TEACHING GUIDE

Module 8: Inequalities in Triangles

A. Learning Outcomes

All activities and inputs in this module that you have to facilitate are aligned with the content and performance standards of the K to 12 Mathematics Curriculum for Grade 8. Ensuring that students undertake all the activities at the specified time with your maximum technical assistance lies under your care. The table below shows how the standards are unpacked.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="3">SUBJECT:
Grade 8 Mathematics
QUARTER:
Third Quarter
STRAND:
Geometry
TOPIC:
Inequalities in Triangles
LESSON:
Inequalities in Triangles</td><td colspan="2">LEARNING COMPETENCIES
• (KNOWLEDGE) State and illustrate the theorems on triangle inequalities such as exterior angle inequality theorem, triangle inequality theorems, hinge theorem and its converse.
• (SKILL) Apply theorems on triangle inequalities to:
a. determine possible measures for the angles and sides of triangles.
b. justify claims about the unequal relationships between side and angle measures.
• (SKILL) Use the theorems on inequalities in triangles to prove statements involving triangle inequalities.</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:

Students will understand that inequalities in triangles can be justified deductively.</td><td>ESSENTIAL QUESTION:

How can you justify inequalities in triangles?</td></tr>
<tr><td colspan="2">TRANSFER GOAL:
Students will on their own justify inequalities in triangles that are evident in the things around us such as in artworks and designs.</td></tr>
</table>

B. Planning for Assessment

To assess learning, students should perform a task to demonstrate their understanding of Inequalities in Triangles. It is expected that students, having been equipped with knowledge and skills on inequalities in triangles, would come up with a product—a design and a miniature model of a folding ladder that can reach as high as 10 feet. This task is found in Activity No. 23 of the module

Assessment Map

To ensure understanding and learning, students should be engaged in different learning experiences with corresponding assessment. The table below shows the assessment at different stages of the learning process. Details of this assessment map will guide you which items in each stage of assessment are under specific domains—Knowledge, Process/Skills, Understanding or Performance. . Be sure to expose students to varied assessment in this module in order to develop their critical thinking and problem solving skills.

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Pre – assessment/ Diagnostic	• Pre-Test Items No. 1,2 and 10	• Pretest Items No. 3, 4, 7, 12, and 13	• Pretest Items No. 5, 6, 8, 9, 11, and 14	• Pretest Items No. 14-20

<table>
<tr>
<td rowspan="2">Formative</td>
<td>Revisiting and Modifying Answers in Activity No. 1</td>
<td>Revisiting and Modifying Answers in Activity No. 3</td>
<td>Revisiting and Modifying Answers in Activity No. 2</td>
<td>• Answering Questions of the following activities:
<table>
<tr><th>Act.</th><th>Items</th></tr>
<tr><td>8</td><td>1-2</td></tr>
<tr><td>9</td><td>8-9</td></tr>
<tr><td>10</td><td>5</td></tr>
<tr><td>21</td><td>1-11</td></tr>
</table>
• Answering Quiz Items
<table>
<tr><th>Quiz</th><th>Items</th></tr>
<tr><td>1</td><td>A 1-3</td></tr>
<tr><td>2</td><td>C</td></tr>
<tr><td>3</td><td>9-12</td></tr>
<tr><td>5</td><td>E 1</td></tr>
</table>
• Answering Questions:
✓ Mathematics in the Kitchen
✓ Mathematics in Art: Geometric Shapes for Foundation Piecing
✓ Mathematics for Eco-Architecture
✓ Mathematics in the Garden</td>
</tr>
<tr>
<td>• Quiz
<table>
<tr><th>Quiz</th><th>Items</th></tr>
<tr><td>1</td><td>A 1-3</td></tr>
<tr><td>2</td><td>A 1-3</td></tr>
<tr><td>3</td><td>A 1, 5</td></tr>
<tr><td>4</td><td>1</td></tr>
</table>
</td>
<td>• Completing the tables of the following activities: 4, 5, 6, 7, 9, 10
• Answering Questions of the following activities:
<table>
<tr><th>Act.</th><th>Items</th></tr>
<tr><td>4</td><td>6</td></tr>
<tr><td>5</td><td>6-7</td></tr>
<tr><td>6</td><td>1-2</td></tr>
<tr><td>7</td><td>1-3</td></tr>
</table>
• Completing the proofs of the following activities: No. 11, 12, 13, 14, 15, 16
• Answering Quiz items
<table>
<tr><th>Quiz</th><th>Items</th></tr>
<tr><td>1</td><td>B</td></tr>
<tr><td>2</td><td>B</td></tr>
<tr><td>3</td><td>A 2-4</td></tr>
<tr><td>5</td><td>C 1-4</td></tr>
</table>
</td>
<td>• Answering Questions of the following activities:
<table>
<tr><th>Act.</th><th>Items</th></tr>
<tr><td>4</td><td>1-5</td></tr>
<tr><td>5</td><td>1-5, 8</td></tr>
<tr><td>7</td><td>4</td></tr>
<tr><td>9</td><td>1-8</td></tr>
<tr><td>10</td><td>1-4</td></tr>
<tr><td>22</td><td>1-3</td></tr>
</table>
• Answering Quiz items
<table>
<tr><th>Quiz</th><th>Items</th></tr>
<tr><td>1</td><td>C</td></tr>
<tr><td>3</td><td>A 6-11</td></tr>
<tr><td>4</td><td>2-3</td></tr>
<tr><td>5</td><td>A 1-4, B 1-5, D</td></tr>
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			• Answering the Questions about Watch-this problems in the following activities: 17, 18, 19, 20 • Solving It's-Your-Turn problems of the following activities: 17, 18, 19, 20	✓ Mathematics in Geography ✓ Mathematics in Architecture: The World's Thinnest House ✓ Mathematics in Art: Color Triangle ✓ Mathematics in Psychology ✓ Mathematics in Fashion ✓ Career in Mathematics: Air Traffic Controller
TYPE	**KNOWLEDGE**	**PROCESS/SKILLS**	**UNDERSTANDING**	**PERFORMANCE**
Summative	• Post-Test Items No. 1,2 & 10	• Pretest Items No. 3, 4, 7, 12, & 13	• Pretest Items No. 5, 6, 8, 9, 11, & 14	• Pretest Items No. 14-20
	Finalizing Answers in Activity No. 1	Finalizing Answers in Activity No. 3	Finalizing Answers in Activity No. 2	**Act 23:** Creation of a design and a miniature model of a folding ladder that can reach as high as 10 feet—allowing its user to gain access to their ceilings/ roofs during floods caused by typhoons or monsoon rains. The standards are as follows: ✓ Product must be efficient, ✓ The design must be creative, ✓ Measurements are accurate ✓ Mathematical Justification of the design is logically clear, convincing, and professionally delivered
			All items in Activity No. 24	

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Self - assessment	Answering Activity No. 1	Answering Activity No. 3	Answering Activity No. 2	Answering questions in More Triangular Designs and Artworks

Assessment Matrix (Summative Test)

What will I assess?	Post-Test Items by Levels of Assessment			
	Knowledge	Process/ Skills	Understanding	Product
	3 items	5 items	6 items	6 items
	15%	25%	30%	30%
	Scoring: One point Each			
Competency No. 1: State and illustrate the theorems on triangle inequalities such as exterior angle inequality theorem	1, 2			
				20
Competency No. 2: Apply theorems on triangle inequalities to determine possible measures for the angles and sides of triangles.		4, 11, 13		
			5, 10	
				15, 16
Competency No. 3: Apply theorems on triangle inequalities to justify claims about the unequal relationships between side and angle measures.	3			
		12, 14		
			6, 8	
				17
Competency No. 4: Use the theorems on inequalities in triangles to prove statements involving triangle inequalities			7, 9	
				18, 19
Competency Nos. 1, 2, 3, and 4	**Activity:** Creation of a Design or Product **Scoring:** By Rubrics			

C. Planning for Teaching-Learning

Introduction:

The unit lesson on Geometry for Grade 8 is to be delivered in the Third Quarter of the school year. Triangle Inequalities is the third chapter of Geometry for Grade 8. Since there are four chapters in this unit, you are expected to facilitate this lesson within 15 days, non-inclusive of extra time student spend for tasks that you may most likely assign for students to do in their independent/cooperative learning time, free time, or after school.

INTRODUCTION AND FOCUS QUESTIONS:

Aside from arresting the attention and interest of the students, the introduction stresses the purpose of studying inequalities in triangles.

The introduction, through the essential question, serves as a steering mechanism of the lesson. All sections and activities in the lesson are geared towards the goal of answering it.

As the learning facilitator, your role is to emphasize the Essential Question in the introduction and to remind the students about it in every section of the module.

Your key role is to underscore that the process of answering the essential question on how inequalities in triangles can be justified will:

- improve their attention to details;
- shape their deductive thinking;
- hone their reasoning skills; and
- polish their mathematical communication.

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LESSONS AND COVERAGE:

This section of the learning module cites the subtopics of Inequalities in Triangles and the competencies that will be covered in the module. Your task is to know these competencies so you can ensure that students shall have learned them at the end of the lesson.

MODULE MAP:

Through the Module Map, you will be able to show to the students that

- inequalities exist in one triangle and in two triangles
- four theorems can be developed, verified, and proved regarding inequalities in one triangle
- two theorems can be developed, verified, and proved regarding inequalities in two triangles

PRE-ASSESSMENT:

This section features the test that diagnoses what students already know about the topic before the actual teaching of the lesson. This feedback information is valuable to you because it directs you on how to proceed as a facilitator of learning. As a result, you are able to provide the appropriate technical assistance students need as the lesson unfolds.

Answer Key to Pre-Test

1. **C** The measure of an exterior angle of a triangle is always greater than either remote interior angle. Basis: Exterior Angle Inequality Theorem.
2. **B** Angle 5 is an exterior angle of triangle TYP because segment PR is an extension of side TP. Basis: Definition of Exterior Angle.
3. **B** Marie was not able to form a triangle because the sum of the two shorter lengths 4 and 5 is not greater than the third side of 9 inches. Basis: Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$).
4. **A** Working Inequality $4x - 3 < 42^o$
   Basis: Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
5. **B** Basis: Converse of Hinge Theorem
6. **B** The included angle between two distances 3 km and 4 km covered by Oliver is 150^o. This is larger than that of Ruel's--140^o. Therefore, Oliver is father because his distance is opposite a larger angle. *Basis: Hinge Theorem.*
7. **B**
8. **C** *Basis: Converse of Hinge Theorem.*
9. **B** Conclusions must be based on complete facts.
10. A $m\angle D = 180 - (86 + 49) = 45$. The shortest side is $\angle D$. Therefore the shortest side is opposite it. *Basis: Triangle Inequality Theorem 2 (Aa→Ss)*
11. **C**
    - Considering the triangle with sides p, q and s: The angle opposite p is 61°. Hence, $s < q < p$.
    - Considering the triangle with sides r, s and t: The angle opposite r is 60°. Hence, $t < r < s$.
      Combining both results, $t < r < s < q < p$
      Basis: Triangle Inequality Theorem (Aa→Ss)

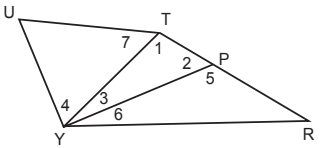
III. PRE - ASSESSMENT

Find out how much you already know about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items. During the checking, take note of the items that you were not able to answer correctly and find out the right answers as you go through this module.

1. The measure of an exterior angle of a triangle is always _____.

   a. greater than its adjacent interior angle.
   b. less than its adjacent interior angle.
   c. greater than either remote interior angle.
   d. less than either remote interior angle.

2. Which of the following angles is an exterior angle of ΔTYP?

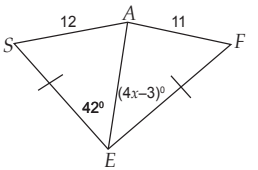


   A. $\angle 4$ B. $\angle 5$ C. $\angle 6$ D. $\angle 7$

3. Each of Xylie, Marie, Angel and Chloe was given an 18-inch piece of stick. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Xylie—6 in, 6 in, 6 in; Marie—4 in, 5 in, 9 in; Angle—7 in, 5 in, 6 in; and Chloe—3 in, 7 in, 5 in. Who among them was not able to make a triangle?

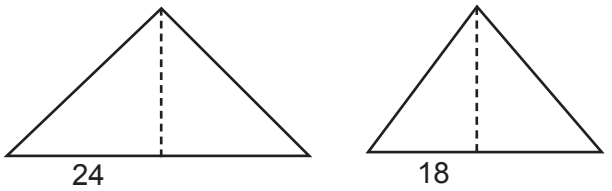
   a. Xylie b. Marie c. Angel d. Chloe

4. What are the possible values for x in the figure?



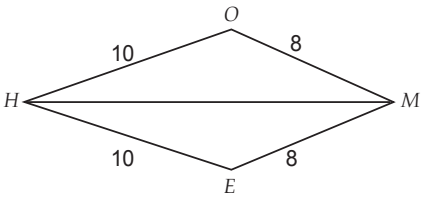
   a. $x < 11.25$ c. $x \leq 11.25$
   b. $x > 11.25$ d. $x \geq 11.25$

12. **C** *Basis: Triangle Inequality Theorem (Ss→Aa)*
13. **C** *Basis: Triangle Inequality Theorem (Aa→Ss)*
14. **B** *Basis: Triangle Inequality Theorem 3* ($S_1 + S_2 > S_3$)
15. **B**
16. **D**
17. **D**



18. **C** III mostly conveys wrong signal to a client.
19. **A**
20. **D**

5. From the inequalities in the triangles shown, a conclusion can be reached using the converse of hinge theorem. Which of the following is the last statement?



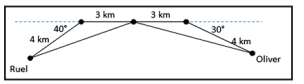
a. $HM \cong HM$
b. $m\angle OHM > m\angle EHM$
c. $HO \cong HE$
d. $m\angle EHM > m\angle OHM$

6. Hikers Oliver and Ruel who have uniform hiking speed walk in opposite directions- Oliver, eastward whereas Ruel, westward. After walking three kilometers each, both of them take left turns at different angles- Oliver at an angle of 30^0 and Ruel at 40^0. Both continue hiking and cover another four kilometers each before taking a rest. Which of the hikers is farther from their point of origin?

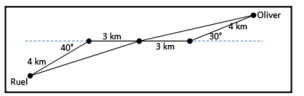
a. Ruel
b. Oliver
c. It cannot be determined.
d. Ruel is as far as Oliver from the rendezvous.

7. Which of the following is the accurate illustration of the problem?

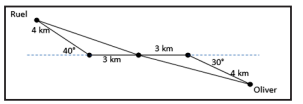
a.



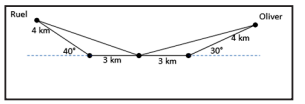
b.



c.

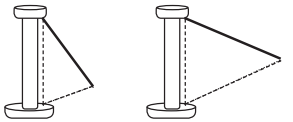


d.



Teacher's Note and Reminders

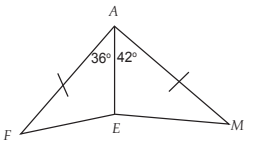
8. The chairs of a swing ride are farthest from the base of the swing tower when the swing ride is at full speed. What conclusion can you make about the angles of the swings at different speeds?



a. The angles of the swings remain constant whether the speed is low or full.
b. The angles of the swings are smaller at full speed than at low speed.
c. The angles of the swings are larger at full speed than at low speed.
d. The angles of the swings are larger at low speed than at full speed.

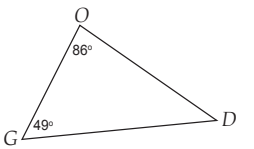
9. Will you be able to conclude that $EM > EF$ if one of the following statements is not established: $\overline{AE} \cong \overline{AE}$, $\overline{AF} \cong \overline{AM}$, $m\angle MAE > m\angle FAE$?

a. Yes, I will.
b. No, I won't.
c. It is impossible to decide.
d. It depends on which statement is left out.



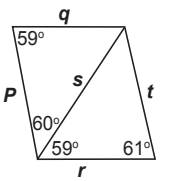
10. Which side of ΔGOD is the shortest?

a. $\overline{GO}$
b. $\overline{DO}$
c. $\overline{DG}$
d. $\overline{GD}$



11. The diagram is not drawn to scale. Which of the following combined inequalities describes p,q,r,s, and t?

a. $p < q < r < s < t$
b. $s < p < q < r < t$
c. $t < r < s < q < p$
d. $q < p < t < r < s$

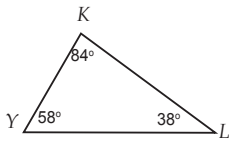


Teacher's Note and Reminders

12. In ΔTRU, TR = 8 cm, RU = 9 cm, and TU = 10 cm. List the angles in order from least to greatest measure.

a. $m\angle T,\ m\angle R,\ m\angle U$
b. $m\angle U,\ m\angle T,\ m\angle R$
c. $m\angle R,\ m\angle T,\ m\angle U$
d. $m\angle U,\ m\angle R,\ m\angle T$

13. List the sides of ΔLYK in order from least to greatest measure.



a. $\overline{LY}, \overline{YK}, \overline{LK}$
b. $\overline{YK}, \overline{YL}, \overline{LK}$
c. $\overline{LY}, \overline{LK}, \overline{KL}$
d. $\overline{YK}, \overline{LK}, \overline{LY}$

14. What is the range of the values of the diagonal d of a lot shaped like a parallelogram if adjacent sides are 10 inches and 14 inches?

a. $4 \geq d \geq 24$
b. $4 < d < 24$
c. $4 \leq d \leq 24$
d. $4 > d > 24$

For items no. 15-20, use the figure shown.

15. A *balikbayan* chose you to be one of the contractors to design an A-frame house maximizing the size of two square lots with dimensions 18 ft and 24 ft on each side. Which of the following is affected by the dimensions of the lot if the owner would like to spend the same amount of money on the roofs?

I. The width of the base of the house frames
II. Design of the windows
III. The height of the houses
IV. The roof angles

a. I and IV
b. III and IV
c. II, III and IV
d. I, II, III, and IV

Teacher's Note and Reminders

16. Which of the following theorems justifies your response in item no. 15?

    I. Triangle Inequality Theorem 1
    II. Triangle Inequality Theorem 2
    III. Triangle Inequality Theorem 3
    IV. Hinge Theorem
    V. Converse of Hinge Theorem

    a. I, II, and III b. IV only c. IV and V d. V only

17. If the owner would like the same height for both houses, which of the following is true?

    I. Roof costs for the larger lot is higher than that of the smaller lot.
    II. The roof of the smaller house is steeper than the larger house.

    a. I only c. neither I nor II
    b. II only d. I and II

18. What considerations should you emphasize in your design presentation so that the *balikbayan* would award you the contract to build the houses?

    I. Kinds of materials to use considering the climate in the area
    II. Height of floor-to-ceiling corner rooms and its occupants
    III. Extra budget needed for top-of-the-line furnishings
    IV. Architectural design that matches the available funds
    V. Length of time it takes to finish the project

    a. I, II, and IV c. I, II, IV, and V
    b. I, IV, and V d. I, II, III, IV, V

19. Why is it not practical to design a house using A-Frame style in the Philippines?

    I. A roof also serving as wall contributes to more heat in the house.
    II. Placement of the windows and doors requires careful thinking.
    III. Some rooms of the house would have unsafe low ceiling.
    IV. An A-Frame design is an unusually artful design.

    a. I and III c. I, II, and III
    b. II and IV d. I, II, III, IV

20. Why do you think an A-Frame House is practical in countries with four seasons?

    A. The design is customary.
    B. An artful house is a status symbol.
    C. The cost of building is reasonably low.
    D. The snow glides easily on steep roofs.

Before engaging the students in the different activities you have to underscore the following to the students:

Mathematical Connection — learning new lessons requires the use of lessons previously learned;
Cooperative Learning — learning is much easier, faster, more meaningful and more fun when working with group mates;
Engagement — learning is maximized through active performance of students in all activities

Activity No.1:
My Decisions Now and Then Later

Let them perform activity No. 1 in at most 5 minutes. Inform the students that there is no right or wrong answer because the activity is only intended to find out their background knowledge on Inequalities in Triangles. Tell them these: Your answers can be modified after tackling the module. Hence, there will be no checking of your responses. Hence, the answer key that follows is used to check their final answers after tackling the module.

Answer Key to Activity No.1

1. D
2. D (can be should be replaced with is always)
3. D
4. A
5. A

Lesson 1 Inequalities in Triangles

What to Know

Let's start the module by doing three activities that will reveal your background knowledge on triangle inequalities.

Activity 1 MY DECISIONS NOW AND THEN LATER

Directions:

1. Replicate the table below on a piece of paper.
2. Under the my-decision-now column of the first table, write **A** if you agree with the statement and **D** if you don't.
3. After tackling the whole module, you will be responding to the same statements using the second table.

	Statement	My Decision Now
1	To form a triangle, any lengths of the sides can be used.	
2	The measure of the exterior angle of a triangle can be greater than the measure of its two remote interior angles.	
3	Straws with lengths 3 inches, 4 inches and 8 inches can form a triangle.	
4	Three segments can form a triangle if the length of the longest segment is greater than the difference but less than the sum of the two shorter segments.	
5	If you want to find for the longest side of a triangle, look for the side opposite the largest angle.	

	Statement	My Decision Later
1	To form a triangle, any lengths of the sides can be used.	
2	The measure of the exterior angle of a triangle can be greater than the measure of its two remote interior angles.	
3	Straws with lengths 3 inches, 4 inches and 8 inches can form a triangle.	
4	Three segments can form a triangle if the length of the longest segment is greater than the difference but less than the sum of the two shorter segments.	
5	If you want to find for the longest side of a triangle, look for the side opposite the largest angle.	

Artistically Yours

Your task is to get students interested in the new lesson. You may start by posing this task: What objects around us are triangular in shape? You and your students will find out that most objects are circular or rectangular.

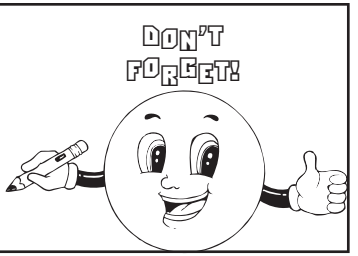
After a 2-minute discussion, divide the class into groups and let them study the pictures and answer the questions in ponder time of Activity No. 2 Artistically Yours for at least three minutes. Let all group representatives report their answers to the questions. Give each representative at most one minute each to be able to maximize time. Process all their answers by unifying all their ideas or supplementing them so it would converge to the expected answers provided.

Invite also the students to discover more triangular designs and artworks by locating them in www.google.com under Images. Instruct them to type any of the following in the search bar: triangular designs, triangular artworks, triangular architecture, triangular art, and more.

Answer Key to Activity No.2

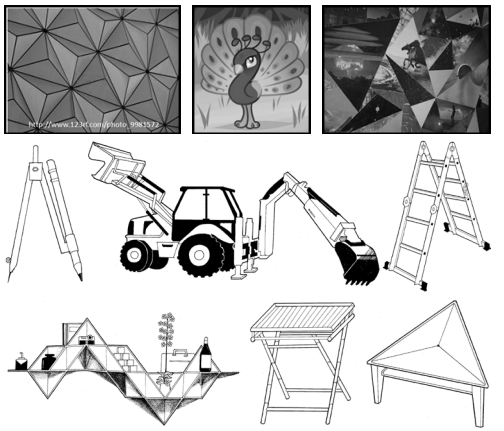
1. Triangles
2. Yes. Some sides are longer than the others and some corners are larger than the others.
3. Possible Answers: Interesting, Practical, creative, artful
4. Because they have not tackled the lesson yet, possible Answer: Inequalities in triangles in these artworks and designs are necessary in order to achieve beauty, artistry, creativity, and usefulness to the designs.

Teacher's Note and Reminders



Activity 2 ARTISTICALLY YOURS!

Direction: Study the artworks below and answer the questions that follow:



QUESTIONS ?

1. What features prevail in the artworks, tools, equipment, and furniture shown?
2. Have you observed inequalities in triangles in the designs? Explain.
3. What is the significance of their triangular designs?
4. How can you justify inequalities in triangles in these designs?

Activity 3 HELLO, DEAR CONCEPT CONTRACTOR!

The figure on the next page is a concept museum of inequalities in triangles. You will be constructing this concept museum throughout this module.

Each portion of the concept museum, mostly triangular, poses a task for you to perform. All tasks are related to knowledge and skills you should learn about inequalities in triangles.

More Triangular Designs and Artworks

1. **Triangular Girl by Caroline Johansson** http://thecarolinejohansson.com/blog/2011/10/triangular-girl-2/
2. **Tile works: Diminishing Triangles** http://sittenintehills64.blogspot.com/2010/05/tile-house-8.html
3. **Repetitive Graduation by Scott Mihalik** http://mathtourist.blogspot.com/2012/02/flight-of-tetrahedra.html
4. **Maths-the best use for golf balls** http://www.whizz.com/blog/fun/maths-best-use-for-golf-balls/
5. **Luxury sailboat** http://edgeretreats.com/
6. **Triangle Card Stand** http://www.behance.net/gallery/TRIANGLE-CARD-STAND/3883741
7. **Triangular Periodic Table** http://www.meta-synthesis.com/webbook/35_pt/pt_database.php?PT_id=40
8. **A triangular approach to fat loss by Stephen Tongue** http://www.flickr.com/photos/32462223@N05/3413593357/in/photostream/
9. **Triangular Petal Card** http://www.flickr.com/photos/32462223@N05/3413593357/in/photostream/

Question:

- Which among these designs and artworks you find most interesting? Explain.
- Which design you would like to pattern from for a personal project?

What is a contractor?

A *contractor* is someone who enters into a binding agreement to build things.

~Wordweb 4.5a by Anthony Lewis~

What is a museum?

Museum is a depository for collecting and displaying objects having scientific or historical or artistic value.

~Wordweb 4.5a by Anthony Lewis~

Activity No.3:
Hello, Dear Concept Contractor!

Your task is to make students understand the activity. To do that, these are the things that you need to do:

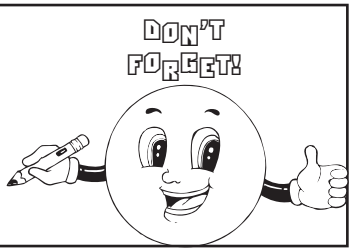
- Check and strengthen their understanding of the definitions of contractor and museum;
- Explain that the finished concept museum will display all the concepts and skills about inequalities in triangles and seeing the tasks at this point provides them an overview of the lesson.
- Point out that building the concept museum takes time—that there's a possibility that they may not be able to do any of the tasks listed on the triangles yet but they know already what to expect to learn. Thus, at the end of the lesson, they will be able to encapsulate all the concepts and skills on inequalities in triangles using the concept museum.
- Let them see that in order for them to completely build the concept museum, they need to perform all the activities in the succeeding sections.

You need to master the concepts and skills of the whole module. To facilitate that, study the completely built concept museum. Note that the students must have built their concept museums at the end of this lesson.

In short, the students have the option not to perform any task yet. The activity is just for presentation in order to direct the students of one goal—to build the concept museum as the lesson unfolds.

The presentation of this activity must take at least five minutes.

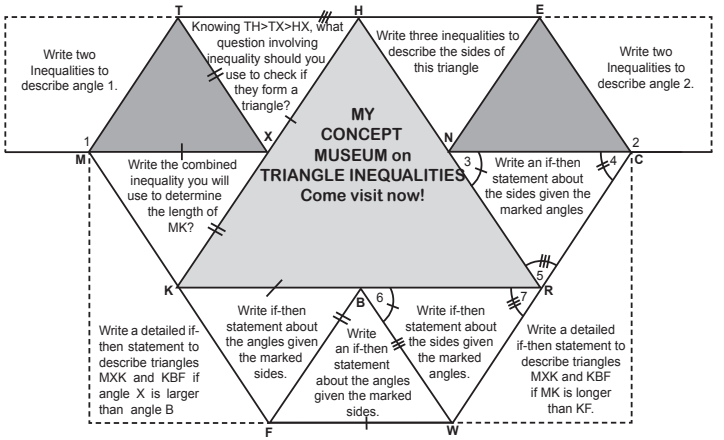
Teacher's Note and Reminders



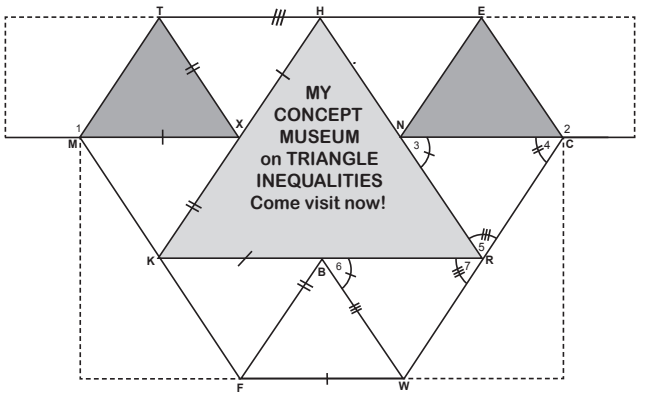
Note that the triangles in this concept museum are not drawn to scale and all sides can be named using their endpoints. Consider using numbers to name the angles of these triangles.

Notice that markings are shown to show which angles are larger and which sides are longer. These markings serve as your hints and clues. Your responses to the tasks must be justified by naming all the theorems that helped you decide what to do.

How many tasks of the concept museum can you tackle now?

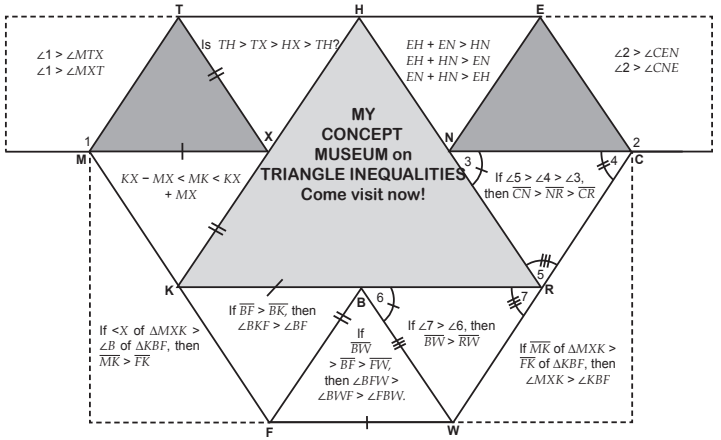


Replicate two (2) copies of the unfilled concept museum. Use the first one for your responses to the tasks and the second one for your justifications.

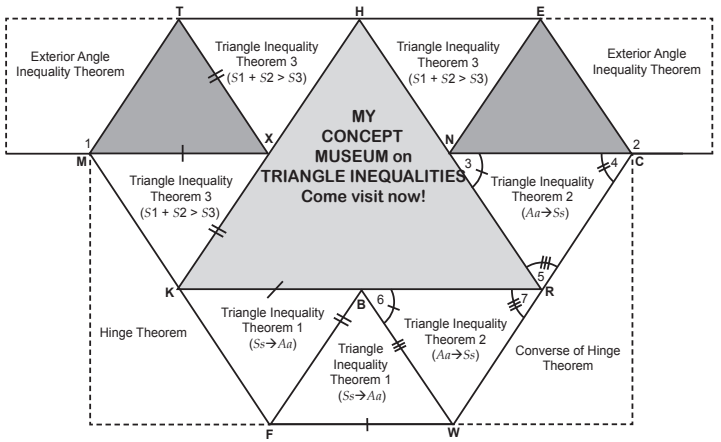


Answer Key to Activity No.3

A. Responses



B. Justifications



Are you excited to completely build your concept museum, Dear Concept Contractor? The only way to do that is by doing all the succeeding activities in the next section of this module. The next section will also help you answer this essential question raised in the activity ***Artistically Yours: How can you justify inequalities in triangles?***

The next lesson will also enable you to do the final project that is inspired by the artworks shown in ***Artistically Yours****. When you have already learned all the concepts and skills related to inequalities in triangles, you will be required to make a model of a folding ladder and justify the triangular features of its design. Your design and its justification will be rated according to these rubrics: accuracy, creativity, efficiency, and mathematical justification.*

What to Process

Your first goal in this section is to develop and verify the theorems on inequalities in triangles. To succeed, you need to perform all the activities that require investigation.

When you make *mathematical generalizations* from your observations, you are actually making *conjectures* just like what mathematicians do. Hence, consider yourself *little mathematicians* as you perform the activities.

Once you have developed these theorems, your third goal is to prove these theorems. You have to provide statements and/or reasons behind statements used to deductively prove the theorems.

The competence you gain in writing proofs enables you to justify inequalities in triangles and in triangular features evident in the things around us.

Before you go through the process, take a few minutes to review and master again the knowledge and skills learned in previous geometry lessons. The concepts and skills on the following topics will help you succeed in the investigatory and proof-writing activities.

1. Axioms of Equality

1.1 Reflexive Property of Equality
- For all real numbers p, $p = p$.

1.2 Symmetric Property of Equality
- For all real numbers p and q, if $p = q$, then $q = p$.

1.3 Transitive Property of Equality
- For all real numbers p, q, and r, if $p = q$ and $q = r$, then $p = r$.

1.4 Substitution Property of Equality
- For all real numbers p and q, if $p = q$, then q can be substituted for p in any expression.

WHAT TO PROCESS:

The **PROCESS** section showcases investigatory activities designed to develop and verify the theorems to learn in the lesson. This is also where students are given the opportunities to practice the concepts and skills learned in the lesson and to write proofs of the theorems.
This section is characterized by student-centered activities as inspired by this saying of Kahlil Gibran:

The teacher who is indeed wise does not bid you to enter the house of his wisdom but rather leads you to the threshold of your mind.

Your task in this section is to make sure that all the group activities that are suggested in the learning module shall be completely delivered.

Your responsibilities involve the following:

1. Conduct a quick but comprehensive review of the pre-requisite skills needed to succeed in the new lesson; and

2. Manage group work.
   2.1 Grouping of students
   Suggestion: Form at least two sets of groupings so students will learn to work with different group mates
   2.2 Time allotment for each group work
   Note: Activities in this section are simplified so that they can be performed in a short span of time. It can be done individually or as a group.
   Suggestion: For the students to finish the whole module on time (within two weeks), you may opt to let groups do the activity in their free time or after class. Let them write their answers to questions in Ponder Time on a piece of manila paper.
   2.3 Do's and don'ts during group work
   2.4 Monitor student behavior during group work to ensure time is spent on the task.
   2.5 Give technical assistance during group work so that group responses to activities and process questions are accurate. In short provide subtle coaching.
   Suggestion: If you let the groups answer the activity as an assignment, give them at least 3-5 minutes to review their answers so you may be able to give them technical assistance if their outputs have errors.

2. Properties of Equality
2.1 Addition Property of Equality
- For all real numbers p, q, and r, if $p = q$, then $p + r = q + r$.

2.2 Multiplication Property of Equality
- For all real numbers p, q, and r, if $p = q$, then $pr = qr$.

3. Definitions, Postulates and Theorems on Points, Lines, Angles and Angle Pairs
3.1 Definition of a Midpoint
- If points P, Q, and R are collinear (P–Q–R) and Q is the midpoint of $\overline{PR}$, then $\overline{PQ} \cong \overline{QR}$.

3.2 Definition of an Angle Bisector
- If $\overrightarrow{QS}$ bisects $\angle PQR$, then $\angle PQS \cong \angle SQR$.

3.3 Segment Addition Postulate
- If points P, Q, and R are collinear (P–Q–R) and Q is between points P and R, then $\overline{PQ} + \overline{QR} \cong \overline{PR}$.

3.4 Angle Addition Postulate
- If point S lies in the interior of $\angle PQR$, then $\angle PQS + \angle SQR \cong \angle PQR$.

3.5 Definition of Supplementary Angles
- Two angles are supplementary if the sum of their measures is 180º.

3.6 Definition of Complementary Angles
- Two angles are complementary if the sum of their measures is 90º.

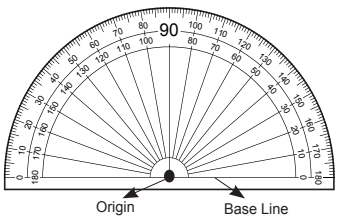
3.7 Definition of Linear Pair
- Linear pair is a pair of adjacent angles formed by two intersecting lines

3.8 Linear Pair Theorem
- If two angles form a linear pair, then they are supplementary.

3.9 Definition of Vertical Angles
- Vertical angles refer to two non-adjacent angles formed by two intersecting lines

3.10 Vertical Angles Theorem
- Vertical angles are congruent.

4. How to Measure Angles using a Protractor



Internet Learning

Mastering the Skill in Estimating Measures of Angles

Interactive:
- **http://www.mathplayground.com/measuringangles.html**
- **http://www.teacherled.com/resources/anglemeasure/anglemeasureload.html**

Games:
- **http://www.bbc.co.uk/schools/teachers/ks2_activities/maths/angles.shtml**
- **http://www.innovationslearning.co.uk/subjects/maths/activities/year6/angles/game.asp**
- **http://www.bbc.co.uk/keyskills/flash/kfa/kfa.shtml**
- **http://resources.oswego.org/games/bananahunt/bhunt.html**
- **http://www.fruitpicker.co.uk/activity/**

2.6 Processing of outputs in group work

Suggestion: Let them post their work for everyone to see. If groups have similar answers, you may decide (or let the class decide) only one or two groups to discuss their answers to questions in Ponder Time. If there are groups with different answers, let the class discuss these answers. Note that a good teacher facilitator minimizes unexpected answers by giving technical assistance to every group before posting outputs.

Your facilitating role is crucial so that students are able to achieve the goal in this section to develop, verify, and prove all six theorems of inequalities in triangles and to continue to unlock triangles in their concept museum.

What to Process

Your first goal in this section is to develop and verify the theorems on inequalities in triangles. To succeed, you need to perform all the activities that require investigation.

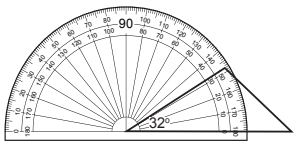
When you make *mathematical generalizations* from your observations, you are actually making *conjectures* just like what mathematicians do. Hence, consider yourself *little mathematicians* as you perform the activities.

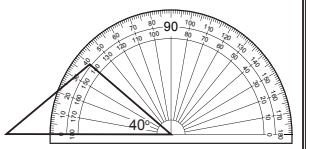
Once you have developed these theorems, your third goal is to prove these theorems. You have to provide statements and/or reasons behind statements used to deductively prove the theorems.

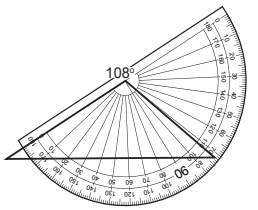
The competence you gain in writing proofs enables you to justify inequalities in triangles and in triangular features evident in the things around us.

Before you go through the process, take a few minutes to review and master again the knowledge and skills learned in previous geometry lessons. The concepts and skills on the following topics will help you succeed in the investigatory and proof-writing

To measure an angle, the protractor's origin is placed over the vertex of an angle and the base line along the left or right side of the angle. The illustrations below show how the angles of a triangle are measured using a protractor.







Mathematical History

Who invented the first advanced protractor?

Capt. Joseph Huddart (1741-1816) of the United States Navy invented the first advanced protractor in 1801. It was a three-arm protractor and was used for navigating and determining the location of a ship

~Brian Brown of www.ehow.com~

To read more about the history of protractor, visit these website links:

- http://www.counton.org/museum/floor2/gallery5/gal3p8.html
- http://www.ablogaboutthistory.com/2011/07/29/the-worlds-first-protractor/

5. Definitions and Theorems on Triangles

5.1 The sum of the measures of the angles of a triangle is 180º.

5.2 Definition of Equilateral Triangle
- An equilateral triangle has three sides congruent.

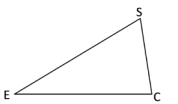
5.3 Definition of Isosceles Triangle
- An isosceles triangle has two congruent sides.
- *Is an equilateral triangle isosceles*? Yes, since it also has two congruent sides.
- Base angles of isosceles triangles are congruent.
- Legs of isosceles triangles are congruent.

5.4 Exterior Angle of a Triangle
- An exterior angle of a triangle is an angle that forms a linear pair with an interior angle of a triangle when a side of the triangle is extended.

5.5 Exterior Angle Theorem
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two interior angles of the triangle.

5.6 Sides and Angles of a Triangle

- $\angle S$ is opposite $\overline{EC}$ and $\overline{EC}$ is opposite $\angle S$.
- $\angle E$ is opposite $\overline{SC}$ and $\overline{SC}$ is opposite $\angle E$
- $\angle C$ is opposite $\overline{ES}$ and $\overline{ES}$ is opposite $\angle C$.

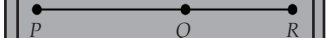


Notes to the Teacher

For the review of all the pre-requisite concepts of the lesson on inequality of triangles, you may decide to present it in a creative manner like making sets of flashcards for each of the following:

Set 1: Axioms of Equality
Set 2: Properties of Equality
Set 3: Definitions, Postulates, and Theorems on Points, Lines, Angles, and Angle Pairs
Set 4: Definitions and Theorems on Triangles
Set 5: Definitions and Postulates on Triangle Congruence
Set 6: Properties of Inequality

Sample Flash Card (Front and Back):

• If points P, Q and R are collinear and Q is the midpoint of $\overline{PR}$, then $\overline{PQ} \cong \overline{QR}$  	Definition of a Midpoint

You may also assign each group of students to prepare a specific set of flash cards using used folders. You may then have a quiz bee for six representatives of a group using the flash cards. If a competitor is the first one to name the axiom, property, definition, theorem or postulate flashed, he then can make a step forward until he/she who reaches the front of the classroom is declared as winner. Another set of representatives is called until everyone has mastered all the axioms, properties, definitions, theorems, and postulates.

Include in your discussion Capt. Joseph Huddard—the inventor of the first advanced protractor. In that connection, invite them to visit presented website links about protractors and those that have interactive activities and games that enable them to master the skill in estimating measures of angles and knowledge of triangle congruence postulates. In this manner, their internet visits would be more educational. Follow up on their Internet activity by asking them to share their insights about learning mathematics online.

Internet Learning Mastering the Triangle Congruence Postulates

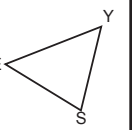
Video
- http://www.onlinemathlearning.com/geometry-congruent-triangles.html

Interactive
- http://www.mrperezonlinemathtutor.com/G/1_5_Proving_Congruent_SSS_SAS_ASA_AAS.html
- http://nlvm.usu.edu/en/nav/frames_asid_165_g_1_t_3.html?open=instructions
- http://www.mangahigh.com/en/maths_games/shape/congruence/congruent_triangles?localeset=en

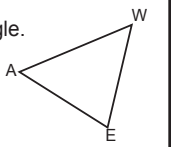
6. Definition and Postulates on Triangle Congruence

6.1 Definition of Congruent Triangles: Corresponding parts of congruent triangles are congruent (CPCTC).

6.2 Included Angle
- Included angle is the angle formed by two distinct sides of a triangle.
  - $\angle YES$ is the included angle of $\overline{EY}$ and $\overline{ES}$
  - $\angle EYS$ is the included angle of $\overline{YE}$ and $\overline{YS}$
  - $\angle S$ is the included angle of $\overline{SE}$ and $\overline{SY}$



6.3 Included Side
- Included side is the side common to two angles of a triangle.
  - $\overline{AW}$ is the included side of $\angle W\mathbf{A}E$ and $\angle E\mathbf{W}A$
  - $\overline{EW}$ is the included side of $\angle A\mathbf{E}W$ and $\angle A\mathbf{W}E$
  - $\overline{AE}$ is the included side of $\angle W\mathbf{A}E$ and $\angle A\mathbf{E}W$



6.4 SSS Triangle Congruence Postulate
6.5 SAS Triangle Congruence Postulate
6.6 ASA Triangle Congruence Postulate

7. Properties of Inequality

7.1 For all real numbers p and q where $p > 0$, $q > 0$:
- If $p > q$, then $q < p$.
- If $p < q$, then $q > p$.

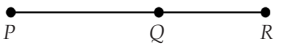
7.2 For all real numbers p, q, r and s, if $p > q$ and $r \geq s$, then $p + r > q + s$.

7.3 For all real numbers p, q and r, if $p > q$ and $r > 0$, then $pr > qr$.

7.4 For all real numbers p, q and r, if $p > q$ and $q > r$, then $p > r$.

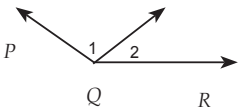
7.5 For all real numbers p, q and r, if $p = q + r$, and $r > 0$, then $p > q$.

The last property of inequality is used in geometry such as follows:



Q is between P and R.

$\overline{PR} \cong \overline{PQ} + \overline{QR}$
Then $\overline{PR} > \overline{PQ}$ and $\overline{PR} > \overline{QR}$.



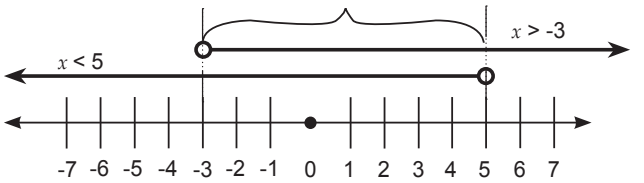
$\angle 1$ and $\angle 2$ are adjacent angles.

$\angle PQR \cong \angle 1 + \angle 2$
Then $\angle PQR > \angle 1$ and $\angle PQR > \angle 2$

Teacher's Note and Reminders

8. How to Combine Inequalities

- Example: How do you write $x < 5$ and $x > -3$ as a combined inequality?



From the number line, we observe that the value of x must be a value between -3 and 5, that is, x **is greater than -3 but less than 5**. In symbols, $-3 < x < 5$.

9. Equality and Congruence

Congruent figures (segments and angles) have equal measures such that:

- If $\overline{PR} \cong \overline{PR}$, then $PR = PR$.
- If $\angle PQS \cong \angle PQS$, then $m\angle PQS = m\angle PQS$.

Note that to make proofs brief and concise, we may opt to use $\overline{PR} \cong \overline{PR}$ or $\angle PQS \cong \angle PQS$ instead of $PR = PR$ or $m\angle PQS = m\angle PQS$. Because the relation symbol used is for congruence; instead of writing, say, reflexive property of equality as reason; we just have to write, reflexive property. Note that some other books sometimes call reflexive property as reflexivity.

10. How to Write Proofs

Proofs in geometry can be written in paragraph or two-column form. A proof in paragraph form is only a two-column proof written in sentences. Some steps can be left out when paragraph form is used so that two-column form is more detailed.

A combination of both can also be used in proofs. The first part can be in paragraph form especially when the plan for proof is to add some constructions first in the illustration. Proving theorems sometimes requires constructions to be made.

The first column of a two-column proof is where you write down systematically every step you go through to get to the conclusion in the form of a statement. The corresponding reason behind each step is written on the second column.

Possible reasons are as follows: Given, by construction, axioms of equality, properties of equality, properties of inequality, definitions, postulates or previously proven theorems.

Activity No.4:
What if It's Longer?

For Activity No. 4, make sure that each student has his/her own protractor. Ask them to define precision, accuracy, and tolerance using their own words. Discuss the meaning of these words related to making measurements by giving Donna Roberts's definitions:

The **precision** of a measuring instrument is determined by the smallest unit to which it can measure. The precision is said to be the same as the smallest fractional or decimal division on the scale of the measuring instrument.

Ask the students: What is the precise unit of a ruler? Answer should be millimeter.

Accuracy is a measure of how close the result of the measurement comes to the "true", "actual", or "accepted" value. Accuracy answers this question: How close is your answer to the accepted value?

Tolerance is the greatest range of variations in measurements that can be allowed.

Tolerance addresses this question: How much error in the answer is acceptable?

Proceed by discussing that it is expected that the measurements they get from measuring the same lengths vary. Explain that their answers are not wrong. Their answers vary because a **measurement made with a measuring device is approximate, not exact**. Discussion of the Greatest Possible Error and Tolerance Interval should follow.

Note that you and your class may decide on a tolerance interval. For the example given in the learning guide, you may decide to

The following steps have to be observed in writing proofs:

- Draw the figure described in the problem. The figure may already be drawn for you, or you may have to draw it yourself.
- Label your drawn figure with the information from the given by
  - ✓ marking congruent or unequal angles or sides,
  - ✓ marking perpendicular, parallel or intersecting lines or
  - ✓ indicating measures of angles and/or sides

The markings and the measures guide you on how to proceed with the proof and it also directs you whether your plan for proof requires you to make additional constructions in the figure.

- Write down the steps carefully, without skipping even the simplest one. Some of the first steps are often the given statements (but not always), and the last step is the statement that you set out to prove.

11. How to Write an Indirect Proof

11.1 Assume that the statement to be proven is not true by negating it.
11.2 Reason out logically until you reach a contradiction of a known fact.
11.3 Point out that your assumption must be false, thus, the statement to be proven must be true.

12. Greatest Possible Error and Tolerance Interval in Measurements

You may be surprised why two people measuring the same angle or length may give different measurements. Variations in measurements happen because measurement with a measuring device, according to Donna Roberts (2012), is approximate. This variation is called uncertainty or error in measurement, but not a mistake. She added that there are ways of expressing error of measurement. Two are the following:

Greatest Possible Error (GPE)

One half of the measuring unit used is the greatest possible error. For example, you measure a length to be 5.3 cm. This measurement is to the nearest tenth. Hence, the GPE should be one half of 0.1 which is equal to 0.05. This means that your measurement may have an error of 0.05 cm, that is, it could be 0.05 longer or shorter.

Tolerance Intervals

Tolerance interval (margin of error) may represent error in measurement. This interval is a range of measurements that will be tolerated or accepted before they are considered flawed.

Supposing that a teacher measures a certain angle x as 36 degrees. The measurement is to the nearest degree, that is, 1. The GPE is one half of 1, that is, 0.5. Your answer should be within this range: $36-0.5 \leq x \leq 36 + 0.5$. Therefore, the tolerance interval or margin of error is $35.5 \leq x \leq 36.5$ or 35.5 to 36.5.

have this margin of error: $36 - 1 \leq x \leq 36 + 1$ or $35 \leq x \leq 37$. Thus, a student's measure maybe 35 or 37 degrees. Still the answer is accepted because you set which range of measurements has to be tolerated.

In the discussion of errors in measurement, let the groups do the activity. Let them post their outputs. Process them and answers to numbers 3, 4, and 5 should be written on cartolina and posted on the display board for math concepts developed.

Once all questions are answered, let the students answer Quiz No. 1. Be sure to explain fully and carefully answers to each item in order to strengthen their understanding of the topic. For the questions under each item in Enrichment, you may let them answer by group. Give it as an assignment to give the students time to think. Follow this procedure in unlocking the answers to all quizzes in this module.

Answer Key to Activity No.4

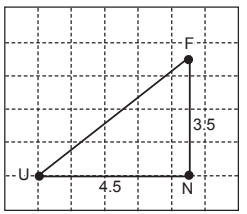
1. Yes, there is.
2. When one side of a triangle is longer than a second side, the angle opposite the first side is larger than the angle opposite the second side.
3. If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.
4. If one side of a triangle is the longest, then the angle opposite it is the largest.
5. If one side of a triangle is the shortest, then the angle opposite it is the smallest.
6. **Note**: Because of GPE and Tolerance Interval, it is your task to give the measure of the sides as accurately as you can.

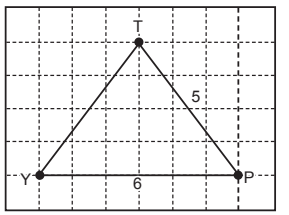
Now that you have already reviewed concepts and skills previously learned that are useful in this module, let us proceed to the main focus of this section—develop, verify, and prove the theorems on inequalities in triangles.

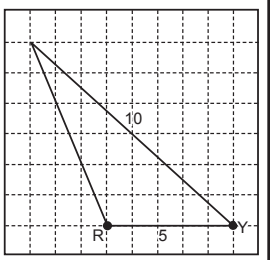
Activity 4 WHAT IF IT'S LONGER?

Materials Needed: protractor, manila paper, ruler
Procedures:

1. Replicate the activity table on a piece of manila paper.
2. Measure using a protractor the angles opposite the sides with given lengths. Indicate the measure in your table.
3. Discover the relationship that exists between the lengths of the sides of triangles and the angles opposite them and write them on your piece of manila paper.



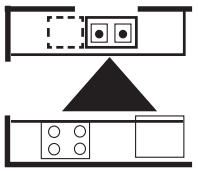




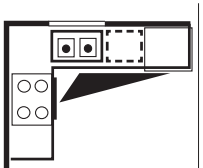
Triangle	Length of Sides		Measures of Angles Opposite the Sides	
ΔFUN	FN	3.5	$m\angle U$	
	NU	4.5	$m\angle F$	
ΔPTY	TP	5	$m\angle Y$	
	PY	6	$m\angle T$	
ΔRYT	RY	5	$m\angle T$	
	TY	10	$m\angle R$	

Mathematics in the Kitchen: The Kitchen Triangle

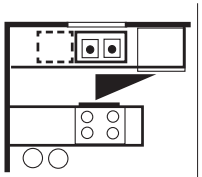
1.



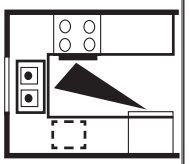
$SR = SC < RC$
$\angle C = \angle C < S$



$RC > RS > SC$
$\angle S > \angle C > R$

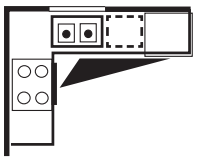


$RC > RS > SC$
$\angle S > \angle C > R$



$RS > RC > SC$
$\angle C > \angle S > R$

2. Possible answer and reasons:



1. It utilizes only two corners of a room.
2. It is more open so two or more people can watch and learn and also help in cooking

Answer Key to Quiz No. 1

A.

	Triangle	Largest Angle	Smallest Angle
1.	ΔAIM	$\angle A$	$\angle I$
2.	ΔRYT	$\angle Y$	$\angle R$
3.	ΔEND	$\angle N$	$\angle D$

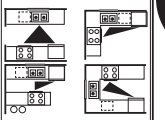
B.

	ΔNAY	ΔFUN	ΔWHT
Sides	$\overline{NY}, \overline{AN}, \overline{AY}$	$\overline{FN}, \overline{FU}, \overline{NU}$	$\overline{HW}, \overline{HT}, \overline{TW}$
Angle	$\angle A, \angle Y, \angle N$	$\angle U, \angle N, \angle F$	$\angle T, \angle W, \angle H$

C.

	Grant: 3 wishes	Grant: 2 wishes
Region to Hit with an Arrow	Region O	Region M

Mathematics in the Kitchen
The Kitchen Triangle



The Kitchen Triangle was developed in the 1950s as a tool to aid designers in creating an effective kitchen layout. The triangle has a corner at the sink, the refrigerator, and the stove, the three essential locations in the kitchen when cooking. Most kitchen plans still include this today. The idea is to have them close enough that they can easily be moved between, but not too far away to reduce the amount of movement while cooking. The general rule is that the triangles perimeter must be at least 12 ft., but should be larger than 26 ft.

The area inside the triangle should be completely open, making movement between each of these easy. Most modern kitchen plans still include this.

See the whole article at http://bathroomphotogallery.com/kitchen-plans.php~

Questions:

1. Suppose R is for Refrigerator, S is for Sink, and C for cooking stove, describe the relationship of the sides and angles of the kitchen triangle in each of the model.
2. If you were to build a house in the future, which kitchen model do you prefer from the figure? Why?

QUESTIONS ?

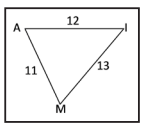
1. Is there a relationship between the length of a side of a triangle and the measure of the angle opposite it?
   ☐ Yes, there is. ☐ No, there isn't.
2. *Making Conjecture*: What is the relationship between the sides of a triangle and the angles opposite them?
   - *When one side of a triangle is longer than a second side, the angle opposite the* _____.
3. Your findings in no. 2 describe the Triangle Inequality Theorem 1. Write it in if-then form.
   - *If one side of a triangle is longer than a second side, then* _____.
4. What is the relationship between the longest side of a triangle and the measure of the angle opposite it?
5. What is the relationship between the shortest side of a triangle and the measure of the angle opposite it?
6. Without using a protractor, determine the measure of the third angles of the triangles in this activity. (**Hint**: T*he sum of the measures of the angles of a triangle is 180°*.)

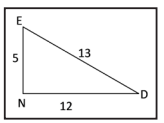
Name of Triangle	Working Equations	Measure of the Third Angle	
ΔFUN		$m\angle N$	
ΔTYP		$m\angle P$	
ΔTRY		$m\angle Y$	

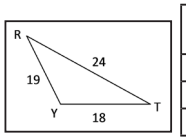
Quiz No. 1

Directions: Write your answer on a separate answer sheet.

A. Name the smallest angle and the largest angle of the following triangles:







	Triangle	Largest Angle	Smallest Angle
1.	ΔAIM		
2.	ΔEND		
3.	ΔRYT		

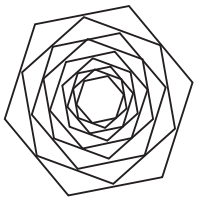
Mathematics in Art: Geometric Shapes for Foundation Piecing

1. Possible Answer: The figures started from the largest regular polygons.

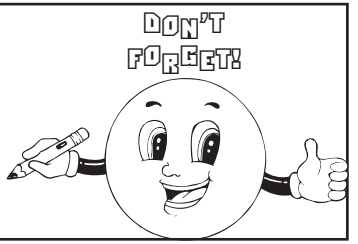
   1.1 Hexagon
   - 1.1.1 The next larger hexagon is determined by doing the following:
     - getting the midpoints of the sides of the original hexagon
     - connecting adjacent midpoints to form segments that serve as sides of the next hexagon.
   - 1.1.2 Repeat steps in 1.1 until the desired smallest hexagon is formed

   1.2 Heptagon
   - 1.2.1 The next larger heptagon is determined by doing the following:
     - placing a point of desired distance from the left endpoint of the sides of the original heptagon
     - connecting adjacent points to form segments that serve as sides of the next heptagon.
   - 1.2.2 Repeat steps in 1.1 until the desired smallest heptagon is formed

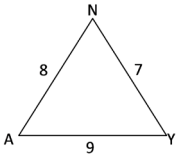
2. Possible Answer:

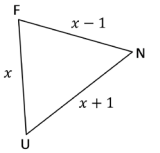


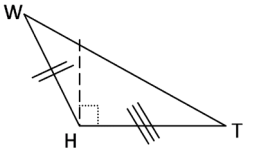
Teacher's Note and Reminders



B. The diagrams in the exercises are not drawn to scale. If each diagram were drawn to scale, list down the sides and the angles in order from the least to the greatest measure.





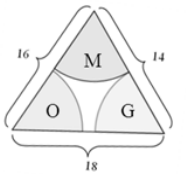


	ΔNAY	ΔFUN	ΔWHT
Sides			
Angle			

C. Your parents support you in your studies. One day, they find out that your topic in Grade 8 Math is on *Inequalities in Triangles*. To assist you, they attach a triangular dart board on the wall with lengths of the sides given.

They say they will grant you three wishes if you can hit with an arrow the corner with the smallest region and two wishes if you can hit the corner with the largest region.

- Which region should you hit so your parents will grant you three wishes?
- Which region should you hit so your parents will grant you two wishes?



	Grant: 3 wishes	Grant: 2 wishes
Region to Hit with an Arrow		

Mathematics in Art
Geometric Shapes for Foundation Piecing by Dianna Jesse

Challenge:
1. Which figure is drawn first in the artworks--the smallest polygon or the largest polygon?
2. Make your own design by changing the positions or the lengths of the sides of the triangles involved in constructing the figure.
3. Would you like to try using the hexagon?

Visit this web link to see the artworks shown: http://diannajessie.wordpress.com/tag/triangular-design/~

Activity No.5:
What if It's Larger?

For Activity No. 5, start up the class by having a review of the different kinds of triangles according to sides and angles. Proceed by asking what triangles are shown in the activity What if it's Larger.

Discuss the GPE and the tolerance interval of measurements. Once these are established, let the groups proceed with the activity. Let them post their outputs, process their outputs; and their answers to the questions in ponder time. The answers to numbers 3, 4, and 5 should be written on cartolina and posted in a display board for math concepts.

After discussion, let them answer Quiz No. 1.

Answer Key to Activity No.5

1. Yes, there is.
2. When one angle of a triangle is larger than a second angle, the side opposite the first angle is longer than the side opposite the second angle.
3. If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.
4. If one angle of a triangle is the largest, then the side opposite it is the longest.
5. If one angle of a triangle is the smallest, then the side opposite it is the shortest.
6.

Name of Triangle	Smallest Angle	Smaller Angle	Largest Angle
ΔLYF	$\angle Y$	$\angle L$	$\angle F$
ΔQUT	$\angle T$	$\angle U$	$\angle Q$
ΔOMG	$\angle M$	$\angle O$	$\angle G$

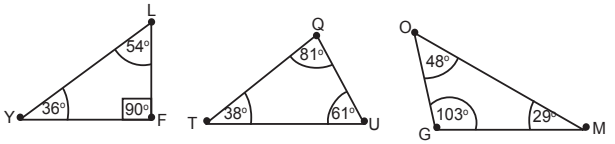
7.

Name of Triangle	Shortest Side	Shorter Side	Longest Side
ΔLYF	$\overline{LY}$	$\overline{FY}$	$\overline{LY}$
ΔQUT	$\overline{TU}$	$\overline{QT}$	$\overline{QU}$
ΔOMG	$\overline{MO}$	$\overline{GM}$	$\overline{GO}$

Activity 5 WHAT IF IT'S LARGER?

Materials Needed: ruler, manila paper
Procedures:

1. Replicate the activity table on a piece of manila paper.
2. Measure using ruler the sides opposite the angles with given sizes. Indicate the lengths (in mm) on your table.
3. Develop the relationship of angles of a triangle and the lengths of the sides opposite them by answering the ponder questions on a piece of manila paper.



Triangle	Measure of the Angles		Lengths of Sides Opposite the Angles	
ΔLYF	$m\angle L$		FY	
	$m\angle Y$		LF	
	$m\angle F$		LY	
ΔQUT	$m\angle Q$		TU	
	$m\angle U$		QT	
	$m\angle T$		QU	
ΔOMG	$m\angle O$		MG	
	$m\angle M$		GO	
	$m\angle G$		MO	

QUESTIONS ?

1. Is there a relationship between the size of an angle and the length of the side opposite it?
   ☐ Yes, there is. ☐ No, there isn't.
2. Making Conjecture: What is the relationship between the angles of a triangle and the sides opposite them?
   - When one angle of a triangle is larger than a second angle, the side opposite the _____.
3. Your findings in no. 2 describe Triangle Inequality Theorem 2. Write it in *if-then* form.
4. What is the relationship between the largest angle of a triangle and the side opposite it?
5. What is the relationship between the smallest angle of a triangle and the side opposite it?

8.

Kind of Triangle	How do you know that a certain side is the longest side?
Acute Δ	The longest side is opposite the largest acute angle.
Right Δ	The longest side is opposite the right angle.
Obtuse Δ	The longest side is opposite the obtuse angle.

Answer Key to Quiz No. 2

A.

	Triangle	Longest Side	Shortest Side
1.	ΔTRY	$\overline{TY}$	$\overline{RY}$
2.	ΔAPT	$\overline{AT}$	$\overline{AP}$
3.	ΔLUV	$\overline{LV}$	$\overline{LU}$

B.

List of Sides in Decreasing Order of Lengths	ΔTRP	ΔZIP	ΔFRE
	$\overline{PT}$, $\overline{PR}$ and $\overline{RT}$	$\overline{PZ}$, $\overline{IZ}$, $\overline{IP}$	$\overline{EF}$, $\overline{FR}$, $\overline{ER}$

C. $\overline{NZ}$

D. Answers vary

Mathematics in Eco-Architecture: Triangular Skyscraper

1. Answers vary
2. Possible Answers:
   - When the lot to build on is triangular in shape.
   - When the owner would like to have a triangular design.

6. Arrange in increasing order the angles of the triangles in this activity according to measurement.

Name of Triangle	Smallest Angle	Smaller Angle	Largest Angle
ΔLYF			
ΔQUT			
ΔOMG			

7. Arrange in decreasing order the sides of the triangles in this activity according to their lengths.

Name of Triangle	Shortest Side	Shorter Side	Longest Side
ΔLYF			
ΔQUT			
ΔOMG			

8. Having learned Triangle Inequality 2, answer the question in the table.

Kind of Triangle	How do you know that a certain side is the longest side?		
Acute Δ			
Right Δ			
Obtuse Δ			

Mathematics for Eco-Architecture
Triangular Skyscraper with Vegetated Mini-Atriums

The triangular form, which in China is symbolic with balance and stability, also allows the building to shade itself, which lowers the amount of energy required to cool the interiors. The signature feature of the entire design is the atrium, which runs the entire height of the building and also allows each level to be illuminated by natural light.

Questions:

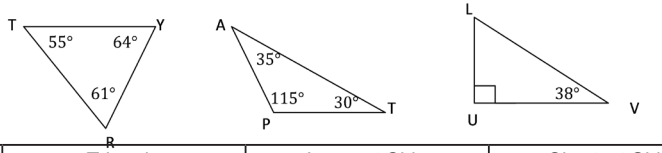
1. Have you seen triangular buildings or structures in your area?
2. When do you think it is best to use a triangular design like the one shown in building a structure?

To find out the reasons why the triangular design is eco-friendly, visit this website link: http://www.ecofriend.com/eco-architecture-triangular-skyscraper-designed-with-vegetated-mini-atriums.html

QUIZ No. 2

Directions: Write your answer on a separate answer sheet. Note that the diagrams in the exercises are not drawn to scale.

A. Name the shortest side and the longest side of the following triangles:



	Triangle	Longest Side	Shortest Side
1.	ΔTRY		
2.	ΔAPT		
3.	ΔLUV		

Triangular Design and Artworks

1. Answers Vary
2. Possible Answers:
   - Triangular Petal Card because it is easy to perform
   - Triangular Card Stand for those who likes wood working
   - Triangular Girl for those who love sketching and drawing
   - Diminishing Triangles for those who love tiling works.

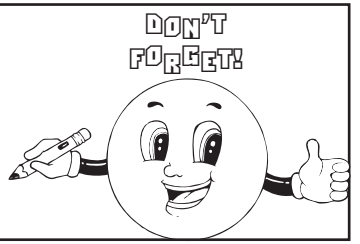
Activity No.6:
When Can You Say "ENOUGH"?

Two days before Activity No. 6 will be tackled, assign the groups to prepare pieces of straws with the following lengths in inches: 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Let them duplicate the 3- and 5-inch straws. Note: If straws are not available, they may use other objects like broom sticks. Make sure that all groups have the materials on the day the activity is scheduled.

Let the group do the activity and record their findings in the table they transferred on the manila paper. Process their outputs and their answers to the questions. Answers should be written on cartolina and posted on the display board of math concepts.

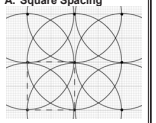
End the activity by asking this: What insight can you share about the title of the activity? After sharing insights, let them answer Quiz No. 3.

Teacher's Note and Reminders



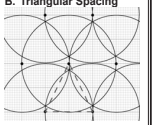
Mathematics in the Garden
How to Space Sprinklers?
~irrigationrepair.com~

A. Square Spacing



Square spacing is the easiest to plot; the downfall is that there will be areas that are going to be covered by all four sprinkler heads causing some over watering. Sprinklers are spaced relatively close when using a square pattern (on average around 50% of the diameter of the throw). This means you will also need more sprinkler heads to cover a given area.

B. Triangular Spacing

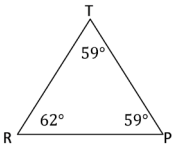


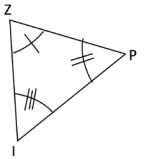
Triangular spacing is plotted using three points which means that more surface area is watered with less overlap .Since you can cover more surface using triangular spacing you will be able to space the sprinkler heads farther apart (usually around 60% of the diameter of the throw). Using a triangular pattern in plotting sprinkler heads can save money because less sprinkler heads are needed to irrigate any given area.

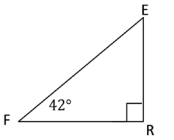
Task:
If you were asked to space sprinklers, which spacing would you use?

To read more about spacing sprinklers, visit this website link: http://www.irrigationrepair.com/how_to_space_rotors_sprays.html

B. List down the sides in order from the longest to the shortest length.

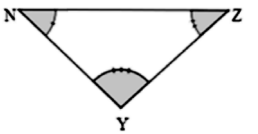






ΔTRP			ΔZIP			ΔFRE		

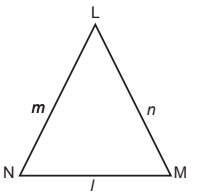
C. Skye buys a triangular scarf with angle measures as described in the figure shown. She wishes to put a lace around the edges. Which edge requires the longest length of lace?



Activity 6
WHEN CAN YOU SAY "ENOUGH!"?

Materials Needed: plastic straws, scissors, manila paper, and ruler
Procedure:

1. Cut pieces of straws with the indicated measures in inches. There are three pieces in each set.
2. Replicate the table in this activity on a piece of manila paper.
3. With each set of straws, try to form triangle LMN.
4. Write your findings on your table and your responses to the ponder questions on a piece of manila paper.



Answer Key to Activity No.6

Sets of Straw Pieces				Do the straws form a triangle or not?		Compare the sum of the lengths of shorter straws ($a + b$) with that of the longest length c			Compare ($b + c$) and a			Compare ($a + c$) and b		
	l	m	n	YES	NO	$l+m$	<,>,=	n	$m+n$	<,>,=	l	$l+n$	<,>,=	m
1.	3	3	7		√	6	<	7	10	>	3	10	>	3
2.	3	3	5	√		6	>	5	8	>	3	8	>	3
3.	4	6	10		√	10	=	10	16	>	4	14	>	6
4.	4	6	9	√		10	>	9	15	>	4	13	>	6
5.	5	5	10		√	10	=	10	15	>	5	15	>	5
6.	5	5	8	√		10	>	8	13	>	5	13	>	5
7.	6	7	11	√		13	>	11	18	>	6	17	>	7
8.	6	7	9	√		13	>	9	16	>	6	15	>	7
9.	4	7	12		√	11	<	12	19	>	4	16	>	7
10.	4	7	10	√		11	>	10	17	>	4	14	>	7

1. Making Conjectures:
   1.1
   - If the sum of the lengths of the two shorter straws is EQUAL to the length of the longest side, a triangle cannot be formed.
   - If the sum of the lengths of the two shorter straws is LESS THAN the length of the longest side, a triangle CANNOT be formed.
   - If the sum of the lengths of the two shorter straws is GREATER THAN the length of the longest side, a triangle CAN be formed.

   1.2
   - When the straws form a triangle, the sum of the lengths of any two straws is greater than the third straw.
   - When the straws do not form a triangle, the sum of the lengths of any two straws is less than or equal to the third straw.
2. Triangle Inequality Theorem 3
   - The sum of the lengths of any two sides of a triangle is greater than the third side.

Sets of Straw Pieces				Do the straws form a triangle or not?		Compare the sum of the lengths of shorter straws ($l + m$) with that of the longest length c			Compare ($m + n$) and l			Compare ($l + n$) and m		
	l	m	n	YES	NO	$l+m$	<,>,=	n	$m+n$	<,>,=	l	$l+n$	<,>,=	m
1.	3	3	7											
2.	3	3	5											
3.	4	6	10											
4.	4	6	9											
5.	5	5	10											
6.	5	5	8											
7.	6	7	11											
8.	6	7	9											
9.	4	7	12											
10.	4	7	10											

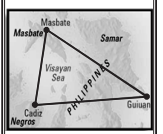
QUESTIONS ?

1. Making Conjectures:

   1.1 What pattern did you observe when you compared the sum of the lengths of the two shorter straws with the length of the longest straw? Write your findings by completing the phrases below:
   - If the sum of the lengths of the two shorter straws is equal to the length of the longest straw _____.
   - If the sum of the lengths of the two shorter straws is less than the length of the longest straw _____.
   - If the sum of the lengths of the two shorter straws is greater than the length of the longest straw _____.

   1.2 What pattern did you observe with the sets of straws that form and do not form a triangle? Complete the phrases below to explain your findings:
   - When the straws form a triangle, the sum of the lengths of any two straws _____.

Mathematics in Geography
Feasible Possible Distance
(McDougal Little Geometry, 2001)



Suppose you know the following information about distances between cities in the Philippine Islands:

Cadiz to Masbate ≈ 159 km
Cadiz to Guiuan ≈ 265 km

Considering the angle formed with Cadiz as the vertex, describe the range of possible distances from Guiuan to Masbate.

Answer Key to Quiz No. 3

1. Description:
   - $\overline{AW} + \overline{EW} > \overline{AE}$
   - $\overline{AW} + \overline{AE} > \overline{EW}$
   - $\overline{AE} + \overline{EW} > \overline{AW}$

2.

	Hints	In Symbols	Simplified Form	Is the simplified form true?	Can a triangle be formed? Justify
1	Is the sum of 8 and 10 greater than 14?	Is 8 + 10 > 14	Is 18 > 14	YES	YES because the sum of any two sides is greater than the third side.
2	Is the sum of 8 and 14 greater than 10?	Is 8 + 14 > 10	Is 22 > 10	YES	
3	Is the sum of 10 and 14 greater than 8?	Is 10 + 14 > 8	Is 24 > 8	YES	

Which question should be enough to find out if a triangle can be formed?

- *The question asking whether 8 +1 0 > 14 should be enough to find out if triangle is formed from the sides because 8 and 10 are the shorter sides.*

3.

	Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1	5 + 8 > 13	13 > 13	NO	YES because the sum of any two sides is greater than the third side.
2	5 + 13 > 8	18 > 8	YES	
3	8 + 13 > 5	21 > 5	YES	

Which question should be enough to find out if a triangle can be formed?

- *The question asking whether 8 + 10 > 14 should be enough to find out if triangle is formed from the sides because 8 and 10 are the shorter sides.*

- When the straws do not form a triangle, the sum of the lengths of any two straws_____.

2. Your findings in this activity describe Triangle Inequality Theorem 3. State the theorem by describing the relationship that exists between the lengths of any two sides and the third side of a triangle.
   - The sum of the lengths of any two sides of a triangle is _____.

QUIZ No. 3

Directions: Write your answer on a separate answer sheet.

1. Describe sides $\overline{AW}$, $\overline{EW}$ and $\overline{AE}$ of ΔAWE using Triangle Inequality Theorem 3.

2. Your task is to check whether it is possible to form a triangle with lengths 8, 10, and 14. Perform the task by accomplishing the table shown. Let the hints guide you.

	Hints	In Symbols	Simplified Form	Is the simplified form true?	Can a triangle be formed? Justify
1	Is the sum of 8 and 10 greater than 14?				
2	Is the sum of 8 and 14 greater than 10?				
3	Is the sum of 10 and 14 greater than 8?				

Which question should be enough to find out if a triangle can be formed?

3. Is it possible to form a triangle with sides of lengths 5, 8, and 13? Complete the table to find out the answer.

	Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1				
2				
3				

Which question should be enough to find out if a triangle can be formed?

4.

	Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1	7 + 9 > 20	16 > 20	7 + 9 > 20	16 > 20 27 > 9 29 > 7
2	7 + 20 > 9	27 > 9	7 + 20 > 9	
3	9 + 20 > 7	29 > 7	9 + 20 > 7	

Which question should be enough to find out if a triangle can be formed?

- *The question asking whether 7+9>20 should be enough to find out if triangle is formed from the sides because 7 and 9 are the shorter sides.*

5.

$\overline{CA} + \overline{AR}$	>	$\overline{CR}$
$\overline{ER} + \overline{AR}$	>	$\overline{AE}$
$\overline{AR}$	<	$\overline{RE} + \overline{AE}$
$\overline{AC} + \overline{CE}$	>	$\overline{CE}$
$\overline{AC}$		$\overline{AE} + \overline{CE}$

6. $s_1 + s_2 > s_3$

7. If t is the third side then, the following should be satisfied:

$7 + 10 > t$ $\quad$ $7 + t > 10$ $\quad$ $t + 10 > 7$

$t < 7+10$ $\quad$ $t < 10 - 7$ $\quad$ $t < 7 - 10$

$t < 17$ $\quad$ $t < 3$ $\quad$ $t < -3$

But length should be greater than zero. The values described must be excluded.

- Therefore, side t may have the following measurements in ft.: {4, 5, 6,…14, 15, 16}
- Side t has lengths between 3 ft and 17 ft.

8. Xylie's estimation, 180 meters, is feasible. The distance of 180 is within the range of (120 – 80 = 40) and (120 + 80 = 200).
9. Because $S_1 < 80 < 120$ and $40 < S_1 < 200$, then $40 < S_1 < 80$
10. Path No. 2: School to Church. Justification: Triangle Inequality Theorem 3
11. Errors:
    - With two marks on $\overline{EF}$, it is the longer side so the angle opposite it must also be the larger angle. However, it is opposite the shortest angle—angle D.

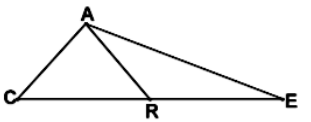
4. Can you form a triangle from sticks of lengths 7, 9, and 20?

	Find out if:	Simplified Forms	Is the simplified form true?	Can a triangle be formed? Justify
1				
2				
3				

Which question should be enough to find out if a triangle can be formed?

5. Study the figure shown and complete the table of inequalities using Triangle Inequality Theorem 3.

$\overline{CA} + \overline{AR}$	>	
$\overline{ER} + \overline{AR}$	>	
	<	$\overline{RE} + \overline{AE}$
$\overline{AC} + \overline{CE}$	>	
		$\overline{AE} + \overline{CE}$



Mathematics in Architecture

World's Thinnest House: Keret House by Jakub Szczesny

The four-feet-wide house is built on a tiny space between two buildings in Warsaw, Poland.

Questions:

1. Explain why Architect Szczesny used triangular design in building the house?
2. Why is it not advisable to use rectangular design?

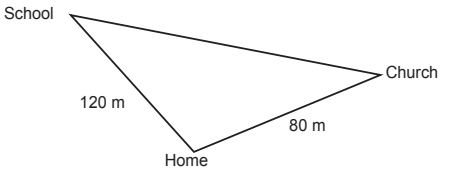
To see more photos of the house, visit this website link: http://design-milk.com/worlds-thinnest-house-keret-house-by-jakub-szczesny/?fb_action_ids=4022332433678&fb_action_types=og.likes&fb_source=aggregation&fb_aggregation_id=288381481237582

6. Using Triangle Inequality Theorem 3, what inequality will you write to check whether segments with lengths s_1, s_2, and s_3 form a triangle if $s_1 < s_2 < s_3$?

7. If two sides of a triangle have lengths 7 feet and 10 feet, what are the possible integral lengths of the third side? Between what two numbers is the third side?

8. The distance Klark walks from home to school is 120 meters and 80 meters when he goes to church from home. Xylie estimates that the distance Klark walks when he goes directly to Church, coming from school, is 180 meters. Realee's estimation is 210 meters. Which estimation is feasible? Justify your answer.

For items no. 8-10, use the figure shown



9. Supposing that the shortest distance among the three locations is the school-church distance, what are its possible distances?

Mathematics in Geography: Feasible Possible Distance

From the map, it is clear that the distance d of Guiuan to Masbate is the longest. Hence, the distance of Guiuan to Masbate must be greater than 265 m but less than the sum of 159 and 265 m, which is 424 m. Therefore, $265 < d < 424$.

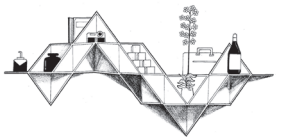
1. Architect Szczesny used a triangular design because it is enough for him to provide a bedroom, a bathroom, and a kitchen.
2. Possible reasons:
   - Unlike a rectangular design, a triangular design has the roof already steep so rainwater or snow will just slide easily.
   - With a rectangular design, it needs two stands as foundations to achieve balance. In that case, where will he place the ladder to the house?

Mathematics in Art: Color Triangle

1. Color combinations
   - Yellow and Blue = Green
   - Red and Yellow = Orange
   - Blue and Red = Violet
2. Possible number of exterior angles
   - Two (2)
   - Four (4)
   - Two (2)

Notes to the Teacher

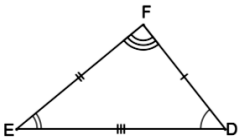
After reviewing their knowledge on exterior angles of triangles, direct their attention to one of the artworks in the activity Artistically Yours. Let them determine the exterior angles and their corresponding remote interior angles.



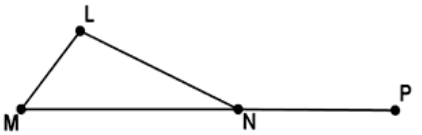
Once done, invite them to go to the internet and perform the interactive activity from http://www.mathwarehouse.com/geometry/triangles/angles/remote-exterior-and-interior-angles-of-a-triangle.php.

10. Which of the following paths to church is the shortest if you are from school? Justify your answer.
    - Path No. 1: School to Home then to Church
    - Path No. 2: School to Church

11. Some things are wrong with the measurements on the sides and angles of the triangle shown. What are they? Justify your answer.

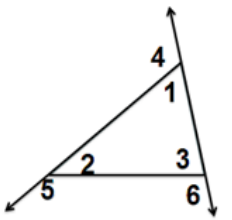


The next activity is about discovering the triangle inequality theorem involving an exterior angle of a triangle. Before doing it, let us first recall the definition of an exterior angle of a triangle.



By extending $\overline{MN}$ of ΔLMN to a point P, $\overline{MP}$ is formed. As a result, $\angle LNP$ forms a linear pair with $\angle LNM$. Because it forms a linear pair with one of the angles of ΔLMN, $\angle LNP$ is referred to as an exterior angle of ΔLMN. The angles non-adjacent to $\angle LNP$, $\angle L$ and $\angle M$, are called remote interior angles of exterior $\angle LNP$.

In the triangle shown, ∠4, ∠5 and ∠6 are exterior angles. The remote interior angles of ∠4 are ∠2 and ∠3; of ∠5, ∠1 and ∠3; of ∠6, ∠1 and ∠2.



Internet Learning
Measures of Interior and Exterior Angles of a Triangle

Interactive:
- http://www.mathwarehouse.com/geometry/triangles/angles/remote-exterior-and-interior-angles-of-a-triangle.php

Activity No. 7:
MEASURE Mania: Exterior or Remote Interior?

For Activity no. 7, explain to them that Grade 8 students should have the passion for getting measurements; hence, the title has mania in it (mania for passion). And one has to be sure of his/her measure, hence, MEaSURE.

Tell the students that in this activity, they will find out the inequality that exists between an exterior angle of a triangle and each of its remote interior angles. But before you proceed, decide on the GPE and Tolerance Interval of your measurements.

Let them get the measurement of the exterior and interior angles of the triangles, compare them and write their findings and answers to ponder questions on a piece of manila paper. Process their outputs. You have to consider the GPE and the Tolerance Interval. Their answer to activity question no. 4 should be written on a piece of cartolina and posted on a display board of math concepts.

End the activity with the students answering Quiz No. 4.

Answer key to Activity No. 7 Questions

1. The answer to each item is: >.
2. The answer to each item is: >.
3. The answer to each item is: >.
4. Conjecture: The measure of an exterior angle of a triangle is greater than the measure of either interior angle.

Answer key of Quiz No. 4

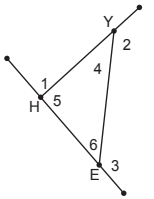
1. Inequalities:

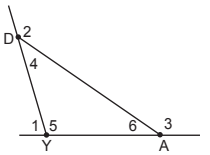
Considering ΔREA	Considering ΔHAM
$m\angle CAR > m\angle E$	$m\angle HAT > m\angle M$
$m\angle CAR > m\angle R$	$m\angle HAT > m\angle H$

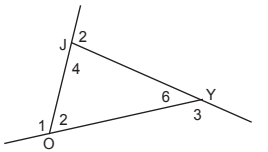
Activity 7 MEASURE MANIA: EXTERIOR OR REMOTE INTERIOR?

Materials Needed: protractor, manila paper, and ruler
Procedures:

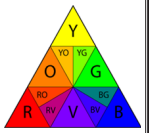
1. Measure the numbered angles of ΔHEY, ΔDAY, and ΔJOY.
2. Replicate the table in this activity on a piece of manila paper.
3. Indicate the measures on your table and write your answers to the ponder questions on a piece of manila paper.







Mathematics in Art
Color Triangle



The Color Triangle makes it easier to determine the resulting color if two colors are combined.

Questions:

1. What is the resulting color with the following combinations?
   - Yellow and Blue
   - Red and Yellow
   - Blue and Red
2. How many possible exterior angles do the following sets of color triangles have?
   - B, R, Y
   - G, O, V
   - YO, YG, RO, RV, BG, BV

To read more about the color triangle, visit this website link:
http://www.atpm.com/9.08/design.shtml

Name of Triangle	MEASURES								
	1st Exterior ∠	Remote Interior ∠s		2nd Exterior ∠	Remote Interior ∠s		3rd Exterior ∠	Remote Interior ∠s	
	∠1	∠4	∠6	∠2	∠5	∠6	∠3	∠4	∠5
ΔHEY									
ΔDAY									
ΔJOY									

2.

$m\angle AED$	>	$m\angle AED$
$m\angle DEB$	>	$m\angle DCE$
$m\angle DEB$	=	$m\angle DBE$
$m\angle CDE$	<	$m\angle DEB$
$m\angle DEB$	<	$m\angle ACD$

3.

ΔDEB	$\angle AED$, $\angle BDF$ or $\angle CDF$
ΔCDG	$\angle AGD$, $\angle CGE$, $\angle CDF$, $\angle BCG$
ΔAGE	$\angle CGE$, $\angle BEG$, $\angle AGD$
ΔBAC	$\angle ACD$ or $\angle GCD$

Teacher's Note and Reminders

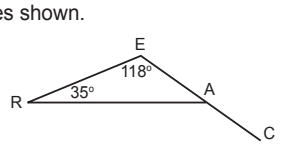
QUESTIONS ?

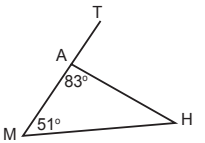
1. Compare the measure of exterior ∠1 with either remote interior ∠4 or ∠6 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔHEY, $m\angle 1$ is _____ $m\angle 6$.
   - In ΔDAY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔDAY, $m\angle 1$ is _____ $m\angle 6$.
   - In ΔJOY, $m\angle 1$ is _____ $m\angle 4$.
   - In ΔJOY, $m\angle 1$ is _____ $m\angle 6$.
2. Compare the measure of exterior ∠2 with either remote interior ∠5 or ∠6 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔHEY, $m\angle 2$ is _____ $m\angle 6$.
   - In ΔDAY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔDAY, $m\angle 2$ is _____ $m\angle 6$.
   - In ΔJOY, $m\angle 2$ is _____ $m\angle 5$.
   - In ΔJOY, $m\angle 2$ is _____ $m\angle 6$.
3. Compare the measure of exterior ∠3 with either remote interior ∠4 or ∠5 using the relation symbols >, <, or =.
   - In ΔHEY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔHEY, $m\angle 3$ is _____ $m\angle 5$.
   - In ΔDAY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔDAY, $m\angle 3$ is _____ $m\angle 5$.
   - In ΔJOY, $m\angle 3$ is _____ $m\angle 4$.
   - In ΔJOY, $m\angle 3$ is _____ $m\angle 5$.
4. *Making Conjecture:* Your comparison between the measure of an exterior angle of a triangle and either interior angle in this activity describes the Exterior Angle Inequality Theorem. With the pattern that you observed, state the exterior angle inequality theorem.
   - *The measure of an exterior angle of a triangle is*_____.

QUIZ No. 4

Directions: Write your answer on a separate answer sheet.

1. Use the Exterior Angle Inequality theorem to write inequalities observable in the figures shown.





Mathematics in Psychology: Robert Sternberg's Triangular Theory of Love

1. Possible Answer:

   - At the angles of the triangles are the Liking (intimacy), Infatuation (passion), and Empty Love (Commitment).
   - The sides are made up of Romantic Love, Companionate Love, and Fatuous Love. Romantic love is a result of passion and intimacy. Companionate love is a result of intimacy and commitment. Fatuous Love is a result of passion and commitment.
   - Consummate Love is at the interior of the triangle and it is a result of passion, intimacy, and commitment.

2. Possible Answer: All are important. However, the most important is commitment because love based on commitment will survive amidst challenges like illness and poverty.

3. Possible Answer: Consummate love because it has all the elements that every human being dreams of.

4. Possible Answers:

   - Consummate love > companionate love > fatuous love > romantic love
   - Commitment > intimacy > passion

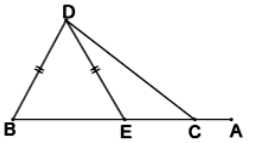
Activity No. 8:
My Grandpa, My Model of a Healthy Lifestyle

A day before the Activity No. 8 is scheduled to be performed, reproduce the desired number of copies of the Grandpa pictures and the pictures of the suggested outputs of the activity. Note that answers may vary so analyze the merit of students' outputs carefully.

Considering ΔREA	Considering ΔHAM

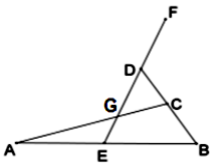
2. Use >, <, or = to compare the measure of angles.

$m\angle AED$		$m\angle CED$
$m\angle DEB$		$m\angle DCE$
$m\angle DEB$		$m\angle DBE$
$m\angle CED$		$m\angle DEB$



3. Name the exterior angle/s of the triangles shown in the figure.

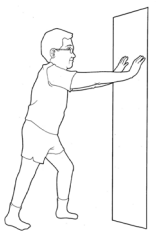
ΔDEB	
ΔCDG	
ΔAGE	
ΔBAC	



You have successfully developed all the theorems on inequalities in one triangle. Activity No. 8 can be performed using them. Good luck!

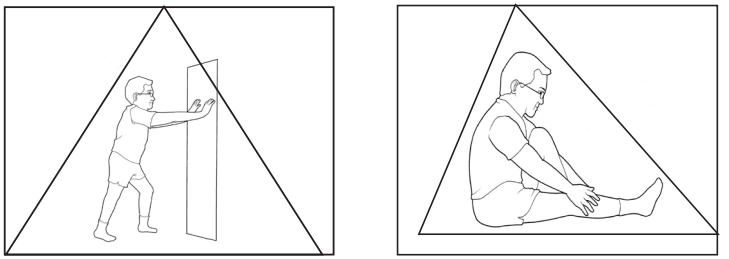
Activity 8 My Grandpa, My Model of Healthy Lifestyle!

Leruana has a triangular picture frame that her grandpa gave her on her 13th birthday. Like her, his grandpa loves triangular shapes. Since it is going to be his grandpa's 65th birthday soon, her birthday gift idea is to have two triangular frames made so she can place in them photos of his grandpa as health exercise instructor. As her woodworker friend, she asks you to do the triangular frames for her. To determine the shapes of the picture frames, how should the photos be cropped?



Possible Answers to Activity 8:

1.



2.

- Any part of the body can't be cropped.
- For the standing grandpa, the line on his back, and for the sitting one, his outstretched legs or the line from the tip of his foot to his head

Activity No. 9:
Clock Wisdom: Pretty One!

For Activity no. 9, it is advisable for you to bring a real clock. You have to show to them the angles formed by the short and long hands at 1PM, 2 PM, 3PM and 4PM.

You have to elicit from the class the measures of the angles formed by the hands of the clock at the aforementioned times. Let the students determine the angle by giving them these clues:

- One complete revolution is 360 degrees
- The whole revolution is subdivided into 12 hours

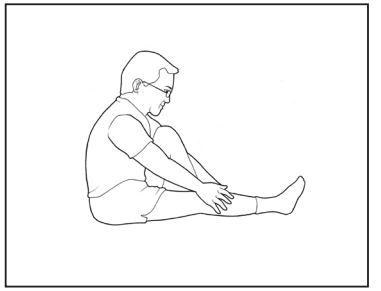
They must realize that each subdivision is (360/12) degrees or 30 degrees. Once the measures of the angles of clock faces A, B, C, and D are determined, let the groups do the activity and answer questions in Ponder Time. Let them post their outputs written on manila paper. Process their outputs. Answers to question no. 6 and 7 (including the drawings of ΔCAT and ΔDOG) of ponder time must be written on a piece of cartolina and posted on a display board of math concepts.

Mathematics in Psychology
Robert Sternberg's Triangular Theory of Love

Questions:

1. Study the triangle intently. What do you understand about the triangular theory of love?
2. Which mean more to you—passion, intimacy or commitment?
3. Which love you would like to have in the future—romantic, fatuous, companionate or consummate love?
4. How do you rank romantic love, fatuous love, companionate love and consummate love using combined inequality?

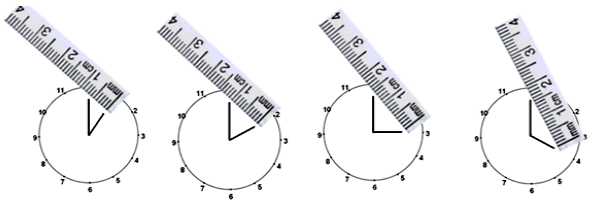
To help you decide, visit http://gentlemencalling.wordpress.com/2012/03/13/for-the-love-of-triangles/



QUESTIONS ?

1. *How do you plan to crop the photographs?*
   - Indicate the vertices of the triangular part of the photos.
   - Mark the sides of the new triangular photos.
2. *What made you decide to have that shape and not something else?*
   - What is your basis for determining the largest corner?
   - What is your basis for determining the longest side?

Activity 9 Clock Wisdom, Pretty One!



A complete revolution around a point is equivalent to 360º. The minute and hour hands of the clock also cover that in a compete revolution.

Materials: ruler and manila paper
Procedure:

1. Replicate the activity table on a piece of manila paper.
2. Study the faces of the clock shown at different hours one afternoon and complete your copy of the activity table.

Answer Key to Questions in Activity 9

1. The short hands of the clock in clock faces A, B, C, and D are equal (=).
2. The short hands of the clock in clock faces A, B, C, and D are equal (=).
3. The angles formed by the hands of the clock are called as included angles.
4. The later in the afternoon the hour is, the larger is the angle.
5. The measure of the distance between the tips of the hands of the clock is influenced by the measure of the included angle at a certain time.
6. If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.
7. If $\overline{AC} \cong \overline{OD}$, $\overline{AT} \cong \overline{OG}$, and $m\angle A > m\angle O$; then $CT > DG$
8. Note: Answers may vary.
9. Some examples: Ladies' fan, door hinge, tail of a peacock, geometric compass, puller, nipper, pliers, pages of a book, arms with the elbow joint as the hinge, legs with knee joint as the hinge, etc.

Activity No. 10:
Roof-y Facts, Yeah!

Before starting group Activity 10, decide for the GPE and Tolerance Interval of the measurements. Proceed to the following: groups working on the activity and answering activity questions while you roam around to give technical assistance; posting of outputs; processing outputs; and writing answers of nos. 3 and 4 (including the drawings ΔRAP and ΔYES) on a piece of cartolina to be posted on the display board in mathematics.

Let the class answer Quiz No. 5 and discuss the solutions and answers for each item.

3. Write also your answers to the ponder questions on a piece of manila paper.
4. Compute for the measure of the angle formed by the hands of the clock given that one revolution for each hand is equivalent to 360°.

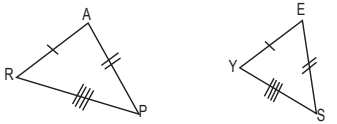
Clock Face	Time (Exact PM Hours)	Measure of angle formed by the hour hand and minute hand	Distance between the tips of the hour hand and minute hands (in mm)
A			
B			
C			
D			

QUESTIONS ?

1. Write your observations on the following:
   - The lengths of the roofs at the left part of both houses __.
   - The Lengths of the roof at the right part of both houses __.
   - The lengths of the roof bases of both houses __.
   - The Roof angles of both houses __.
2. What influences the measures of the roof angles of both houses? Justify.
3. *Making a Conjecture:* Your findings describe the Converse of Hinge Theorem (This is otherwise known as SSS Triangle Inequality Theorem). How will you state this theorem if you consider the two corresponding roof lengths as two sides of two triangles, the roof bases as their third sides, and the roof angles as included angles? State it in *if-then* form.

   If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is greater than the third side of the second, then _____.
4. Using the Converse of Hinge Theorem, write an if-then statement to describe the appropriate sides and angles of ΔRAP and ΔYES.

HOUSE	Roof Lengths at the Right (in cm)		Roof Lengths at the Left (in cm)		Lengths of Roof Base (in cm)		Roof Angle	
A								
B								



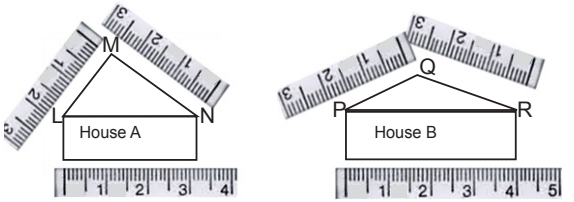
5. With both houses having equal roof lengths, what conclusion can you make about their roof costs?

Answer Key to Questions in Activity 10

1. Observations:
   - The lengths of the roofs at the left part of both houses are equal.
   - The lengths of the roofs at the right part of both houses are equal.
   - The lengths of the roof bases of both houses differ in lengths. Roof base of house A is shorter than the roof base of House B.
2. The measures of roof angles are affected by the length of the roof bases. If the roof base is longer, the roof angle is also larger.
3. If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second triangle.
4. If $\overline{AR} \cong \overline{EY}$, $\overline{AP} \cong \overline{ES}$, and $PR > SY$; then $m\angle A > m\angle E$.
5. Roof costs for House A is the same as roof costs for House B.

Teacher's Note and Reminders

Activity 10 ROOF-Y FACTS, YEAH!



Materials Needed: protractor, manila paper, and ruler

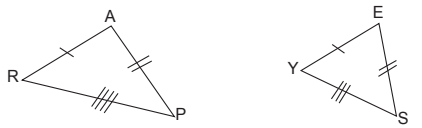
Procedure: Study the house models and complete your copy of the activity table. For ponder questions, write your answers on a piece of manila paper.

HOUSE								
A								
B								

QUESTIONS ?

1. Write your observations on the following:
   - The lengths of the roofs at the left part of both houses __.
   - The Lengths of the roof at the right part of both houses __.
   - The lengths of the roof bases of both houses __.
   - The Roof angles of both houses __.
2. What influences the measures of the roof angles of both houses? Justify.
3. *Making a Conjecture:* Your findings describe the Converse of Hinge Theorem (This is otherwise known as SSS Triangle Inequality Theorem). How will you state this theorem if you consider the two corresponding roof lengths as two sides of two triangles, the roof bases as their third sides, and the roof angles as included angles? State it in *if-then* form.

   If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is greater than the third side of the second, then _____.
4. Using the Converse of Hinge Theorem, write an if-then statement to describe the appropriate sides and angles of ΔRAP and ΔYES.



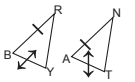
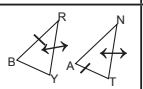
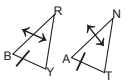
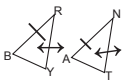
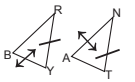
5. With both houses having equal roof lengths, what conclusion can you make about their roof costs?

Answer Key to Quiz No. 5

A.

Statements			Justification
1. If $\overline{AC} \cong \overline{AD}$ and $m\angle 1 = m\angle 2$, then $\overline{BC}$	=	$\overline{BD}$	Hinge Theorem
2. If $\overline{BC} \cong \overline{BD}$ and $\overline{AC} > \overline{AD}$,then $m\angle 4$	>	$m\angle 3$	Converse of Hinge Theorem
3. If $\overline{AD} \cong \overline{AC}$ and $m\angle 2 < m\angle 1$,then $\overline{BD}$	<	$\overline{BC}$	Hinge Theorem
4. If $\overline{BD} \cong \overline{BC}$ and $\overline{AD} > \overline{AC}$,then $m\angle 3$	>	$m\angle 4$	Converse of Hinge Theorem

B.

	GIVEN FACTS			FOR MARKINGS	CONCLUSION	JUSTIFICATION
1.	$BY = AT$	$BR = AN$	$m\angle B > m\angle A$		$RY > NT$	Hinge Theorem
2.	$BR = AT$	$RY = NT$	$m\angle R > m\angle N$		None	∠N is not an included angle
3.	$BY = AT$	$BR = AN$	$RY > NT$		$m\angle B > m\angle A$	Converse of Hinge Theorem
4	$BR = AN$	$RY = NT$	$BY > AT$		$m\angle R > m\angle N$	Hinge Theorem
5.	$RY = NT$	$BY = AN$	$m\angle N < m\angle Y$		$AT > BR$	Converse of Hinge Theorem

C. 1. $AC > DF$ 3. $m\angle RAT > m\angle YAT$
2. $HI > GI$ 4. $m\angle FAE > m\angle MAE$

D. $2m - 1 > m + 4$. Therefore, $m > 5$.

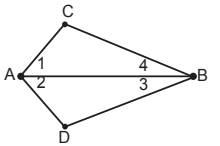
E. Sample Answer:

The hinge of the compass makes it possible to adjust the distance between the tips of the compass point and the pencil point. Adjustments determine the desired lengths of radii for circles to be drawn.

QUIZ No. 5

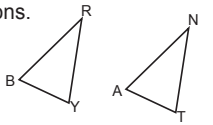
Directions: Write your answer on a separate answer sheet.

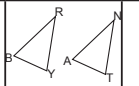
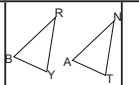
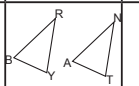
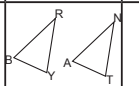
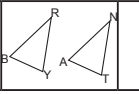
A. Use the symbol <, > or = to complete the statements about the figure shown. Justify your answer.



Statements			Justification
1. If $\overline{AC} \cong \overline{AD}$ and $m\angle 1 = m\angle 2$, then $\overline{BC}$		$\overline{BD}$	
2. If $\overline{BC} \cong \overline{BD}$ and $\overline{AC} > \overline{AD}$,then $m\angle 4$		$m\angle 3$	
3. If $\overline{AD} \cong \overline{AC}$ and $m\angle 2 < m\angle 1$,then $\overline{BD}$		$\overline{BC}$	
4. If $\overline{BD} \cong \overline{BC}$ and $\overline{AD} > \overline{AC}$,then $m\angle 3$		$m\angle 4$	

B. Make necessary markings on the illustration based on the given. What conclusion can you make, if there is any, given the facts about the two triangles? Provide justifications to your conclusions.



	GIVEN FACTS			FOR MARKINGS	CONCLUSION	JUSTIFICATION
1.	$BY = AT$	$BR = AN$	$m\angle B > m\angle A$			
2.	$BR = A\,T$	$RY = NT$	$m\angle R > m\angle N$			
3.	$BY = AT$	$BR = AN$	$RY > NT$			
4	$BR = AN$	$RY = NT$	$BY > AT$			
5.	$RY = NT$	$BY = AN$	$m\angle N < m\angle Y$			

E. Mathematics in Fashion: Ladies' Fan

1. It is important when it is hot and there is no air conditioning unit like in churches.
2. When the fan is not opened completely, the distance between the tips of the side frame of the fan is shorter than when the fan is opened completely.

From the prior investigations, we have discovered the following theorems on triangle inequalities:

Inequalities in One Triangle:

Triangle Inequality Theorem 1 (Ss → Aa)
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Triangle Inequality Theorem 2 (Aa → Ss)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

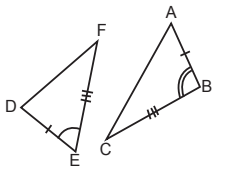
Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle

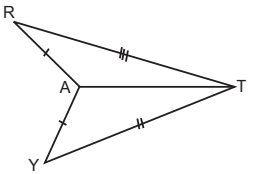
Inequalities in Two Triangles:

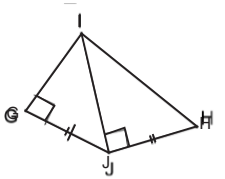
Hinge Theorem or SAS Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

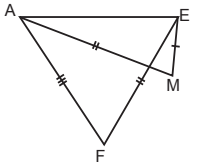
Converse of Hinge Theorem or SSS Inequality Theorem:
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

C. Using Hinge Theorem and its converse, write a conclusion about each figure.

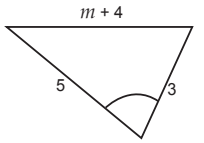
1. 

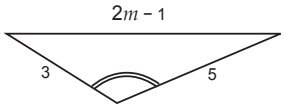
3. 

2. 

4. 

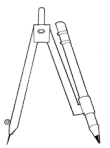
D. Using Hinge Theorem and its converse, solve for the possible values of m.

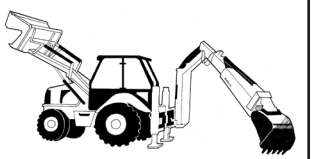


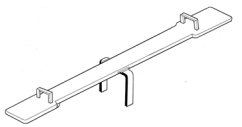


E. Enrichment Activities

1. *Hinges in Tools and Devices*
Hinges are used to fasten two things together and allow adjustment, rotation, twisting or pivoting. Choose at least one of the following hinged devices and explain how it works.







Explain to the students that the next activities of the PROCESS section are on writing proofs of the theorems on inequalities in triangles.

Inspire the students to think clearly and systematically together as a group. Explain to them that in writing the proofs of theorems, focus and collaboration are the instruments for their success.

How can we prove these theorems?

Writing proofs is an important skill that you will learn in geometry. It will develop your observation skills, deductive thinking, logical reasoning, and mathematical communication. Guide questions are provided to help you succeed in the next activities.

In writing proofs, you have to determine the appropriate statements and give reasons behind these statements. There are cases when you only have to complete a statement or a reason. Make use of hints to aid you in your thinking.

Be reminded that theorems may be proven in different ways. The proofs that follow are some examples of how these theorems are to be proven.

For activity 11-16, you are required to use a piece of manila paper for each proof.

Make sure that a day before the activities in writing proofs are scheduled, groups already have enough number of pieces of manila paper for the activity where tables for statements and reasons are already prepared.

You may opt to let the students prepare metastrips (each piece is 1/3 of bond paper cut lengthwise) and pentel pen or ball pen so that they only have to write each statement or reason on a metastrip and attach it on the appropriate row and column.

Your technical assistance is crucial in the proof-writing activities so roam around purposely. Most of your assistance involves your directing them to refer the review points in this module.

2. *Mathematics in Fashion: Ladies' Fan*
From the sixteenth century up to the late 1800s throughout the whole of Europe, each fashionable lady had a fan and because of its prominence, it was considered as a "woman's scepter"—tool for communicating her thoughts.

http://www.victoriana.com/Fans/historyofthefan.html

Questions:
1. Do you think that fan is an important fashion item?
2. Describe the concept of inequality in triangles that is evident about a ladies' fan.

From the prior investigations, we have discovered the following theorems on triangle inequalities:

Inequalities in One Triangle:

Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle

Inequalities in Two Triangles:

Hinge Theorem or SAS Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

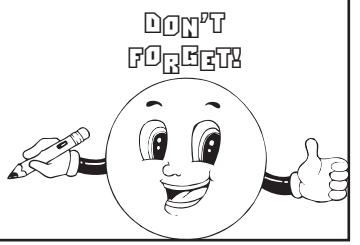
Converse of Hinge Theorem or SSS Inequality Theorem:
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

Answer Key to Activity 11: Proving Triangle Inequality Theorem 1

A.

Statements	Reasons
1. $\overline{LM} \cong \overline{LP}$	By construction
2. ΔLMP is isosceles	Definition of Isosceles Triangle
3. $\angle 1 \cong \angle 2$	Base angles of isosceles triangles are congruent.
4. $\angle LMN \cong \angle 1 + \angle 3$	Angle Addition Postulate
5. $\angle LMN > \angle 1$	Property of Inequality
6. $\angle LMN > \angle 2$	Substitution Property
7. $\angle 2 + \angle MPN = 180$	Linear Pair Postulate
8. $\angle MPN + \angle N + \angle 3 = 180$	The sum of the interior angles of a triangle is 180.
9. $\angle 2 + \angle MPN = \angle MPN + \angle N + \angle 3$	Substitution/Transitive Property
10. $\angle 2 = \angle N + \angle 3$	Subtraction Property
11. $\angle 2 > \angle N$	Property of Inequality
12. $\angle LMN > \angle N$	Transitive Property

Teacher's Note and Reminders



How can we prove these theorems?

Writing proofs is an important skill that you will learn in geometry. It will develop your observation skills, deductive thinking, logical reasoning, and mathematical communication. Guide questions are provided to help you succeed in the next activities.

In writing proofs, you have to determine the appropriate statements and give reasons behind these statements. There are cases when you only have to complete a statement or a reason. Make use of hints to aid you in your thinking.

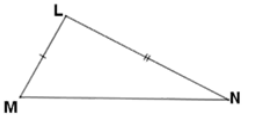
Be reminded that theorems may be proven in different ways. The proofs that follow are some examples of how these theorems are to be proven.

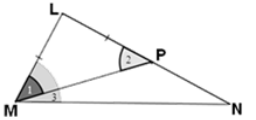
For activity 11-16, you are required to use a piece of manila paper for each proof.

Activity 11 PROVING TRIANGLE INEQUALITY THEOREM 1

Triangle Inequality Theorem 1 ***($Ss \rightarrow Aa$)***
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

Given: ΔLMN; $\overline{LN} > \overline{LM}$
Prove: $\angle LMN > \angle LNM$

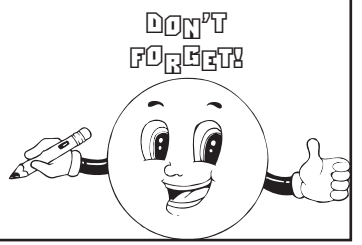




Proof: There is a need to make additional constructions to prove that $\angle LMN > \angle LNM$. With compass point on L and with radius $\overline{LM}$, mark a point P on $\overline{LN}$ and connect M and P with a segment to form triangle.

Statements	Reasons
1. How do you describe the relationship between $\overline{LM}$ and $\overline{LP}$?	By construction
2. Based on statement 1, what kind of a triangle is ΔLMP?	Definition of Isosceles Triangle
3. Based on statement 1, how do you describe $\angle 1$ and $\angle 2$?	Converse of Isosceles Triangle Theorem

Teacher's Note and Reminders



4.	Study the illustration and write a statement about $\angle LMN$ if the reason is the one given.	Angle Addition Postulate
5.	Basing on statement 4, write an inequality statement focusing on $\angle 1$.	Property of Inequality
6.	Using statement 3 in statement 5: $\angle LMN > \angle 2$	Substitution Property
7.	Study the illustration and write an operation statement involving $\angle MPN$, $\angle N$, and $\angle 3$	Sum of the interior angles of a triangle is 180°.
8.	Study the illustration and write an operation statement involving $\angle 2$ and $\angle MPN$	Linear Pair Theorem
9.	$\angle 2 + \angle MPN \cong \angle MPN + \angle N + \angle 3$	What property supports the step wherein we replace the right side of statement 8 with its equivalent in statement 7?
10.	What will be the result if $\angle MPN$ is deducted away from both sides of statement 9?	
11.	Basing on statement 10, write an inequality statement focusing on $\angle N$.	Property of Inequality
12	Based on statement 6 and 11: If $\angle LMN > \angle 2$ and $\angle 2 > \angle N$, then	Property of Inequality

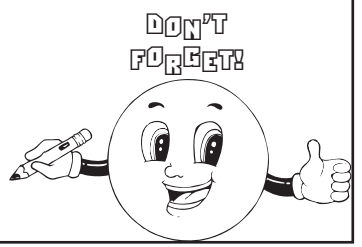
Congratulations! You have contributed much in proving Triangle Inequality Theorem 1. In the next activity, you will see that Triangle Inequality Theorem 1 is used in proving Triangle Inequality Theorem 2.

Answer Key to Activity 12: Indirect Proof of Triangle Inequality Theorem 2

A.

Statements	Reasons
1. $MN \not> LM$ such that $MN = LM$ or $MN < LM$	1. Temporary Assumption
2. Considering $MN = LM$: If $MN = LM$, then ΔLMN is an isosceles triangle.	2. Definition of isosceles triangles
Consequently, $\angle L = \angle N$.	Base angles of isosceles triangles are congruent.
The assumption that $MN = LM$ is false.	The conclusion that $\angle L \cong \angle N$ contradicts the given that $m\angle L > m\angle N$.
3. Considering $MN < LM$: If $MN < LM$, then $m\angle L < m\angle N$.	3. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
The Assumption that $MN < LM$ is False	The conclusion that $m\angle L < m\angle N$ contradicts the given that $m\angle L > m\angle N$
4. Therefore, $MN > LM$ must be True	4. The assumption that $MN \not> LM$ contradicts the known fact that $m\angle L > m\angle N$.

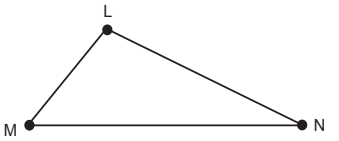
Teacher's Note and Reminders



Activity 12 INDIRECT PROOF OF TRIANGLE INEQUALITY THEOREM 2

Triangle Inequality Theorem 2 (Aa→Ss)
If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

Given: ΔLMN; $m\angle L > m\angle N$
Prove: $MN > LM$



Indirect Proof:

Assume: $MN \not> LM$

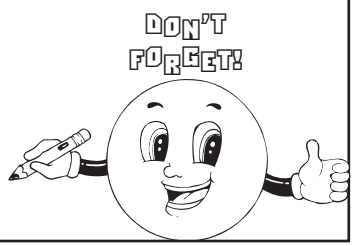
Statements	Reasons
1. $MN = LM$ or $MN < LM$	1. Assumption that $MN \not> LM$
2. Considering $\overline{MN} \cong \overline{LM}$: If $\overline{MN} \cong \overline{LM}$, then ☐	2. Definition of ☐
Consequently, what can you say about $\angle L$ and $\angle N$? ☐	☐ of isosceles triangles are congruent.
The Assumption that $\overline{MN} \cong \overline{LM}$ is ☐ True ☐ False	The conclusion that $\angle L \cong \angle N$ ☐ the given that $m\angle L > m\angle N$.
3. Considering $MN < LM$: If $MN < LM$, then ☐	3. Base angles of isosceles triangles are congruent.
The Assumption that $MN < LM$ is ☐ True ☐ False	The conclusion that $m\angle L < m\angle N$ contradicts the given that ☐
4. Therefore, $MN > LM$ must be ☐ True ☐ False	4. The ☐ that $MN \not> LM$ contradicts the known fact that $m\angle L > m\angle N$.

Amazing! You have helped in proving Triangle Inequality Theorem 2. Let us proceed to prove Triangle Inequality Theorem 3 using a combination of paragraph and two-column form. You will notice that Triangle Inequality Theorem 2 is used as reason in proving the next theorem.

Answer Key to Activity 13: Proving Triangle Inequality Theorem 3

Statements	Reasons
1. $LP = LN$	By construction
2. ΔLNP is an isosceles triangle.	Definition of isosceles triangle
3. $\angle LNP \cong \angle LPN$	Base angles of isosceles triangle are congruent.
4. $\angle LPN \cong \angle MPN$	Reflexive Property
5. $\angle LNP \cong \angle MPN$	Transitive Property
6. $\angle MNP \cong \angle LNM + \angle LNP$	Angle Addition Postulate
7. $\angle MNP \cong \angle LNM + \angle MPN$	Substitution Property
8. $\angle MNP > \angle MPN$	Property of Inequality
9. $MP > MN$	Triangle Inequality Theorem 2 ($Aa \rightarrow Sa$)
10. $LM + LP = MP$	Segment Addition Postulate
11. $LM + LP > MN$	Substitution Property
12. $LM + LN > MN$	Substitution Property

Teacher's Note and Reminders

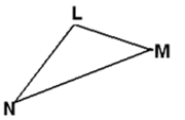


Activity 13 PROVING TRIANGLE INEQUALITY THEOREM 3

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

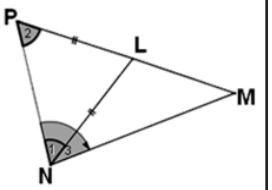
Given: ΔLMN where $\overline{LM} < \overline{LN} < \overline{MN}$

Prove: $\overline{MN} + \overline{LN} > \overline{LM}$
$\overline{MN} + \overline{LM} > \overline{LN}$
$\overline{LM} + \overline{LN} > \overline{MN}$



Proof:

- Notice that since $\overline{MN} > \overline{LN}$ and that $\overline{MN} > \overline{LM}$, then it's obvious that $\overline{MN} + \overline{LM} > \overline{LN}$ and $\overline{MN} + \overline{LN} > \overline{LM}$ are true.
- Hence, what remains to be proved is the third statement: $\overline{LM} + \overline{LN} > \overline{MN}$



Let us construct LP as an extension of $\overline{LM}$ such that L is between M and P, $\overline{LP} \cong \overline{LN}$ and ΔLNP is formed.

Statements	Reasons
1. Write a statement to describe $\overline{LP}$ and $\overline{LN}$. ____	1. By construction
2. Describe ΔLNP. ____	2. ____
3. Describe $\angle LNP$ and $\angle LPN$ ____	3. Bases of isosceles triangles are congruent.
4. The illustration shows that $\angle LPN \cong \angle MPN$	4. Reflexive Property of Equality
5. If $\angle LNP \cong \angle LPN$ (statement 3) and $\angle LPN \cong \angle MPN$ (statement 4), then ____	5. Transitive Property of Equality
6. From the illustration, $\angle MNP \cong \angle LNM + \angle LNP$	6. ____

Answer Key to Activity 14: Proving the Exterior Angle Inequality Theorem

Statements	Reasons
1. $\overline{LQ} \cong \overline{NQ}$; $\overline{MQ} \cong \overline{QR}$	1. By construction
2. $\angle 3 \cong \angle 4$	2. Vertical Angles are congruent.
3. $\Delta LQM \cong \Delta NQR$	3. SAS Triangle Congruence Postulate
4. $\angle MLN \cong \angle 1$	4. Corresponding parts of congruent triangles are congruent
5. $\angle LNP \cong \angle 1 + \angle 2$	5. Angle Addition Postulate
6. $\angle LNP > \angle 1$	6. Property of Inequality
7. $\angle LNP > \angle MLN$	7. Substitution Property of Equality

Teacher's Note and Reminders

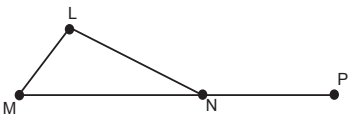
7.	Using statement 5 in statement 6, $\angle MNP \cong \angle LNM + \angle MPN$	7.
8.	From statement 7, $\angle MNP > \angle MPN$	8. Property of Inequality
9.	Using statement 8 and the illustration, write a statement with the reason given.	9. Triangle Inequality Theorem 2
10.	From the illustration, what operation involving $\overline{LM}$ and $\overline{LP}$ can you write?	10. Segment Addition Postulate
11.	Write a statement using statement 10 in statement 9	11. Substitution Property of Inequality
12.	Write a statement using statement 1 in statement 11	12. Substitution Property of Equality

Hurray! Triangle Inequality Theorem 3 is already proven. Let us proceed to writing the proof of Exterior Angle Inequality Theorem.

Activity 14 PROVING THE EXTERIOR ANGLE INEQUALITY THEOREM

Exterior Angle Inequality Theorem
The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle

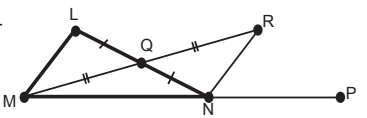
Given: ΔLMN with exterior angle $\angle LNP$
Prove: $\angle LNP > \angle MLN$



Proof:

Let us prove that $\angle LNP > \angle MLN$ by constructing the following:

1. midpoint Q on $\overline{LN}$ such that $\overline{LQ} \cong \overline{NQ}$
2. $\overline{MR}$ through Q such that $\overline{MQ} \cong \overline{QR}$



Answer Key to Activity 15: Proving the Hinge Theorem

Statements	Reasons
1. $\overline{CN} \cong \overline{CH} + \overline{HN}$	1. Segment Addition Postulate
2. $\overline{CN} \cong \overline{CH} + \overline{WH}$	2. Substitution Property of Equality
3. In ΔCHW, $\overline{CH} + \overline{WH} > \overline{CW}$	3. Triangle Inequality Theorem 3: The sum of any two sides of a triangle is greater than the third side.
4. $\overline{CN} > \overline{CW}$	4. Substitution Property of Equality (Using statement 2 in 3)
5. $\overline{CN} > \overline{LT}$	5. Substitution Property of Equality (Using statement in construction 1 in statement 4)

Teacher's Note and Reminders

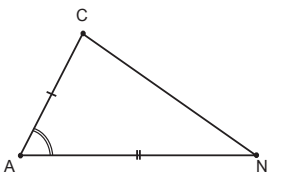
Statements	Reasons
1. $\overline{LQ} \cong \overline{NQ}$; $\overline{MQ} \cong \overline{QR}$	1.
2. What relationship exists between $\angle 3$ and $\angle 4$?	2.
3. Basing on statements 1 and 2, describe two triangles from the illustration:	3. What triangle congruence postulate supports statement 3?
4. Basing on statement 3, $\angle MLN \cong$	4.
5. Basing on the illustration, $\angle LNP \cong$	5. Angle Addition Postulate
6. Basing on statement 5, $\angle LNP > \angle 1$	6.
7. Using statement 4 in statement 6,	7. Substitution Property

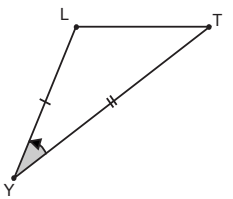
Indeed, the measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

Activity 15 PROVING THE HINGE THEOREM

Hinge Theorem or SAS Triangle Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Given: ΔCAN and ΔLYT; $\overline{CA} \cong \overline{LY}$, $\overline{AN} \cong \overline{YT}$, $\angle A > \angle Y$
Prove: $\overline{CN} > \overline{LT}$





Answer Key to Activity 16:
Indirect Proof of the Converse of Hinge Theorem

Statements	Reasons
1. $\angle D \cong \angle U$ or $\angle D < \angle U$	1. Assumption that $\angle D \ncong \angle U$
2. Considering $\angle D \cong \angle U$: It's given that $\overline{OD} \cong \overline{LU}$, $\overline{DG} \cong \overline{UV}$. If $\angle D \cong \angle U$, then $\Delta ODG \cong \Delta LUV$.	2. SAS Triangle Congruence Postulate
From the congruence, $\overline{OG} \cong \overline{LV}$	Corresponding parts of congruent triangles are congruent
The Assumption that $\angle D \cong \angle U$ is false.	OG ≅ LV contradicts the given that $\overline{OG} > \overline{LV}$
3. Considering $\angle D < \angle U$: If $\angle D < \angle U$, then $\overline{OG} < \overline{LV}$.	3. SAS Inequality Theorem or Hinge Theorem
The assumption that $\angle D < \angle U$ is false.	$\overline{OG} < \overline{LV}$ contradicts the given that $\overline{OG} > \overline{LV}$
4. Therefore, $\angle D > \angle U$ must be true.	4. Assumption that $\angle D \ncong \angle U$ is proven to be false.

After proving the theorems on inequalities in triangles, you are now highly equipped with skills in writing both direct and indirect proofs. Moreover, you now have a good grasp on how to write proofs in paragraph and/or two-column form.

You will be undergoing more complex application problems involving inequalities in triangles in the next section.

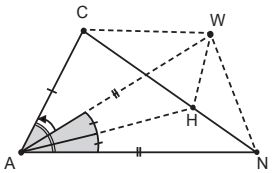
Dear Concept Contractor, your task is to revisit your concept museum. How many more tasks can you tackle? Which concepts you have built previously need revision? Check also your decisions in Activity No.1. Would you like to change any decision?

How can you justify inequalities in triangles? *Do you have a new insight on how to address this essential question raised in the activity* ***Artistically Yours?***

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

Proof:

1. Construct AW such that :
   - $\overline{AW} \cong \overline{AN} \cong \overline{YT}$
   - $\overline{AW}$ is between $\overline{AC}$ and $\overline{AN}$, and
   - $\angle CAW \cong \angle LYT$.



Consequently, $\Delta CAW \cong \Delta LYT$ *by SAS Triangle Congruence Postulate. So,* $\overline{CW} \cong \overline{LT}$ *because corresponding parts of congruent triangles are congruent.*

2. Construct the bisector $\overline{AH}$ of $\angle NAW$ such that:
   - H is on $\overline{CN}$
   - $\angle NAH \cong \angle WAH$

Consequently, $\Delta NAH \cong \Delta WAH$ by SAS Triangle Congruence Postulate because $\overline{AH} \cong \overline{AH}$ by reflexive property of equality and $\overline{AW} \cong \overline{AN}$ from construction no. 1. So, $\overline{WH} \cong \overline{HN}$ *because corresponding parts of congruent triangles are congruent.*

Statements	Reasons
1. From the illustration: $\overline{CN} \cong \overline{CH} + \overline{HN}$	1.
2. $\overline{CN} \cong \overline{CH} + \overline{WH}$	2.
3. In ΔCHW, $\overline{CH} + \overline{WH} > \overline{CW}$	3.
4. Using statement 2 in 3: $\overline{CN} > \overline{CW}$	4.
5. Using statement in construction 1 in statement 4: $\overline{CN} > \overline{LT}$	5.

Bravo! The Hinge Theorem is already proven. Notice that the use of paragraph form on the first part of the proof of the Hinge Theorem shortens the proof process.

Activity 16 INDIRECT PROOF OF THE CONVERSE OF HINGE THEOREM

Converse of Hinge Theorem or SSS Triangle Inequality Theorem
If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

Appreciate the students' interest and diligence in tackling the proof-writing activities. Praise is a great form of motivation.

At this point, let them revisit their answers in Activities No. 1, 2, and 3.

WHAT TO REFLECT AND UNDERSTAND:

The **REFLECT AND UNDERSTAND** section exhibits activities designed to intensify the students' understanding of the lesson. These activities are mostly on solving non-routine problems, writing proofs, and wrapping up of all the concepts and skills learned in the lesson.

Like in the Process/Do section, your task in this section is to manage group work in the problem-solving activities. It is suggested that you have to play an active role in the discussion of the solutions of the model problems so that students will have an extensive grasp on the thinking processes undertaken in solving the problems.

Your guidance and supervision of the students in the It's-Your-Turn problem solving activities of this section lead to the attainment of the following goals:

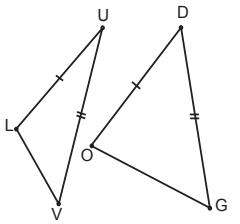
- solve problems that require application of the knowledge and skills in inequalities in triangles;
- solve problems that require writing proofs; and
- unlock all the triangles in their concept museum.

Discuss comprehensively the solutions and answers to It's-Your-Turn problems enhance their understanding and reinforce their learning.

For Activity No. 19, invite them to make a research on the task under Career in Mathematics—Air Traffic Controller.

Given: ΔODG and ΔLUV; $\overline{OD} \cong \overline{LU}$, $\overline{DG} \cong \overline{UV}$, $\overline{OG} > \overline{LV}$

Prove: $\angle D > \angle U$



Indirect Proof:

Assume: $\angle D \not> \angle U$

Statements	Reasons
1. $\angle D \cong \angle U$ and $\angle D < \angle U$	1. Assumption that ☐
2. Considering $\angle D \cong \angle U$: It's given that $\overline{OD} \cong \overline{LU}$, $\overline{DG} \cong \overline{UV}$. If $\angle D \cong \angle U$, then $\Delta ODG \cong \Delta LUV$.	2. ☐ Triangle Congruence Postulate
$\overline{OG} \cong \overline{LV}$	☐
The Assumption that $\angle D \cong \angle U$ is false.	☐
3. Considering $\angle D < \angle U$: If $\angle D < \angle U$, then ☐	3. Hinge Theorem
☐	$\overline{OG} < \overline{LV}$ contradicts the given that $\overline{OG} > \overline{LV}$
4. ☐	4. Assumption that $\angle D \not> \angle U$ is proven to be false.

After proving the theorems on inequalities in triangles, you are now highly equipped with skills in writing both direct and indirect proofs. Moreover, you now have a good grasp on how to write proofs in paragraph and/or two-column form.

You will be undergoing more complex application problems involving inequalities in triangles in the next section.

Dear Concept Contractor, your task is to revisit your concept museum. How many more tasks can you tackle? Which concepts you have built previously need revision? Check also your decisions in Activity No.1. Would you like to change any decision?

How can you justify inequalities in triangles? *Do you have a new insight on how to address this essential question raised in the activity* ***Artistically Yours?***

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

Answer Key to Activity 17: Show Me the Angles!!!

Answer key to Activity No. 17 Watch-This Questions!

1. The value of x is solved first because knowing its value leads to determining the values of the angles of the triangular frame.
2. The sum of the angles is equated to 180 because the sum of the angles of a triangle is always 180 degrees.
3. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
4. Even without the actual measurements, we are sure that our answer is correct because we have used the theorems we developed, verified and proved.

Note: Explain to the students that when we justify our answer using theorems or postulates, we are justifying deductively. So, deductively, it is sure that the answer is correct.

It's Your Turn!

$$
\begin{aligned}
m\angle S + m\angle E + m\angle A &= 180 \\
58 + (2x - 1) + (4x - 3) &= 180 \\
58 + 2x + 4x - 1 - 3 &= 180 \\
6x + 54 &= 180 \\
6x &= 180 - 54 \\
6x &= 126 \\
x &= 21
\end{aligned}
\qquad
\begin{aligned}
m\angle E &= 2x - 1 \\
&= 2(21) - 1 \\
&= 42 - 1 \\
&= 41
\end{aligned}
\qquad
\begin{aligned}
m\angle A &= 4x - 3 \\
&= 4(21) - 3 \\
&= 84 - 3 \\
&= 81
\end{aligned}
$$

Since $\angle A > m\angle E > m\angle S$, then the longest side is opposite $\angle A$, $\overline{ES}$, and the shortest side is opposite $\angle S$, $\overline{AE}$.

What to Understand

Having developed, verified, and proved all the theorems on triangle inequalities in the previous section, your goal now in this section is to take a closer look at some aspects of the topic. This entails you to tackle on more applications of the theorems on triangle inequalities.

Your goal in this section is to use the theorems in identifying unknown inequalities in triangles and in justifying them.

The first set of activities showcases model examples that will equip you with ideas and hints on how to conquer problems of the same kind but already have twists. When it is your turn to answer, **you have to provide justifications** to every step you take as you solve the problem. The model examples provide questions for you to answer. Your answers are the justifications.

The second set of activities requires you to use the theorems on inequalities in triangles in solving problems that require you to write proofs.

There are no limits to what the human imagination can fathom and marvel. Fun and thrill characterize this section. It is also where you will wrap up all the concepts you learned on Triangle Inequalities.

Activity 17 SHOW ME THE ANGLES!!!

Watch this!

For extra fun, groups of students in a class are tasked to create algebraic expressions to satisfy the measures of the angles of their triangular picture frame project. If the measure of the angles are as follows: $m\angle A = 5x - 3$, $m\angle C = 2x + 5$, $m\angle E = 3x - 2$, arrange the sides of the frame in increasing order.

Solution:

To solve for x:	**Solving for $m\angle A$**	**Solving for $m\angle C$**	**Solving for $m\angle E$**
$(5x - 3) + (2x + 5) + (3x - 2) = 180$	$m\angle A = 5x - 3$	$m\angle C = 2x + 5$	$m\angle E = 3x - 2$
$5x + 2x + 3x - 3 + 5 - 2 = 180$	$= 5(18) - 3$	$= 2(18) + 5$	$= 3(18) - 2$
$10x - 5 + 5 = 180$	$= 90 - 3$	$= 36 + 5$	$= 54 - 2$
$10x = 180$	$= \mathbf{87}$	$= \mathbf{41}$	$= \mathbf{52}$
$x = 18$			

Therefore, listing the sides in increasing order should follow this order: Sides opposite $\angle C$, $\angle E$, and $\angle A$. That is, $\overline{AE}$, $\overline{AC}$, and $\overline{CE}$.

Answer Key to Activity 18:
Believe Me, There are Lots of Possibilities!

1. Triangle Inequality Theorem 3($S1 + S2 > S3$)
2. Even without actually drawing all the possible lengths of the third side to form a triangle with known sides 11 and 17, we are convinced that our answer is correct because its basis, Triangle Inequality Theorem 3, is a theorem that we have developed, verified and proved. Deductively, we are convinced that our answer is correct.
3. Relationship: 6 is the difference when 17 is subtracted from 11.
4. Relationship: 28 is the sum of 11 and 17.
5. $l - s < t < l + s$
6. There is an infinite number of possible lengths for the third side t.

Note: Remind them of their lesson on the set of rational numbers or fractions between 0 and 1 in Grade 7. Because a fraction between 0 and 1 can be in the form 1/M, M can be any value greater than 1. Hence, M can be 1, 000, 000 or more. Thus, there are infinite fractions between 0 and 1. Notice that this concept is also applicable to lengths between 6 and 28.

It's Your Turn!

The lengths of the sides of a triangle are $16 - k$, 16 and $16 + k$. What is the possible range of values of k?

Using the Triangle Inequality Theorem 3, let us find the range of values for k:

$16 + (16 + k) > 16$
$16 + 16 + k > 16$
$32 + k > 16$
$k > 16 - 32$
$k > -16$
$-16 < k$

$(16 - k) + 16 > 16 + k$
$16 - k + 16 > 16 + k$
$32 - k > 16 + k$
$32 - 16 > k + k$
$16 > 2k$
$8 > k$
$k < 8$

Writing them as a combined inequality, the answer is $-16 < k < 8$.

QUESTIONS ?

1. Why is the value x being solved first?
2. Why is the sum of the angles being equated to 180°?
3. What theorem justifies the conclusion that the increasing order of the sides is $\overline{AE}$, $\overline{AC}$, and $\overline{CE}$?
4. What makes us sure that our answer is correct considering that we have not exactly seen the actual triangle and have not used tools to measure the lengths of its sides and the measures of its angles?

It's Your Turn!

Angle S of the triangular picture frame of another group is 58°. The rest of the angles have the following measures: $m\angle E = 2x - 1$, $m\angle A = 4x - 3$. Determine the longest and the shortest side. Give justifications.

Activity 18 BELIEVE ME, THERE ARE LOTS OF POSSIBILITIES!

Watch this!

Problem:

You are tasked to draw a triangle wherein the lengths of two sides are specified. What are the possible lengths for the third side of the triangle you will draw if two sides should be 11 and 17, respectively? How many possible integer lengths has the third side?

Solution:

Since the third side is unknown, let's represent its length by t.

Inequality 1	Inequality 2	Inequality 3
$11 + 17 > t$ $28 > t$ $t < 28$ t must be less than **28**	$11 + t > 17$ $t > 17 - 11$ $t > 6$ t must be greater than **6**	$17 + t > 11$ $t > 11 - 17$ $t > -6$ Values of t to be disregarded

The resulting inequalities show that t must be between 6 and 28, that is, 6 as the lower boundary and 28 as the higher boundary. Using combined inequality, the order by which they will be written should be 6, t, then 28.

Therefore,

- the possible lengths for the third side is **$6 < t < 28$**.
- the set of possible integer lengths for the third side of the triangle is described as follows: {7, 8, 9, …,27}. Hence, there are 27 – 6 = **21** possible integer lengths for the third side.

Answer Key to Activity 19:
And You Thought Only Surveyors Trace, Huh!

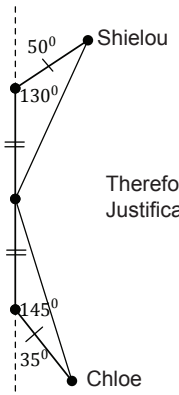
1. 110 and 70 degrees form a linear pair, also 90 and 90 degrees.
2. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
3. Probably yes. However, solving it is easier through a detailed illustration. Solving it mentally would be tedious because there are plenty of information in the problem
4. Had we not seen the illustration and had we not known about Triangle Inequality Theorem 2, the answer would be that their distance from the centre of the oval is the same because they have travelled the same length from start to where they stop.
5. We are convinced that the conclusion is true because the basis is Triangle Inequality Theorem 2 that we developed, verified and proved to be true. Deductively, we are convinced that the conclusion is true.

It's Your Turn!

The lengths of the sides of a triangle are $16 - k$, 16 and $16 + k$. What is the possible range of values of k?

Using the Triangle Inequality Theorem 3, let us find the range of values for k:

1.



Therefore, Chloe is farther from the rotunda.
Justification is the Hinge Theorem.

QUESTIONS ?

1. *What theorem justifies the three inequalities being written about the sides?*
2. *Are you convinced that $6 < t < 28$ is accurate even if you have not tried drawing all the possible lengths of the third side to form a triangle with 11 and 17? Why?*
3. *Do you observe a relationship existing between 6 in **$6 < t < 28$** and the two known lengths 11 and 17? Describe the relationship.*
4. *Do you observe a relationship existing between 28 in **$6 < t < 28$** and the two known lengths 11 and 17? Describe the relationship.*
5. *If the known lengths are l and s, where l is longer and s is shorter, what should be the formula in solving for the unknown third side t?*
6. *There are 21 possible integer lengths for the third side when two respective sides of a triangle have lengths 11 and 17. Can you count all the possible lengths other than the integer lengths? Explain.*

It's Your Turn!

Problem:

The lengths of the sides of a triangle are $16 - k$, 16, and $16 + k$. What is the range of the possible values of k? Create a table of the possible integer lengths of the sides of the triangle. Is $16\text{-}k$ always the shortest length? Develop a general formula for lengths with this description. Provide justifications.

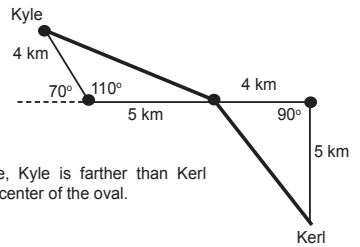
Activity 19 AND YOU THOUGHT ONLY SURVEYORS TRACE, HUH!

Watch this!

Problem:

Kerl and Kyle play with their roller skates at the town oval. From the centre of the oval, Kerl skates 4 meters east and then 5 meters south. Kyle skates 5 meters west. He then takes a right turn of 70° and skates 4 meters. Who is farther from the centre of the oval?

Solution:



Therefore, Kyle is farther than Kerl from the center of the oval.

2. Enrichment Activity

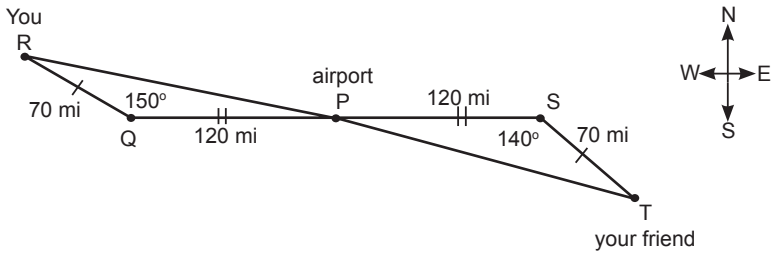
Career in Mathematics: Air Traffic Controller

Sample Research:
http://www.nexuslearning.net/books/ml-geometry/Chapter5/ML%20Geometry%205-6%20Indirect%20Proof%20and%20Inequalities%20in%20Two%20Triangles.pdf

You and a friend are flying separate planes. You leave the airport and fly 120 miles due west. You then change direction and fly W 30° N for 70 miles. (W 30° N indicates a north-west direction that is 30° north of due west.) Your friend leaves the airport and flies 120 miles due east. She then changes direction and flies E 40° S for 70 miles. Each of you has flown 190 miles, but which plane is farther from the airport?

SOLUTION

Begin by drawing a diagram, as shown below. Your flight is represented by ΔPQR and your friend's flight is represented by ΔPST.



Because these two triangles have two sides that are congruent, you can apply the Hinge Theorem to conclude that $\overline{RP}$ is longer than $\overline{TP}$.

Therefore, your plane is farther from the airport than your friend's plane.

QUESTIONS ?

1. How are 110° and 90° produced?
2. What theorem justifies the conclusion that Kyle is farther than Kerl from the center of the oval?
3. Would this problem be answered without a detailed illustration of the problem situation? Explain.
4. Had the illustration of the problem not drawn, what would have been your initial answer to what is asked? Explain.
5. We have not actually known Kerl and Kyle's distances from the center of the oval but it is concluded that Kyle is farther than Kerl. Are you convinced that the conclusion is true? Explain.

It's Your Turn!

1. *Problem:*
From a boulevard rotunda, bikers Shielou and Chloe who have uniform biking speed, bike 85 meters each in opposite directions—Shielou, to the north and Chloe, to the south. Shielou took a right turn at an angle of 50° and Chloe, a left turn at 35°. Both continue biking and cover another 60 meters each before taking a rest. Which biker is farther from the rotunda? Provide justifications.

2. *Enrichment Activity*
Career in Mathematics: Air Traffic Controller

Air traffic controllers coordinate the movement of air traffic to make certain that planes stay a safe distance apart. Their immediate concern is safety, but controllers also must direct planes efficiently to minimize delays.

They must be able to do mental math quickly and accurately. Part of their job is directing aircraft at what altitude and speed to fly.

Task:

Make a research of problems related to the work of air traffic controllers. Solve it and present it in class

Answer Key to Activity 20:
Trust Yourself, You're a Geometrician!

1. Side HT is the longest because triangle HIT looks bigger in the figure.
   Note: Explain to the students that the problem says that the diagram is not drawn to scale so answers based on the drawing without considering the given data would be faulty. It is advised that you integrate it with this saying: Do not judge a book by its cover—that outward appearances can be misleading.
2. It is necessary to consider each right triangle individually because a side of one triangle is also a side to another triangle—the triangles have common sides.
3. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
4. Deductively, we are convinced that the conclusion is true.

Answer Key to It's Your Turn! Problem

1. Side HT is the longest because triangle HIT looks bigger in the figure can be misleading.
2. It is necessary to

Considering ΔMAT:
AT=13, MT=14, MA=15
For angles opposite them
$\angle M < \angle MAT < \angle T$

Considering ΔMEA:
ME=9, EA=12, AM=15
For angles opposite them
$\angle 2 < \angle M < \angle 1$

Considering Exterior Angle 1 of ΔTEA:
$\angle 1 > \angle T$ or $\angle T < \angle 1$

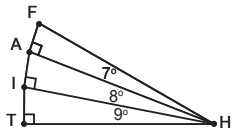
$\angle 2 < \angle M < \angle MAT < \angle T < \angle 1$

Therefore, the answer is as follows:

$\angle 2 < \angle M < \angle MAT < \angle T < \angle 1$

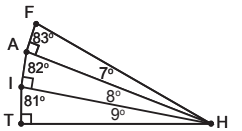
Activity 20 TRUST YOURSELF, YOU'RE A GEOMETRICIAN!

Watch this!
Problem:



The diagram is not drawn to scale. Which of the lengths $\overline{HF}$, $\overline{HA}$, $\overline{HI}$, and $\overline{HT}$ of polygon FAITH is the longest? Which is the shortest?

Solution:



Considering ΔHIT:
$\overline{HT} < \overline{HI}$

Considering ΔHAI:
$\overline{HI} < \overline{HA}$

Considering ΔHFA:
$\overline{HA} < \overline{HF}$

$\overline{HT} < \overline{HI} < \overline{HA} < \overline{HF}$

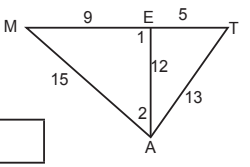
Therefore, the longest side is $\overline{HF}$ and the shortest side is $\overline{HT}$.

QUESTIONS

1. By just looking at the original figure, which side do you think is the longest? There is a misconception to explain why HT would have been the initial choice as having the longest side. Explain.
2. Why is it necessary to consider each right triangle in the figure individually?
3. What theorem justifies the choice of the longer side in each triangle?
4. Notice that the diagram is not drawn to scale. However, we are still able to tell which side is the longest and which side is the shortest. Are you convinced that your answer is true? Explain.

It's Your Turn!
Problem:

The diagram is not drawn to scale. Using $\angle 1$, $\angle 2$, $\angle T$, $\angle M$, and $\angle MAT$, complete the combined inequalities below:



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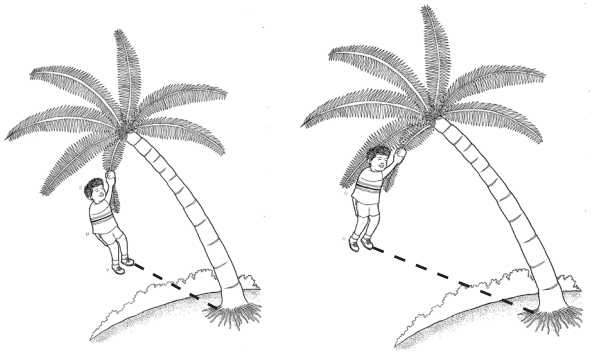
Answer Key to Activity 21: I Believe I can Fly

1. Sides: coco trunk, distance of the kid from the bottom of the coco trunk, length of the coco leaf stalk.
2. The inequalities that exist are the following:
   - The distance of the kid from the bottom of the coco trunk at different speeds.
   - The angle determined by the coco trunk and the coco leaf stalk at different speeds
3. Comparison:
   - The distance of the kid from the bottom of the coco trunk is longer when he swings at full speed and shorter when he swings at low speed.
   - The angle determined by the coco trunk and the coco leaf stalk is larger when he swings at full speed and smaller when he swings at low speed
4. I can justify them deductively using the hinge theorem and its converse.
5. (Answers may vary)
6. Possible answer: Using vines like Tarzan, swing rides in amusement parks)
7. Possible answer: Erecting a post covered with rubber or leather and using big rope for a swing ride
8. Possible disadvantages: Height of swing towers and lengths of swings would not be proportional and can cause accidents
9. Possible answers: Efficient (Strong and Stable), Safe, Well-Built, Attractive
10. Possible answers: Prepare a design to determine the specifications and raw materials; let the best workers make it; have it tested for quality.
11. Yes, so that tools and equipment are efficient, long-lasting, and safe to use.

Activity 21 I BELIEVE I CAN FLY



The figure shows two pictures of a kid swinging away from the coco trunk while holding on a stalk of coco leaf. Compare the distances of the kid from the bottom of the coco trunk in these pictures. Note that the kid's distance from the bottom of the coco trunk is farthest when he swings at full speed.

QUESTIONS ?

1. Name the sides of the triangle formed as the kid swings away holding on to the stalk of coco leaf.
2. An inequality exists in the two triangles shown. Describe it.
3. Compare the angles formed by the coco leaf stalk and the coco trunk at the kid's full speed and low speed.
4. How can you justify the inequality that exists between these triangles?
5. Many boys and girls in the province have great fun using coco leaf stalks as swing rides. Have you tried a coco leaf swing ride?
6. Aside from coco leaf swing rides, what other swing rides do you know in your area or from your knowledge or experience?
7. If you were asked to improvise a swing ride in your community, how would you design the swing ride? Explain.
8. Concepts on inequalities in triangles are useful in improvising a swing ride. What are the disadvantages if a designer of a swing ride does not apply these concepts?
9. What are the qualities of a good improvised swing ride?
10. What are the things you should do to attain these qualities?
11. Should all designers of tools and equipment comply with standards standards and guidelines in designing them? Why?

Answer Key to Activity 22:
You are Now Promoted as PROOFessor!

1.

	Statements	Reasons
1	$\overline{HO} \cong \overline{EP}$	Given
2	$\overline{HP} \cong \overline{HP}$	Reflexive Property of Equality
3	$\angle OHP > \angle EPH$	Given
4	$\overline{OP} > \overline{EH}$	Hinge Theorem

2.

	Statements	Reasons
1	$\angle 1 \cong \angle 2$	Given
2	ΔFIH is isosceles	Base angles of isosceles triangles are congruent.
3	$\overline{FI} \cong \overline{HI}$	Legs of isosceles triangles are congruent.
4	I is the midpoint of AT	Given
5	$\overline{AI} \cong \overline{TI}$	Definition of a Midpoint
6	$\angle 3 > \angle 4$	Given
7	$\overline{HT} > \overline{FA}$	Hinge Theorem

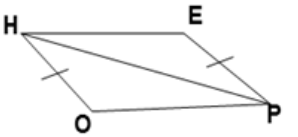
3.

		Reasons
1	$\angle VAE \cong \angle VEA$	Given
2	ΔAVE is an isosceles triangle.	Base angles of isosceles triangles are congruent
3	$\overline{AV} \cong \overline{EV}$	Legs of isosceles triangles are congruent.
4	$\overline{FV} \cong \overline{FV}$	Reflexive Property
5	$\overline{AF} > \overline{EF}$	Given
6	$\angle AVF \cong \angle EVF$	Converse of Hinge Theorem

Activity 22 YOU ARE NOW PROMOTED AS PROOFESSOR!

1. Write the statements supported by the reasons on the right side of the two-column proof.

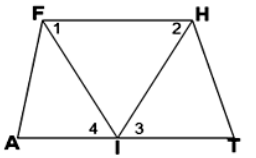
Given: $\overline{HO} \cong \overline{EP}$, $\angle OHP > \angle EPH$
Prove: $\overline{OP} > \overline{EH}$



	Statements	Reasons
1		Given
2		Reflexive Property of Equality
3		Given
4		Hinge Theorem

2. Make necessary markings to the congruent angles and sides as you analyze the given and the meanings behind them. Write the reasons for the statements in the two-column proof.

Given: I is the midpoint of $\overline{AT}$, $\angle 1 \cong \angle 2$, $\angle 3 > \angle 4$
Prove: $\overline{HT} > \overline{FA}$



	Statements	Reasons
1	$\angle 1 \cong \angle 2$	
2	ΔFIH is isosceles	
3	$\overline{FI} \cong \overline{HI}$	
4	I is the midpoint of AT	
5	$\overline{AI} \cong \overline{TI}$	
6	$\angle 3 > \angle 4$	
7	$\overline{HT} > \overline{FA}$	

In this section, the discussion focuses mainly on using the triangle inequality theorems in solving both real-life problems and problems that require writing proofs.

Considering the application and proof-writing problems found in this module, share your insights on the following questions:

- Can you solve these problems without accurate illustrations and markings on the triangles?
- Can you solve these problems without prior knowledge related to triangles and writing proofs?
- Has your knowledge in algebra helped you in solving the problems?
- Have the theorems on triangle inequalities helped you in writing proofs of theorems?

*Having tackled all concepts and skills to be learned on inequalities in triangles, revisit your decisions in Activity No.1 and write your responses to the statements under **My Decisions Later**. Are there changes to your responses? Explain.*

*What would be your reply to the essential question "**how can you justify inequalities in triangles"**?*

Now that you have a deeper understanding of the topic, it is high time for you to put your knowledge and skills to practice before you do the tasks in the next section.

Be sure to discuss the answers to the questions at the END of WHAT TO REFLECT AND UNDERSTAND.

At this point, the students should be able to answers all the questions in Activity Nos. 1, 2, & 3.

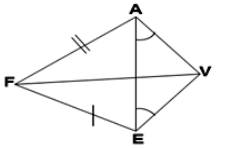
They should be able to answer the essential question "How can you justify inequalities in triangles?" The answer should be: Inequalities in triangles can be justified deductively. When asked how, they are expected to point out the theorems on inequalities in triangles.

What to Transfer

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will enable you to demonstrate your understanding of inequalities in triangles.

3. Write the statement or reason in the two-column proof.

Given: ∠VAE ≅ ∠VEA, $\overline{AF} > \overline{EF}$
Prove: ∠AVF ≅ ∠EVF



		Reasons
1	$\angle VAE \cong \angle VEA$	
2	ΔAVE is an isosceles triangle.	
3		Legs of isosceles triangles are congruent.
4	$\overline{FV} \cong \overline{FV}$	
5		Given
6	$\angle AVF \cong \angle EVF$	

In this section, the discussion focuses mainly on using the triangle inequality theorems in solving both real-life problems and problems that require writing proofs.

Considering the application and proof-writing problems found in this module, share your insights on the following questions:

- Can you solve these problems without accurate illustrations and markings on the triangles?
- Can you solve these problems without prior knowledge related to triangles and writing proofs?
- Has your knowledge in algebra helped you in solving the problems?
- Have the theorems on triangle inequalities helped you in writing proofs of theorems?

*Having tackled all concepts and skills to be learned on inequalities in triangles, revisit your decisions in Activity No.1 and write your responses to the statements under **My Decisions Later**. Are there changes to your responses? Explain.*

*What would be your reply to the essential question "**how can you justify inequalities in triangles"**?*

Now that you have a deeper understanding of the topic, it is high time for you to put your knowledge and skills to practice before you do the tasks in the next section.

You have to explain to the students that concepts and skills learned in inequalities and triangles become meaningful only when they can transfer their learning to real life situations such as performing a task where they would be able to produce something.

Discuss to them the details of Activity No. 23 before letting them start. Give students adequate time to plan and create their outputs before allowing them to present their work in class.

After presentations, clinch the lesson by letting them share insights on the activity questions.

Answer Key to Activity 23:
Disaster Preparedness: Making It through the Rain

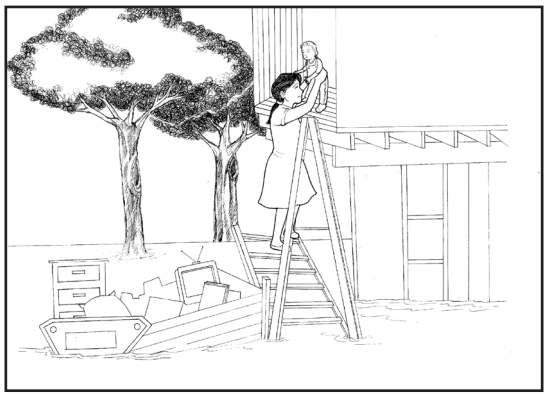
1. Possible Answers: Meaningful, Challenging, Fun, Interesting, Difficult
3. Possible Answers: Good planning makes the work faster; Cooperation and collaboration make the task easier and lighter; Mathematics is important in performing real-life tasks.
3. Yes because the concepts of hinge theorem and its converse is used in designing the range of distances between the feet of the folding ladder to make it stable enough for climbing.
4. The task makes me realize that mathematics is indeed important in performing real life tasks and in creating real life tools and equipment.
5. Note: Refer to the Product Column of Assessment Map for the answers
6. Example: Constructing an A-Frame House on different sizes of lot.

What to Transfer

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will enable you to demonstrate your understanding of inequalities in triangles.

Activity 23 DISASTER PREPAREDNESS: MAKING IT THROUGH THE RAIN

Goal: to design and create a miniature model of a folding ladder
Role: a design engineer
Audience: company head



Situation: The lessons learned from the widespread flooding in many parts of the country during typhoons and monsoon season include securing tools and gadgets needed for safety. More and more people are shopping for ladders that could reach as high as 10 feet, long enough for people to gain access to their ceiling or their roof. There is a high demand for folding ladders for they can be stored conveniently. Being the design engineer of your company, your boss asks you to submit a miniature model of that ladder and justify the design.

Product: design of a folding ladder that can reach up to 10-feet height and its miniature

Standards: accurate, creative, efficient, and well-explained/well-justified

Teacher's Note and Reminders

RUBRIC

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the geometric concepts specifically on triangle inequalities.	The computations are accurate and show the use of geometric concepts specifically on triangle inequalities.	The computations are erroneous and show some use of the concepts on triangle inequalities.	The computations are erroneous and do not show the use of the concepts on triangle inequalities.	
Creativity	The overall impact of the presentation of highly impressive and the use of technology is highly commendable.	The overall impact of the presentation is impressive and the use of technology is commendable.	The overall impact of the presentation is fair and the use of technology is evident.	The overall impact of the presentation is poor and the use of technology is non-existent.	
Efficiency	The miniature is very effective and flawlessly done. It is also attractive.	The miniature is effective and flawless.	The miniature has some defects.	The miniature has many defects.	
Mathematical Justification	Justification is logically clear, convincing, and professionally delivered. The concepts learned on triangle inequalities are applied and previously learned concepts are connected to the new ones.	Justification is clear and convincingly delivered. Appropriate concepts learned on triangle inequalities are applied.	Justification is not so clear. Some ideas are not connected to each other. Not all concepts on triangle inequalities are applied.	Justification is ambiguous. Only few concepts on triangles inequalities are applied.	

Answer Key to Activity 24:
Final Construction of Concept Museum

Please refer to answer key of Activity 3, Hello, Dear Concept Contractor.

Teacher's Note and Reminders

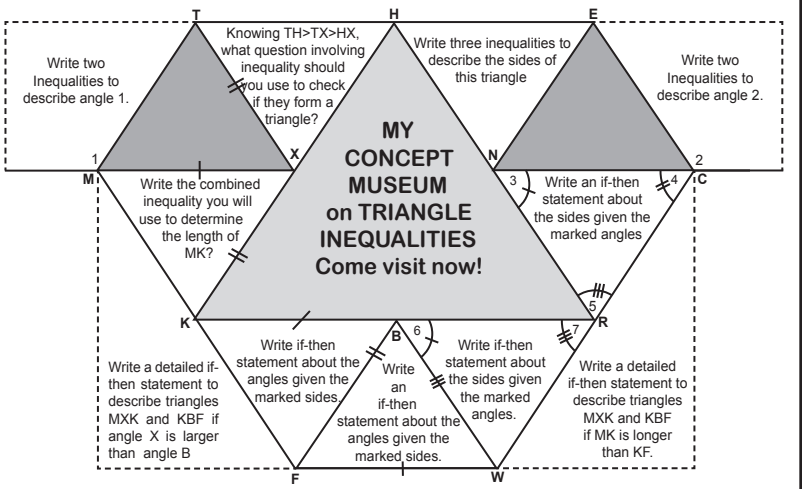
QUESTIONS ?

1. How do you find the experience of designing?
2. What insights can you share from the experience?
3. Has the activity helped you justify inequalities in triangles? How?
4. How did the task help you see the real world use of the concepts on inequalities in triangles?
5. Aside from designing a folding ladder, list down the real-life applications of concepts learned in Inequalities in Triangles from this module.
6. Can you think of other real-life applications of this topic?

SUMMARY

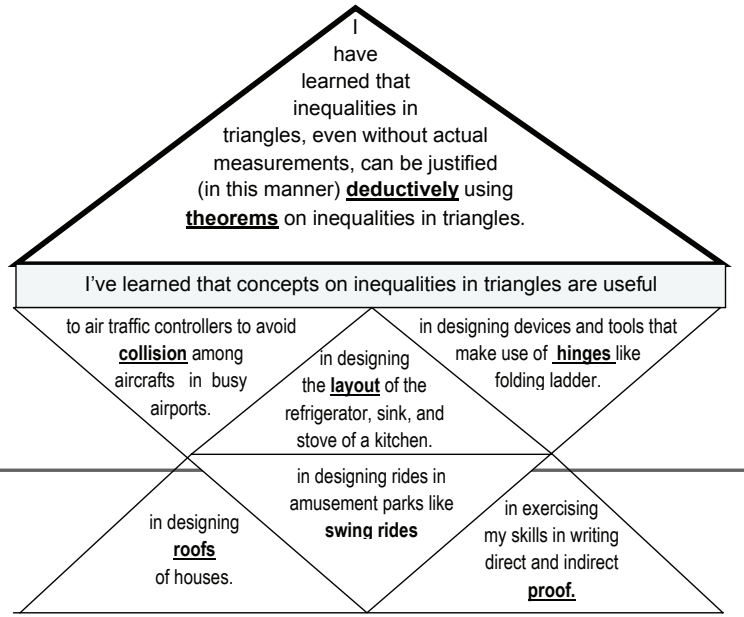
Activity 24 **FINAL CONSTRUCTION OF CONCEPT MUSEUM**

Directions: After learning all the concepts and skills on Inequalities in Triangles, take a final visit to your responses in Activity No.3— *Hello, Dear Concept Contractor*—of this module and make some modifications of or corrections to your responses and their corresponding justifications.



Answer Key to Activity 24:
Final Construction of Concept Museum

Please refer to answer key of Activity 3, Hello, Dear Concept Contractor.

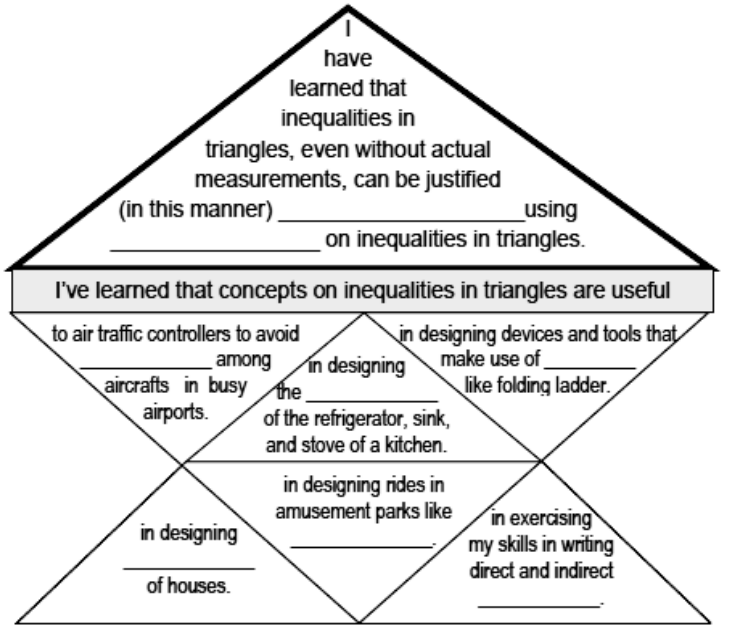


*You have completed the lesson on **Inequalities in Triangles**. Before you go to the next geometry lesson on **Parallelism and Perpendicularity**, you have to answer a post-assessment and a summative test.*

Before giving the post-assessment, let the groups to review their answers to the items in pre-assessment and make corrections. Let them explain their letters of choices. You may ask them to explain why other choices are wrong.

Activity 25 CONCEPTS I'VE LEARNED THAT LAST FOREVER

Direction: Fill in the blanks with the right words to make the statements complete.



*You have completed the lesson on **Inequalities in Triangles**. Before you go to the next geometry lesson on **Parallelism and Perpendicularity**, you have to answer a post-assessment and a summative test.*

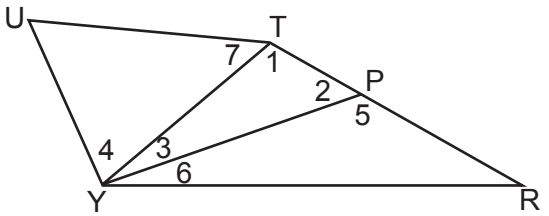
POST-ASSESSMENT:

Let's find out how much you already learn about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items.

1. Which of the following is not an inequality theorem for one triangle?

   a. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   b. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
   c. Exterior Angle Inequality Theorem
   d. Hinge Theorem

2. Which of the following angles is an exterior angle of ΔRPY?



   a. $\angle 2$ b. $\angle 3$ c. $\angle 4$ d. $\angle 7$

3. Study the figure in no. 2. Notice that $m\angle 5 > m\angle 3$ and $m\angle 5 > m\angle 1$. Which theorem justifies these observations?

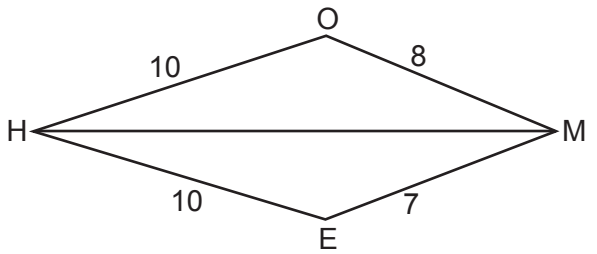
   a. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   b. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
   c. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
   d. Exterior Angle Inequality Theorem

4. Chris forms triangles by bending a 16-inch wire. Which of the following sets of wire lengths successfully form a triangle?

I. 4 in, 5 in, 6 in
II. 4 in, 4 in, 8 in
III. 4 in, 5 in, 7 in
IV. 3 in, 4 in, 9 in

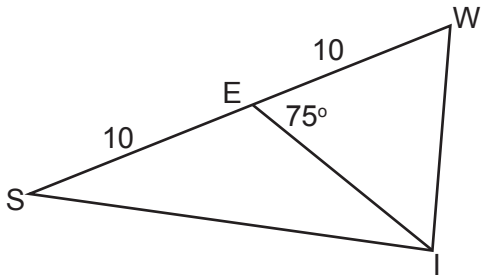
a. I, II b. III, IV c. II, IV d. I, III

5. From the inequalities in the triangles shown, Jarold concluded that $\angle OHM > \angle EHM$. Which theorem on inequalities in triangle justifies his answer?



a. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
b. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
c. Converse of Hinge Theorem
d. Hinge Theorem

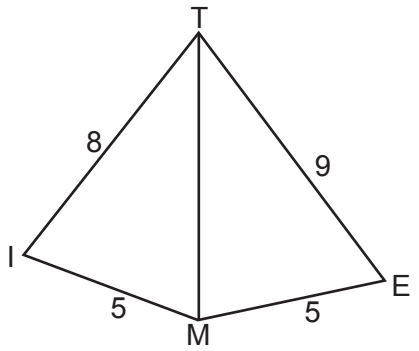
6. Kyle has proved that $\overline{IS} > \overline{IW}$. Which of the following statements is NOT part of his proof?



a. $\overline{ES} \cong \overline{EW}$
b. $\angle WEI + \angle SEI = 180$
c. $\overline{EI} \cong \overline{EI}$
d. $\angle W < \angle S$

7. What theorem should Kyle use to justify his proved statement in no. 5?
   a. Hinge Theorem
   b. Converse of Hinge Theorem
   c. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   d. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

8. Chloe studies the triangles in the figure carefully. Which should be her final conclusion?



   a. $\overline{TM} \cong \overline{TM}$
   b. $\overline{ET} > \overline{IT}$
   c. $IM \cong \overline{EM}$
   d. $\angle EMT > \angle ITM$

9. Which theorem justifies Chloe's conclusion in no. 8?
   a. Hinge Theorem
   b. Converse of Hinge Theorem
   c. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   d. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

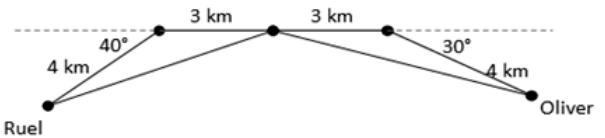
10. In ΔGUD, GU = DU and GD > DU. Which of the following statements may NOT be true?
   a. GU < GD − DU
   b. $m\angle U > m\angle D$
   C. $m\angle U > m\angle G$
   D. $m\angle D = m\angle G$

11. In ΔTRY, if TR = 3, RY = 5, and TY = 2, which statement is true?
    a. $m\angle R > m\angle Y$ c. $m\angle Y > m\angle T$
    b. $m\angle R > m\angle T$ d. $m\angle T > m\angle R$

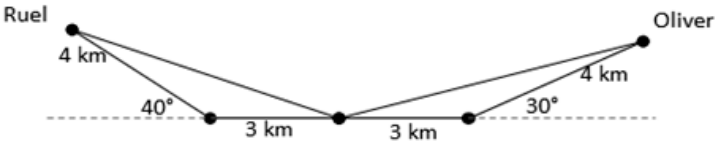
12. Which theorem justifies the then-statement in no. 11?
    a. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    b. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
    c. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
    d. Exterior Angle Inequality Theorem

13. From a rendezvous, hikers Oliver and Ruel who have uniform hiking speed walk in opposite directions—Oliver, eastward whereas Ruel, westward. After walking three kilometers each, both of them take right turns at different angles—Oliver at an angle of 30° and Ruel at 40°. Both continue hiking and cover another four kilometers each before taking a rest. To find out who is farther from the rendezvous, select the illustration that describes appropriately the problem.

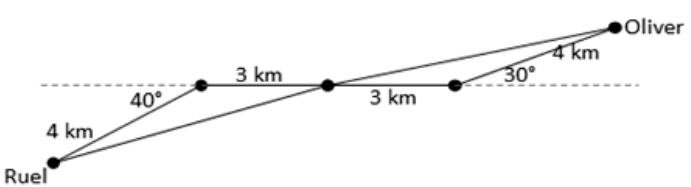
a.



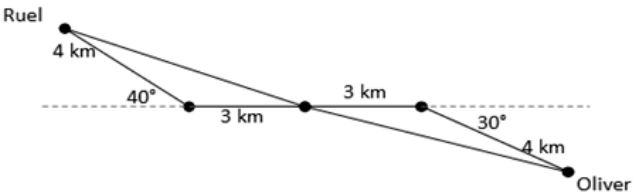
c.



b.



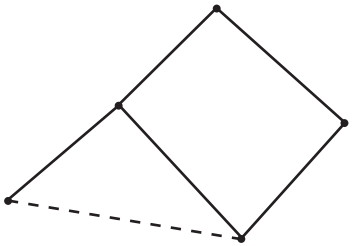
d.



14. Which theorem of inequality in triangles helps you in determining who is farther from the rendezvous?
    A. Hinge Theorem
    B. Converse of Hinge Theorem
    c. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    d. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

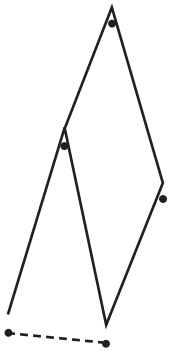
For items no. 15-20, use the situation described.

Your friend asks for your suggestion on how to raise the height of his tent without changing the amount of area it covers.

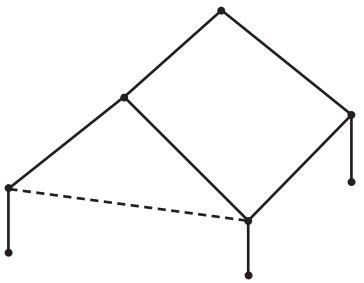


Original Tent

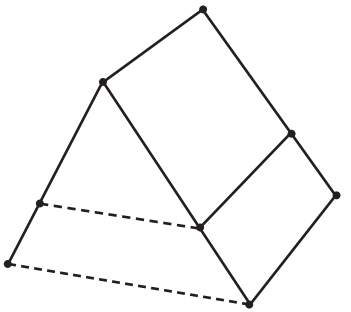
15. Which of the following designs meet the qualifications of your friend?



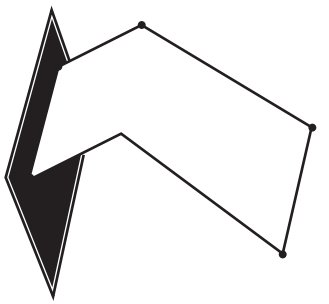
I



II



III



IV

a. I and III b. II and III c. III and IV d. II, III, and IV

16. Which design/s is/are contradictory to your friend's specifications?
    a. I only b. IV only c. I and II d. I and IV

17. Which design requires more tent material?
    a. I b. II c. III d. IV

18. The modified tents have equal heights. Which design is the most practical and easiest to assemble?
    a. I b. II c. III d. IV

19. What theorem of inequality in triangles justifies design no. IV?
    a. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    b. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
    c. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
    d. Exterior Angle Inequality Theorem

20. Which insights have you learned from the tent designs?
    I. The steeper the roof of a tent, the less area it covers.
    II. The larger the roof angle of a tent, the wider the area it covers.
    III. Modifying a tent design does not always require money.

    a. III only b. I, II c. I, III d. I, II, III

Answer Key to Post-Assessment:

1.	D	6.	B	11.	D	16.	A
2.	A	7.	A	12.	A	17.	C
3.	D	8.	D	13.	D	18.	D
4.	D	9.	B	14.	A	19.	C
5.	C	10.	A	15.	D	20.	D