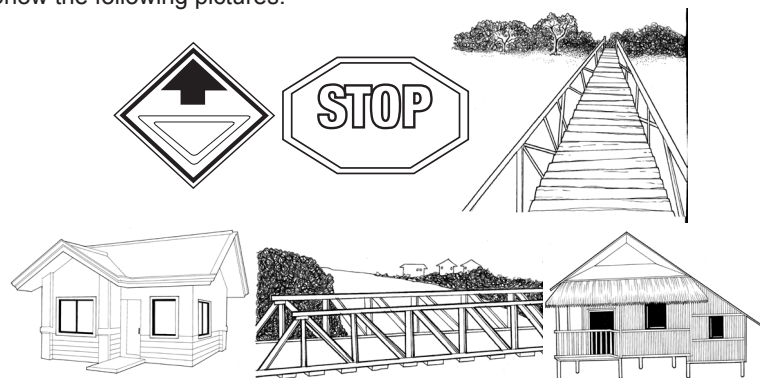
The wonders of Geometry are present everywhere, in nature and in structures. Designs and patterns having the same size and same shape play important roles especially on the stability of buildings and bridges. What ensures the stability of any structures?

Hook: In coming to school, have you met Polygon? Name it and indicate where you met it. (Answers vary, Rectangles, windows, 20 peso bill from my pocket, triangles from bridges and buildings and houses etc.)

Provide the students opportunities to recall congruent figures. Define congruent triangles by doing activities 1 and 2.

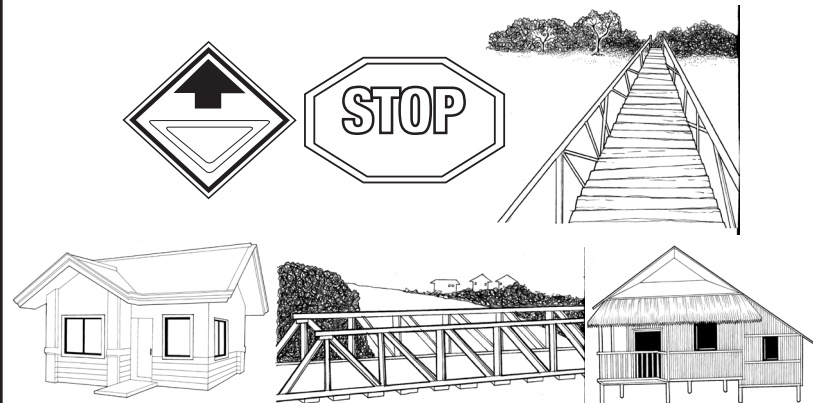
Activity 1: Picture Analysis

Show the following pictures:



From the picture shown, ask the students the given questions in Activity 1. Answers will be presented to the class and be discussed.

Teacher's Note and Reminders



Activity 1 PICTURE ANALYSIS

From the picture shown, your group will answer the following questions:

1. How will you relate the picture to your ambition?
2. If you were an architect or an engineer, what is your dream project?
3. What can you say about the long bridge in the picture? How about the tall building? (presence of congruent triangles, its stability, uses of bridges for economic progress,
4. Why are there triangles in the structures? Are the triangles congruent? When are two triangles congruent?
5. Why are bridges and buildings stable?

Answers will be recorded.

You gave your initial ideas on congruent triangles and the stability of bridges and buildings.

Let's now find out how others would answer the question and compare their ideas to our own. We will start by doing the next activity.!

What to Process



In order to have an idea of congruent figures that will lead to the definition of congruent triangles; let the student perform activity 2. See to it that everybody participates. The teacher is allowed to play music as they perform the activity.

Teacher's Note and Reminders



After the students have chosen and named the two triangles, explain to them the investigation they will make that will lead them to the formal definition of congruent triangles. Students will fill up the activity sheet.

What to Process



Let's begin by finding out what congruent triangles are.

Activity 2

FIND YOUR PARTNER

Let us see your knowledge about congruent fi.

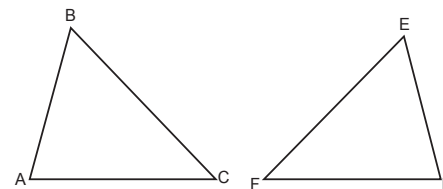
Instruction

Your group (with 10 members) will be given five pairs of congruent figures, each shape for each member. At the count of three, find your partner by matching the shape that you have with another's shape.



1. Why/How did you choose your partner?
2. Describe the two figures you have.
3. What can you say about the size and shape of the two figures?
4. We say that congruent figures have the same size and the same shape. Verify that you have congruent figures.

For each group you pick up a pair of congruent triangles



Name your triangles as $\triangle ABC$ and $\triangle DEF$ as shown in the figure.

Investigate: Matching vertices of the two triangles

First Match: $ABC \leftrightarrow EDF$ (A corresponds to E, B corresponds to D, C corresponds to F)

Second Match: $ABC \leftrightarrow EFD$

Third Match: $ABC \leftrightarrow DEF$

In which of the above pairings are the two triangles congruent? Fill up the activity sheet on the next page.

Let the students state the definition of congruent triangles based on the investigation made. Allow them to answer the following questions on their journal:

- In what pairing or match do the two triangles coincide?
- What are congruent triangles?
- How many pairs of corresponding parts are congruent?
- Illustrate $\triangle TNX \cong \triangle HOP$. Put identical markings on congruent corresponding parts.
- Where do you see congruent triangles?

Ask the students to answer Exercise 1.



Teacher's Note and Reminders



Group No. _____

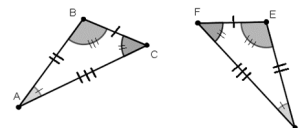
Match	Corresponding sides	Congruent or not congruent?	Corresponding Angles	Congruent Or not congruent?
First				
Second				
Third				

Two triangles are congruent if their vertices can be paired so that corresponding sides are congruent and corresponding angles are congruent.

$\triangle ABC \cong \triangle DEF$ Read as "triangle ABC is congruent to triangle DEF."

\cong symbol for congruency

\triangle symbol for triangle.



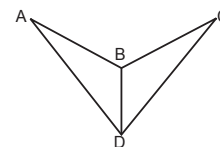
The congruent corresponding parts are marked identically.

Can you name the corresponding congruent sides? Corresponding congruent angles?

Answer the questions below in your journal:

- ✓ What are congruent triangles?
- ✓ How many pairs of corresponding parts are congruent if two triangles are congruent?
- ✓ Illustrate $\triangle TNX \cong \triangle HOP$ Put identical markings on congruent corresponding parts.
- ✓ Where do you see congruent triangles?

Exercise 1



1. $\triangle ABD \cong \triangle CBD$, Write down the six pairs of congruent corresponding parts
2. Which triangles are congruent if $\overline{MA} \cong \overline{KF}$, $\overline{AX} \cong \overline{FC}$, $\overline{MX} \cong \overline{KC}$; $\angle M \cong \angle K$, $\angle A \cong \angle F$, $\angle X \cong \angle C$. Draw the triangles.

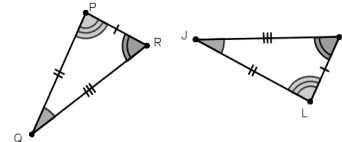
Challenge the students to prove triangle congruence by showing lesser number of corresponding congruent parts.
Discuss first with the students the parts of a triangle in terms of relative position then let them answer Exercise 2 and do Activities 3-5.

Teacher's Note and Reminders



3. Which of the following shows the correct congruence statement for the figure below?

- a. $\triangle PQR \cong \triangle KJL$
- b. $\triangle PQR \cong \triangle LJK$
- c. $\triangle PQR \cong \triangle LKJ$
- d. $\triangle PQR \cong \triangle JLK$



You can now define what congruent triangles are. In order to say that the two triangles are congruent, we must show that all six pairs of corresponding parts of the two triangles are congruent.

Let us see how can we verify two triangles congruent using fewer pairs of congruent corresponding parts.

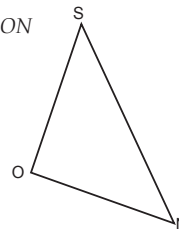
Topic 2: Triangle Congruence Postulates



Before we study the postulates that give some ways to show that the two triangles are congruent given less number of corresponding congruent parts, let us first identify the parts of a triangle in terms of their relative positions.

Included angle is the angle between two sides of a triangle.
Included side is the side common to two angles of a triangle.

In $\triangle SON$



$\angle S$ is included between \overline{SN} and \overline{SO} .
 $\angle O$ is included between \overline{OS} and \overline{ON} .
 $\angle N$ is included between \overline{NS} and \overline{NO} .
 \overline{SO} is included between $\angle S$ and $\angle O$.
 \overline{ON} is included between $\angle O$ and $\angle N$.
 \overline{SN} is included between $\angle S$ and $\angle N$.

Always have a drill and review identifying included side, included angle before you ask the students to do the activities.

From Activity 3, let the students make their own generalization on SAS congruence postulate. Ask them to answer Exercise 3

Teacher's Note and Reminders



Exercise 2

Given $\triangle FOR$, can you answer the following questions even without the figure?

1. What is the included angle between \overline{FO} and \overline{OR} ?
2. What is the Included angle between \overline{FR} and \overline{FO} ?
3. What is the included angle between \overline{FR} and \overline{RO} ?
4. What is the included side between $\angle F$ and $\angle R$?
5. What is the included side between $\angle O$ and $\angle R$?
6. What is the included side between $\angle F$ and $\angle O$?

Activity 3 LESS IS MORE

SAS (Side-Angle-Side) Congruence Postulate

1. Prepare a ruler, a protractor, a pencil and a bond paper.
2. Work in group of four.
3. Follow the demonstration by the teacher.
 - a. Draw a 7- inch segment.
 - b. Name it \overline{BE} .
 - c. Using your protractor make angle B equal to 70° degrees.
 - d. From the vertex draw \overline{BL} measuring 8 inches long.
 - e. How many triangles can be formed?
 - f. Draw $\triangle BEL$
 - g. Compare your triangle with the triangles of the other members of the group. Do you have congruent triangles?
 - h. Lay one triangle on top of the others. Are all the corresponding sides congruent? How about the corresponding angles?
 - i. What can you say about any pair of congruent triangles ?

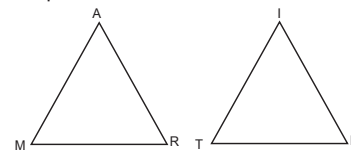
SAS (Side-Angle-Side) Congruence Postulate

If the two sides and an included angle of one triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

$$\text{If } \overline{MA} \cong \overline{TI}, \angle M \cong \angle T, \overline{MR} \cong \overline{TN}$$

Then $\triangle MAR \cong \triangle TIN$ by SAS Congruence Postulate

Mark the congruent parts.



Let the student state the generalization on ASA congruence, deduced from the activity they have just performed.

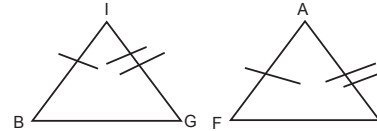
Teacher's Note and Reminders



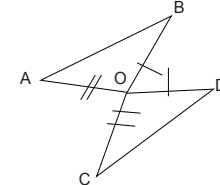
Exercise 3

Complete the congruence statement using the SAS congruence postulate.

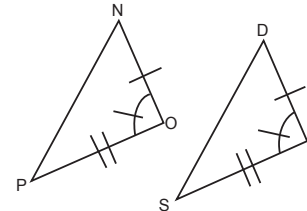
1. $\triangle BIG \cong \triangle$ _____



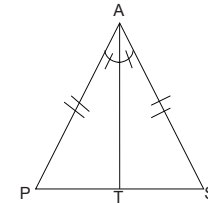
3. $\triangle ABO \cong \triangle$ _____



2. $\triangle PON \cong \triangle$ _____



4. $\triangle PAT \cong \triangle$ _____



After showing that the two triangles are congruent showing only two sides and the included angle of one triangle to two sides and included angle of another triangle, you try another way by doing activity 4

Activity 4 TRY MORE

ASA (Angle-Side Angle) Congruence

Prepare the following materials; pencil, ruler, protractor, a pair of scissors

Working independently, use a ruler and a protractor to draw $\triangle BOY$ with two angles and the included side having the following measures: $m\angle B = 50$, $m\angle O = 70$ and $\overline{BO} = 18$ cm

1. Draw \overline{BO} measuring 18 cm
2. With B as vertex, draw angle B measuring 50.
3. With O as vertex, draw angle O measuring 70.
4. Name the intersection as Y .
5. Cut out the triangle and compare it with four of your classmates.
6. Describe the triangles.
7. Put identical marks on the congruent corresponding sides and angles.
8. Identify the parts of the triangles which are given congruent.

From the Activity 5, ask the students to state their own generalization on SSS Congruence postulate.
Let them answer Exercise 4.

Teacher's Note and Reminders



ASA (Angle-Side-Angle) Congruence Postulate

If the two angles and the included side of one triangle are congruent to the corresponding two angles and an included side of another triangle, then the triangles are congruent.

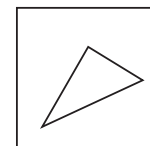
If $\angle A \cong \angle E$, $\overline{AB} \cong \overline{ME}$, $\angle B \cong \angle M$, then $\triangle JAY \cong \triangle MEL$
Draw the triangles and mark the congruent parts.

Activity 5 SIDE UP

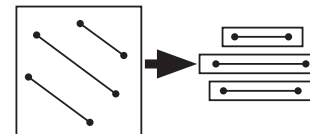
SSS (Side-Side-Side) Congruence Postulate

You need patty papers, pencil, a pair of scissors

1. Draw a large scalene triangle on your patty paper.



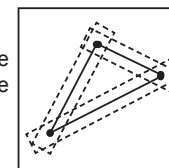
2. Copy the three sides separately onto another patty paper and mark a dot at each endpoint. Cut the patty paper into three strips with one side on each strip.



3. Arrange the three segments into a triangle by placing one endpoint on top of the another.



4. With a third patty paper, trace the triangle formed. Compare the new triangle with the original triangle. Are they congruent?



5. Try rearranging the three segments into another triangle. Can you make a triangle not congruent to the original triangle? Compare your results with the results of others near you.

Now that the students can show triangles congruent with

- two corresponding sides and an included angle
- two angles and an included side
- three pairs of corresponding sides congruent, they are now ready to prove two triangles congruent deductively.

Teacher's Note and Reminders

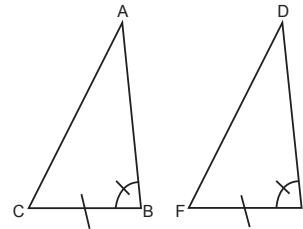
SSS (Side-Side-Side) Congruence Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

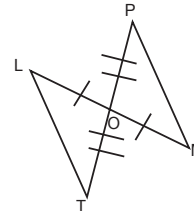
If $\overline{EC} \cong \overline{BP}$, $\overline{ES} \cong \overline{BJ}$, $\overline{CS} \cong \overline{PJ}$, then $\triangle ESC \cong \triangle BJP$, draw the triangles and mark the congruent parts., then answer exercise 4.

Exercise 4

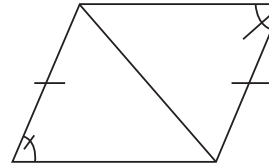
Corresponding congruent parts are marked. Indicate the additional corresponding parts needed to make the triangles congruent by using the specified congruence postulates.



- a. ASA _____
b. SAS _____



- a. SAS _____
b. SSS _____



- a. SAS _____
b. ASA _____



Now that you can show triangles congruent with

- two corresponding sides and an included angle
- two angles and an included side
 - three pairs of corresponding sides congruent, you are now ready to prove two triangles congruent deductively.

Allow students to study and discuss with their group the proof in No.1 then later let them complete the proof in no. 2. Do Activity 6 Let's Do IT.

Tell the students that proving triangle congruence will lead them to prove more theorems that will be helpful for them in the next lesson.

Teacher's Note and Reminders



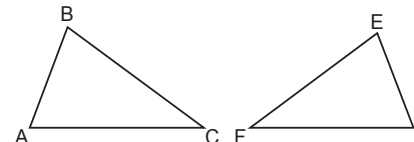
Topic 3: Proving Triangle Congruence

Activity 6 LET'S DO IT

Let's find out how we can apply the Congruence Postulates to prove two triangles congruent. Study the following example and answer exercise 5.

Given: $\overline{AB} \cong \overline{DE}$
 $\angle B \cong \angle E$
 $\overline{BC} \cong \overline{EF}$

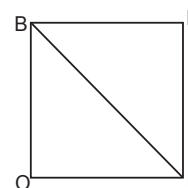
Prove: $\triangle ABC \cong \triangle DEF$



	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\angle B \cong \angle E$	2. Given
3. $\overline{BC} \cong \overline{EF}$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. SAS Postulate

Exercise 5

Try this



Given: $\overline{BE} \cong \overline{LO}$, $\overline{BO} \cong \overline{LE}$

Prove: $\triangle BEL \cong \triangle LOB$

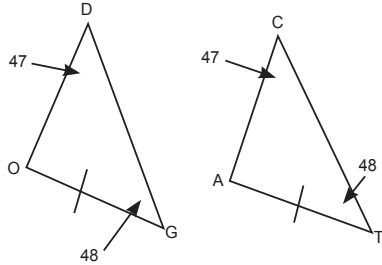
Complete the Proof:

Statements	Reasons
1.	1. Given
2. $\overline{BO} \cong \overline{LE}$	2.
3.	3.
4. $\triangle BEL \cong \triangle LOB$	4.

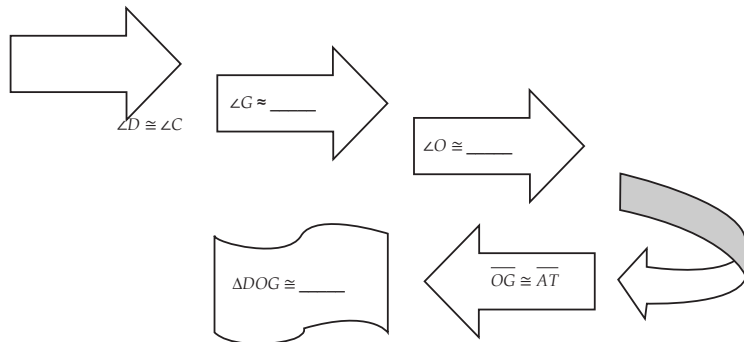
Discuss this with the students by showing more examples.

Let's try to prove a theorem on congruence,

Given the triangles below, a pair of corresponding sides are congruent, and two pairs of corresponding angles have the same measure.



Work in Pairs to discuss the proof of the theorem by completing the flow chart

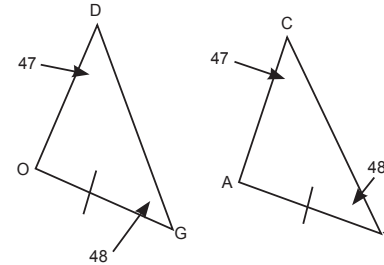


Supply the reason for each

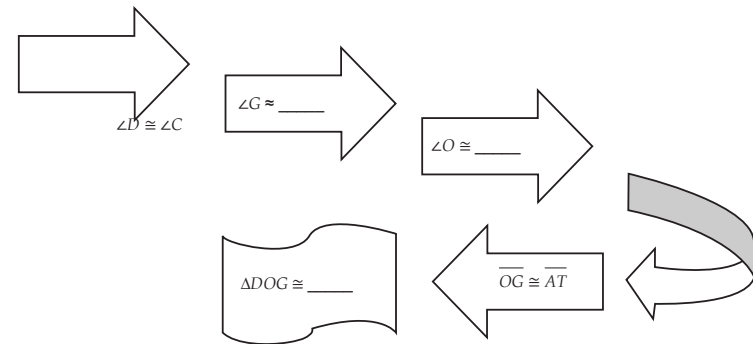
Let the students work in pairs to discuss the proof of the theorem by completing the flow chart.

Let's try to prove a theorem on congruence,

Given the triangles below, a pair of corresponding sides are congruent, and two pairs of corresponding angles have the same measure.



Work in Pairs to discuss the proof of the theorem by completing the flow chart



Supply the reason for each

When you completed the proof, review the parts of the two triangles which are given congruent. Have you realized that you have just proved the AAS congruence Theorem?

Tell the students that when they completed the proof review what parts of the triangles are shown congruent. Let them realized that they have just shown the proof of a theorem on congruence, the AAS congruence theorem.

Ask the students to study the example. Tell them that with the use of SAS, ASA, SSS postulates and AAS congruence theorem, they can now prove two triangles congruent, segments and angles congruent that can lead to prove more theorems especially on right triangles. Let them do activity 7.

Teacher's Note and Reminders



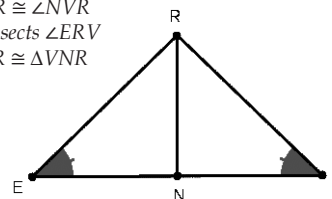
AAS (Angle-Angle-Side) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.

Example:

Given: $\angle NER \cong \angle NVR$
 \overline{RN} bisects $\angle ERV$

Prove: $\triangle ENR \cong \triangle VNR$



Statements	Reasons
1. $\angle NER \cong \angle NVR$	1. Given
2. \overline{RN} bisects $\angle ERV$	2. Given
3. $\angle ENR \cong \angle VNR$	3. Definition of angle bisector
4. $\overline{RN} \cong \overline{RN}$	4. Reflexive Property
5. $\triangle ENR \cong \triangle VNR$	5. AAS Postulate

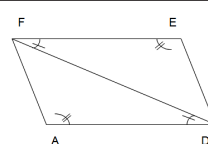
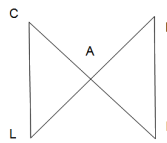
Exercise 6

Complete the congruence statement by AAS congruence.

Figure	Congruence Statement
	$\triangle BOX \cong \underline{\hspace{2cm}}$
	$\triangle GAS \cong \underline{\hspace{2cm}}$

Before you ask the students to do Activity 7, tell them that within their group, they will recall the parts of a right triangle by performing this activity.

Teacher's Note and Reminders

	$\triangle FED \cong \underline{\hspace{2cm}}$
 \overline{CM} bisects \overline{BL} at A $\angle L \cong \angle B$	$\triangle BAM \cong \underline{\hspace{2cm}}$

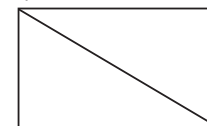
How are we going to apply the congruence postulates and theorem in right triangles? Let us now consider the test for proving two right triangles congruent.

Activity 7 **KEEP RIGHT**

Recall the parts of a right triangle with your groupmates .

1. Get a rectangular sheet of paper.

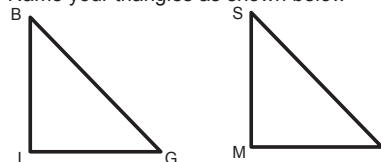
2. Divide the rectangle along a diagonal.



3. Discuss with your group and illustrate the the sides and angles of a right triangle using your cut outs

- What do you call the side opposite the right angle?
- What do you call the perpendicular sides?
- How many acute angles are there in a right triangle?

4. Name your triangles as shown below



Ask each group to present the proof to the class using a two column form, a flow chart, or in paragraph form to deduce the theorem.

Now give the students a chance to prove on their own the next theorems on right triangles congruence.
Ask each group to make a power-point presentation using a flowchart to prove the following theorems.

Teacher's Note and Reminders



5. If $\triangle BIG$ and $\triangle SML$ are right triangles, $\angle I$ and $\angle M$ are right, $\overline{BI} \cong \overline{SM}$, $\overline{IG} \cong \overline{ML}$, prove $\triangle BIG \cong \triangle SML$.

6. Discuss the proof with your group.

7. Answer the following questions:

- What kind of triangles did you prove congruent?
- What parts of the right triangles are given congruent?
 - Complete the statement: If the _____ of one right triangle are congruent to the corresponding _____ of another right triangle, then the triangles are _____.

Since all right angles are congruent you can now use only two pairs of corresponding parts congruent in order to prove two triangles congruent. The proof you have shown is the proof of the LL Congruence Theorem.

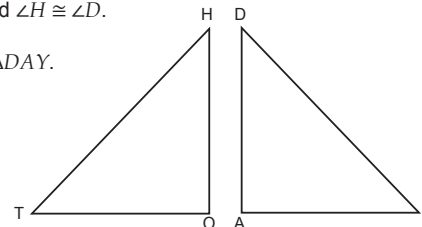
LL Congruence Theorem

If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

The LL Congruence Theorem was deduced from SAS Congruence Postulate.

Consider the right triangles HOT and DAY with right angles at O and A , respectively, such that $\overline{HO} \cong \overline{DA}$, and $\angle H \cong \angle D$.

Prove: $\triangle HOT \cong \triangle DAY$.



Each group will present the proof to the class either by two column form or using flow chart or paragraph form to deduce the theorem:

LA (leg-acute angle) Congruence Theorem

If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

Now it's your turn to prove the other two theorems on the congruence of right triangles.

Allow students to present their work the next day. After the discussion, let them answer Exercise 7.

Teacher's Note and Reminders



Activity 8 IT'S MY TURN

Each group will make a power point presentation using flowchart to prove the following theorems.

HyL (Hypotenuse-leg) Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and a leg of another triangle, then the triangles are congruent.

HyA (Hypotenuse-Acute angle) Congruence Theorem

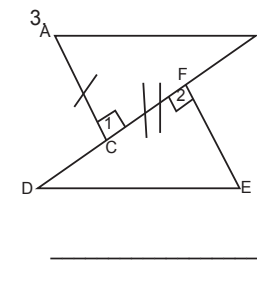
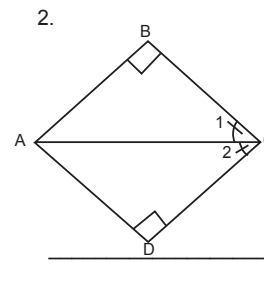
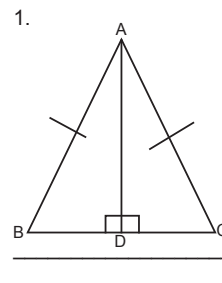
If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

Guide:

1. Draw the figure
2. What is given and what is to be proved?
3. Write the proof in two-column form.

Exercise 7

In each figure, congruent parts are marked. Give additional congruent parts to prove that the right triangles are congruent and state the congruence theorem that justifies your answer.



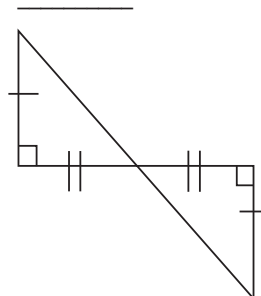
After studying the congruence postulates and theorems the students are now ready to apply them. How can they prove that the two angles or two segments are congruent?
Ask them to perform Activity 9.

Teacher's Note and Reminders

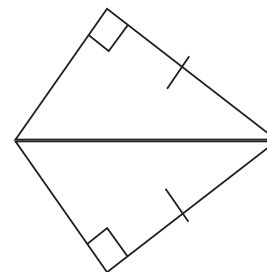


State a congruence theorem. on right triangles.

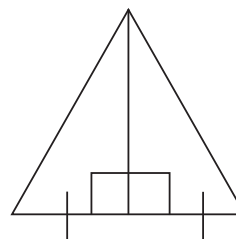
4.



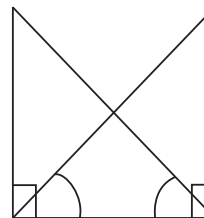
5.



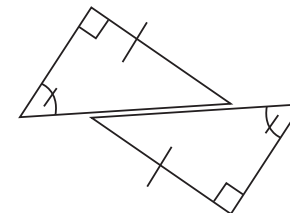
6.



7.



8.



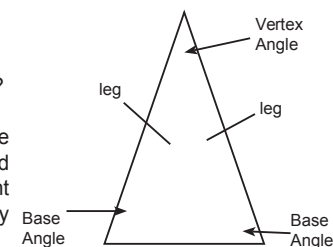
Topic 4: Application of Triangle Congruence

After studying the congruence postulates and theorems you are now ready to apply them. How can you prove that two angles or two segments are congruent?
If they are parts of congruent triangles we can conclude that they are congruent. Let us see how.

Activity 9 WHAT ELSE?

Do you still remember what an isosceles triangle is?

A triangle is isosceles if two of its sides are congruent. The congruent sides are its legs. the third side is the base, the angles opposite the congruent sides are the base angles and the angle included by the legs is the vertex angle.



After the discussion on parts of isosceles triangle, Ask the students to complete the proof of the exercise below.

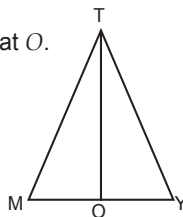
Consider $\triangle TMY$ with $\overline{TM} \cong \overline{TY}$

Is $\angle M \cong \angle Y$?

You find out by completing the proof.

Remember that if they are corresponding parts of congruent triangles then they are congruent.

1. Draw the bisector \overline{TO} of $\angle T$ which intersects \overline{MY} at O .
2. _____ \cong _____ by definition of a bisector
3. _____ \cong _____ given
4. _____ \cong _____ (Why) _____
5. _____ \cong _____ SAS
6. $\angle M \cong \angle Y$ _____



As an exercise, ask the student to prove the converse of isosceles triangle theorem:

Challenge the students to prove that: **An equilateral triangle is equiangular.** They will be guided by the figure and questions below. Let them answer Exercise .

Guide the students to prove: **The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.** Follow the suggested procedure. The students can do this in group and output will be presented to the class.

Teacher's Note and Reminders



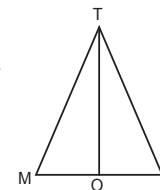
Consider $\triangle TMY$ with $\overline{TM} \cong \overline{TY}$

Is $\angle M \cong \angle Y$?

You find out by completing the proof.

Remember that if they are corresponding parts of congruent triangles then they are congruent.

1. Draw the bisector \overline{TO} of $\angle T$ which intersects \overline{MY} at O .
2. _____ \cong _____ by definition of a bisector
3. _____ \cong _____ given
4. _____ \cong _____ (Why) _____
5. _____ \cong _____ SAS
6. $\angle M \cong \angle Y$ _____



Isosceles Triangle Theorem:

If two sides of a triangle are congruent then the angles opposite these sides are congruent.

How about the converse of isosceles triangle theorem: **If two angles of a triangle are congruent then the sides opposite these angles are congruent.**

Exercise 8

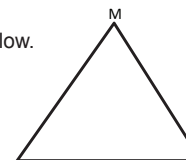
Prove the converse of the isosceles triangle theorem with your group.

Discuss with your group the proof of the statement: **An equilateral triangle is equiangular.**

Use the figure and be guided by the questions below.

Given: $\triangle MIS$ is equilateral

Prove: $\triangle MIS$ is equiangular



In order to prove that $\triangle MIS$ is equiangular you must prove first that $\angle M \cong \angle I \cong \angle S$

1. $\overline{MI} \cong \overline{MS}$ Why?
2. What kind of triangle is $\triangle MIS$?
3. What angles are congruent? Why?
4. $\overline{MI} \cong \overline{MS}$ Why?
5. What angles are congruent? Why?
6. $\angle M \cong \angle I \cong \angle S$ Why?

How will you show that each angle of an equilateral triangle measures 60° ?

Guide Questions:

- a. What is the sum of the measures of the angles of a triangle?
- b. What is true about equilateral triangle?

Guide the students to prove the following: **The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.** Follow the suggested procedure. The students can do this in group and output will be presented to the class.

Procedure:

- Draw an Isosceles $\triangle ABC$.
- Draw the bisector BE of the vertex $\angle B$ which intersects \overline{AC} at E .
- Prove that the two triangles $\triangle BEA$ and $\triangle BEC$ are congruent.
- Show that E is the midpoint \overline{AC} .
- Show \overline{BE} is perpendicular to \overline{AC} at E . (Remember that segments are perpendicular if they form right angles.)

In this section, discussion was on Congruent Triangles. Go back the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on the next section.

Your goal in this section is to take a closer look at some aspects of the topic. And keep in mind the question: **How does knowledge in triangle congruence will help you to solve real-life problems?**

What to Understand



Ask the student to reflect on the different lessons tackled in the module by answering the questions.

Exercise 9

- What is the difference between an equilateral triangle and isosceles triangle?
- One angle of an isosceles triangle measures 60° . What are the measures of the other two angles?
- An angle of an isosceles triangle is 50° . What are the measures of the other two angles? Is there another possible triangle?

Discuss the proof of: **The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.** Do this with your group.

Procedure:

- Draw an Isosceles $\triangle ABC$.
- Draw the bisector BE of the vertex $\angle B$ which intersects \overline{AC} at E .
- Prove that the two triangles $\triangle BEA$ and $\triangle BEC$ are congruent.
- Show that E is the midpoint \overline{AC} .
- Show \overline{BE} is perpendicular to \overline{AC} at E . (Remember that segments are perpendicular if they form right angles.)

Your work will be presented in class.

Theorem: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

In this section, the discussion was on Congruent Triangles

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand



Your goal in this section is to take a closer look at some aspects of the topic. And keep in your mind the question: **"How does knowledge in triangle congruence will help you to solve real life problems?"**

Students can write their answers in their journal, then do Activity 10

Teacher's Note and Reminders



Questions:

- When are two triangles congruent?
- What are the conditions for triangle congruence.?
- How can we show congruent triangles through paper folding?
- Say something about Isosceles triangle.
- Is equilateral triangle isosceles?
- Is equilateral triangle equiangular?
 - What can you say about the bisector of the vertex angle of an isosceles triangle?

Activity 10 FLY FLY FLY



During the Math Fair, one of the activities is a symposium in which the delegates will report on an inquiry about an important concept in Math. You will report on how congruent triangles are applied in real-life. Your query revolves around this situation;

1. Design at most 5 different paper planes using congruent triangles.
2. Let it fly and record the flying time and compare which one is the most stable.
3. Point out the factors that affect the stability of the plane.
4. Explain why such principle works.
5. Draw out conclusion and make recommendations.

Procedure:

1. Each group will prepare 5 paper planes
2. Apply your knowledge on triangle congruence.
3. Follow steps 2 to 5.
4. What is the importance of congruent triangles in making paper planes?

Activity 11 SARANGOLA NI PEPE



Another application of congruent triangles is on stability of your kites.

Show us how triangle congruence works.

In the upcoming City Festival, there will be a kite flying. You are to submit a certain design of kite and an instruction guide of how it operates. The designer who can come up with a kite which can fly the longest wins a prize.

Present the mechanics on how you come up with such a design.

Another challenge to your students is this task which they can do at home.

Submit a journal on how you proved two triangles are congruent. Did you enjoy these lesson on triangle congruence?

Take a picture of triangles in your house. Identify how each of these congruences could help a builder construct furnitures. Make a portfolio of these pictures and discussion.

Teacher's Note and Reminders



Activity 12

3 = 2 = 1 CARD



Since you are done with the concepts and activities about triangle congruence, now let us summarize it by completing the table below:

3 things you have learned

2 things which are interesting

1 question you still have in mind

What to Transfer



Performance GRASPS TASK

(S) One of the moves of the City Council for economic development is to connect a nearby island to the mainland with a suspension bridge for easy accessibility of the people. Those from the island can deliver their produce and those from the mainland can enjoy the beautiful scenery and beaches of the island.

(R) As one of the engineers of the DPWH who is commissioned by the Special Project Committee, (G) you are tasked to present (P) a design/blueprint (P) of a suspension bridge to the (A) City Council together with the City Engineers. (S) Your presentation will be evaluated according to its accuracy, practicality, stability and mathematical reasoning.





Teacher's Note and Reminders



Now that you are done check your work with the rubric below.

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the concepts of triangle congruence.	The computations are accurate and show the use of the concepts of triangle congruence.	Some computations are erroneous and show the use of some concepts of triangle congruence.	The computations are erroneous and do not show the use of the concepts of triangle congruence.	
Creativity	The design is comprehensive and displays the aesthetic aspects of the mathematical concepts learned.	The design is presentable and makes use of the concepts of geometric representations.	The design makes use of the geometric representations but not presentable.	The design doesn't use geometric representations and not presentable.	
Stability	The design is stable, comprehensive and displays the aesthetic aspects of the principles of triangle congruence.	The design is stable, presentable and makes use of congruent triangles.	The design makes use of triangles but not stable.	The design does not use triangles and is not stable.	
Mathematical reasoning	The explanation is clear, exhaustive or thorough and coherent. It includes interesting facts and principles.	The explanation is clear and coherent. It covers the important concepts.	The explanation is understandable but not logical.	The explanation is incomplete and inconsistent.	
				OVERALL RATING	

Another challenge to you is this task for you to accomplish at home

Submit a journal on how you proved two triangles congruent. Did you enjoy the lesson on triangle congruence?

Take a picture of triangles in the house. Identify how each of these congruences could help a builder to construct a furniture. Make a portfolio of these pictures and discussion.



Teacher's Note and Reminders



SUMMARY

Designs and patterns having the same size and the same shape are seen in almost all places. You can see them in bridges, buildings, towers, in furniture even in handicrafts and fabrics

Congruence of triangles has many applications in real world. Architects and engineers use triangles when they build structures because they are considered to be the most stable of all geometric figures. Triangles are oftentimes used as frameworks, supports for many construction works. They need to be congruent.

In this module you have learned that:

- Two triangles are congruent if their vertices can be paired such that corresponding sides are congruent and corresponding angles are congruent.
- The three postulates for triangle congruence are:
 - a. *SAS Congruence* – if two sides and the included angle of one triangle are congruent respectively two sides and the included angle of another triangle then the triangles are congruent.
 - b. *ASA Congruence* – if two angles and the included side of one triangle are congruent respectively two angles and the included side of another triangle then the triangles are congruent.
 - c. *SSS Congruence* – if the three sides of one triangle are congruent respectively three sides of another triangles then the triangles are congruent.
- AAS Congruence Theorem – if the two angles and the non-included side of one triangle are congruent to the two angles and the non-included side of another triangle then the triangles are congruent.
- The congruence theorems for right triangles are:
 - a. *LL Congruence* – if the legs of one right triangle are congruent respectively to the legs of another right triangle, then the triangles are congruent.
 - b. *LA Congruence* – if a leg and an acute angle of one triangle are congruent respectively to a leg and an acute angle of another right triangle, then the triangles are congruent.
 - c. *HyL Congruence* – if the hypotenuse and a leg of one right triangle are congruent respectively to the hypotenuse and a leg of another right triangle, the triangles are congruent.
 - d. *HyA Congruence* – if the hypotenuse and an acute angle of one right triangle are congruent respectively to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.
- Isosceles Triangle Theorem – If two sides of a triangle are congruent then the angles opposite these sides are congruent.
- Converse of Isosceles Triangle Theorem – if two angles of a triangle are congruent then the sides opposite these angles are congruent.
- An equilateral triangle is equiangular.
- The measure of each angle of an equilateral triangle is 60° .

Teacher's Note and Reminders

